Rare-Event Simulation and Sensitivity Analysis for Potential En-Route Wake Encounters

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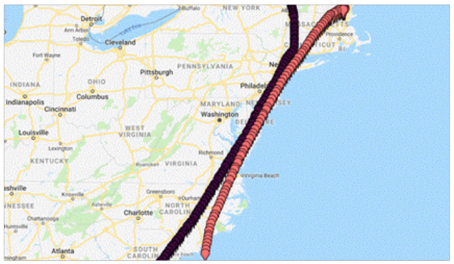
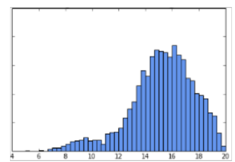
Motivation

- Objective: Establish a toolset for estimating potential wake encounters en-route
- Can be used to evaluate:
  - Impact of changes to fleet mix and disparity in aircraft sizes
  - Impact of separation reductions
  - Capacity benefits for dynamic wake separation concepts
  - Effectiveness of mitigation procedures

Research Overview

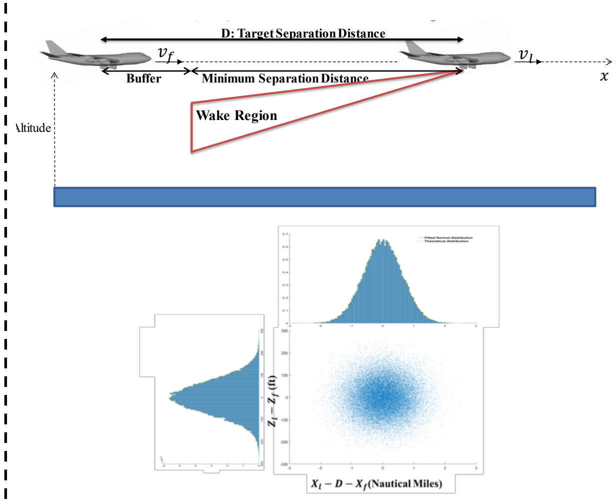
<u>Flight track data for lead/follow pairs</u>





baseline distribution parameters
μ, σ_z, σ_f, etc.

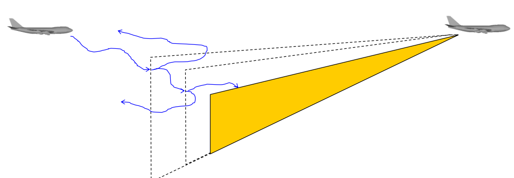
<u>Stochastic en-route aircraft and wake model</u>



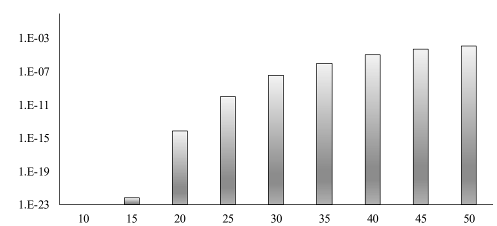
<u>Sensitivity analysis</u>

vary μ, σ_z, σ_f, etc.

<u>Rare-event simulation engine</u>

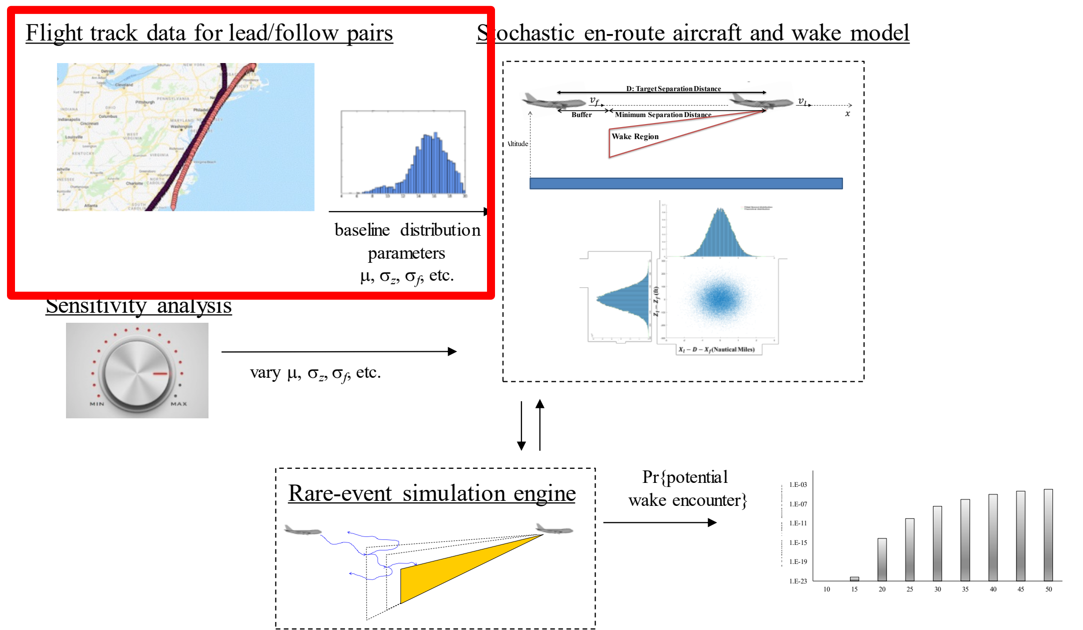


Pr{potential wake encounter}

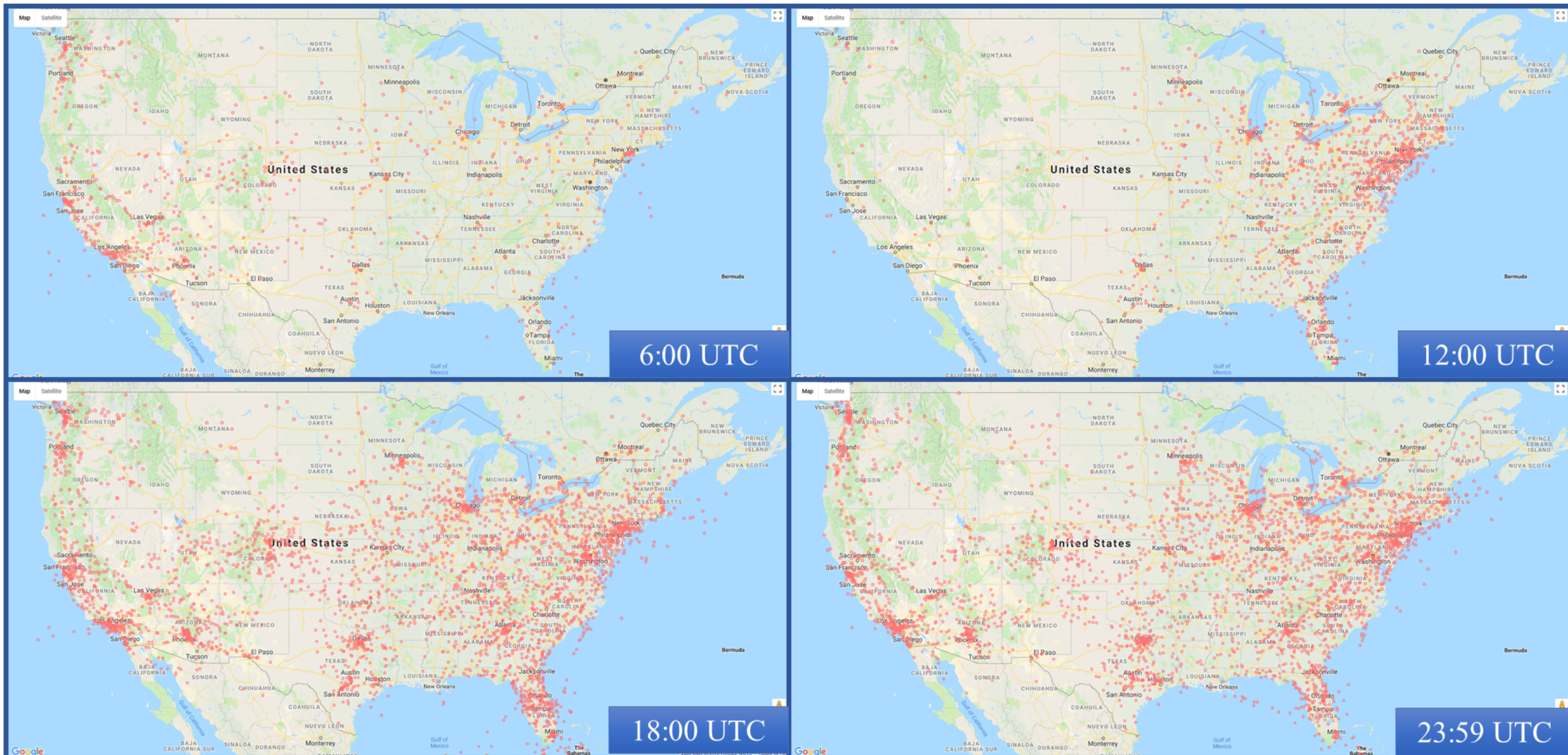


Data Collection Objectives

- Characterize in-trail flight tracks
  - Distribution of along-track separation
  - Distribution of speed
  - Distribution of altitude
- Distributions used to quantify stochastic-distribution parameters in potential wake-encounter simulation model



Snapshots of Aircraft Position



Source: ADS-B Exchange

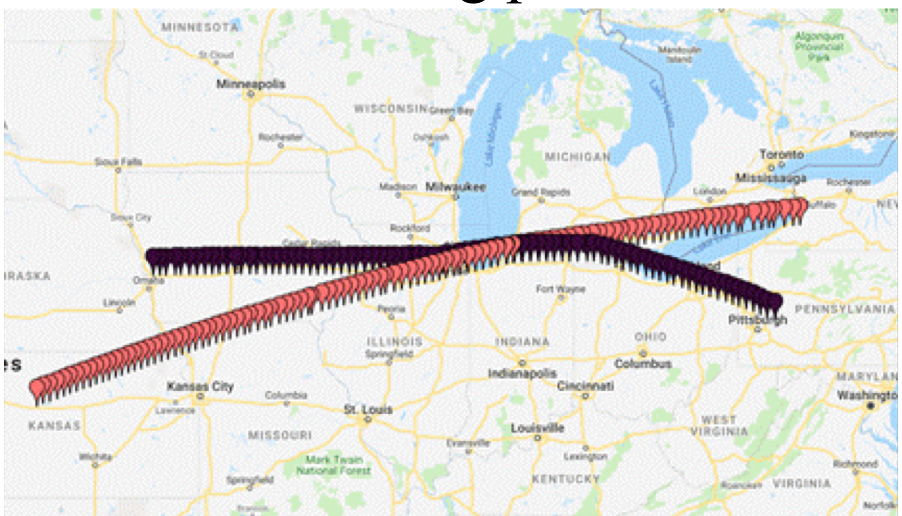
Data Summary

- Identify trailing aircraft pairs from ADS-B exchange
  - Flying at the same altitude above 30,000 ft
  - Have a distance less than 20 nautical miles for at least 10 minutes.
  - Other filters applied to identify in-trail pairs
- U.S. data collected
  - 3 weeks in July 2017
  - 2 weeks in February 2018
  - 3,531 trailing pairs identified
- Europe data collected
  - 2 weeks in July 2017
  - 1,942 pairs were identified

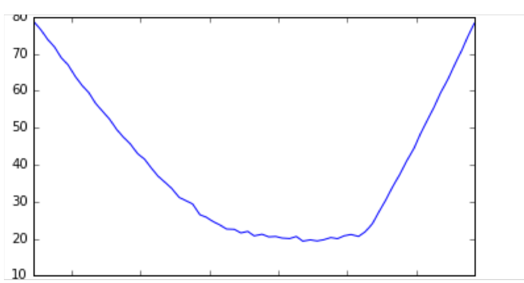
Caveat: Analysis does not consider trailing aircraft one flight level below

Examples of Non-Trailing Pairs

Crossing pairs

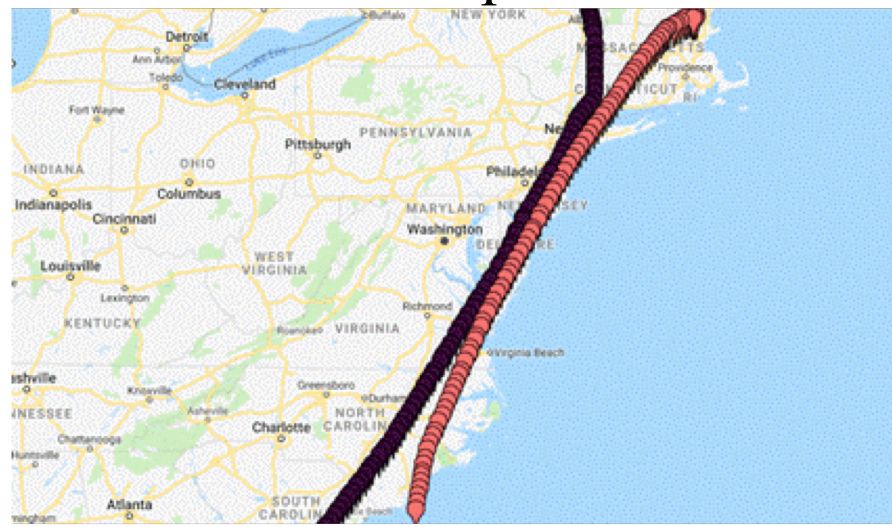


Lateral Distance (nm)

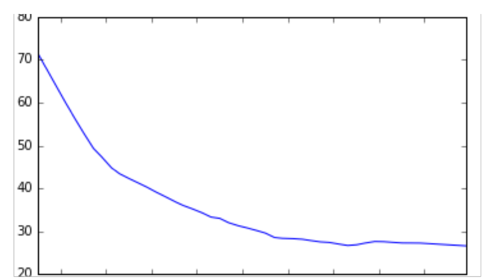


Time

Parallel pairs

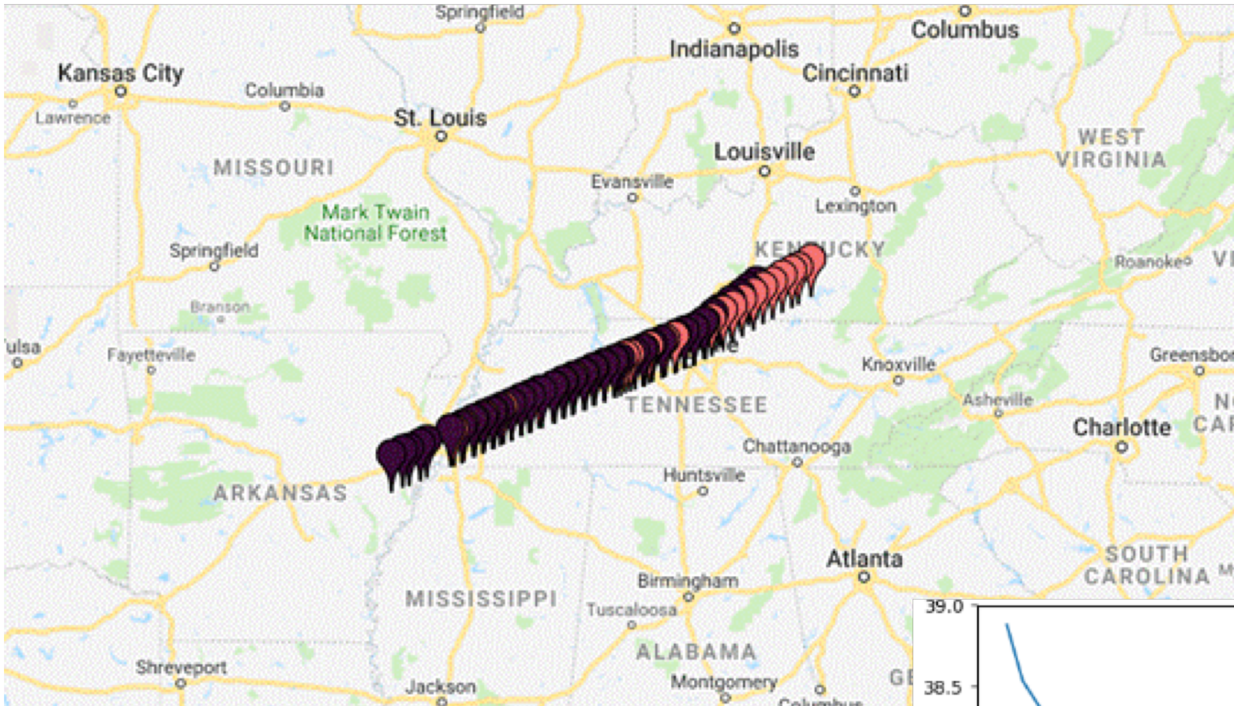


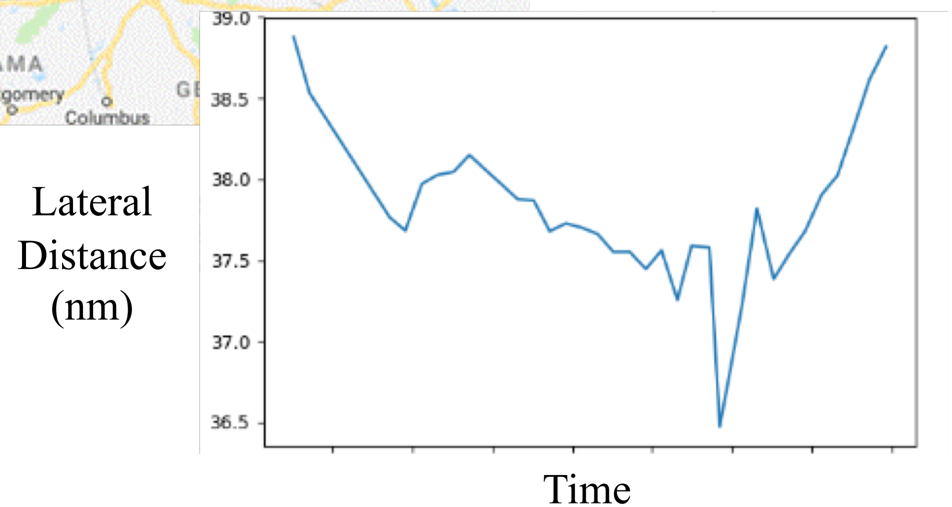
Lateral Distance (nm)



Time

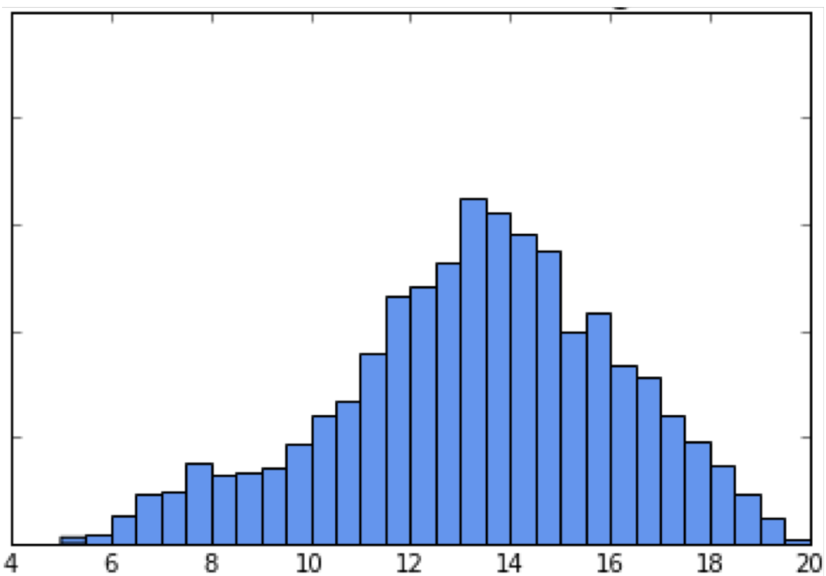
Example of Trailing Pair





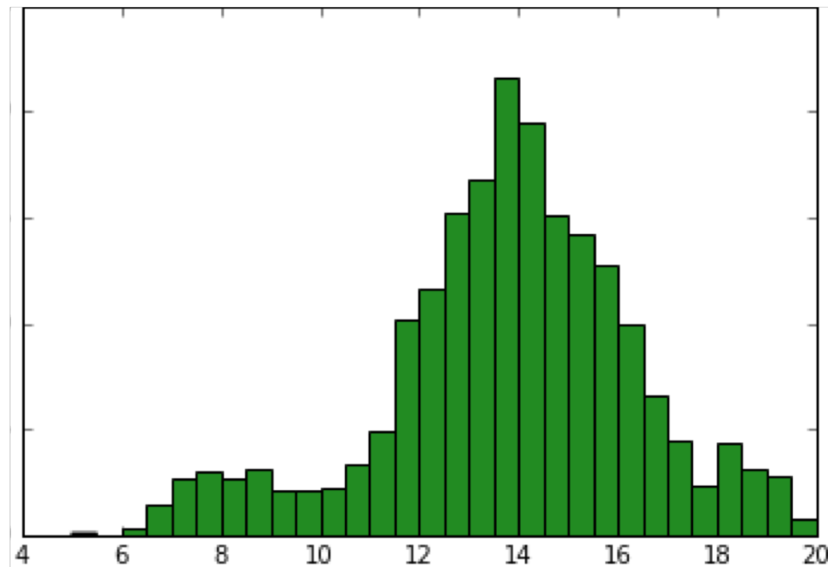
Minimum Distance Between Pairs

United States



Distance (nm)

Europe

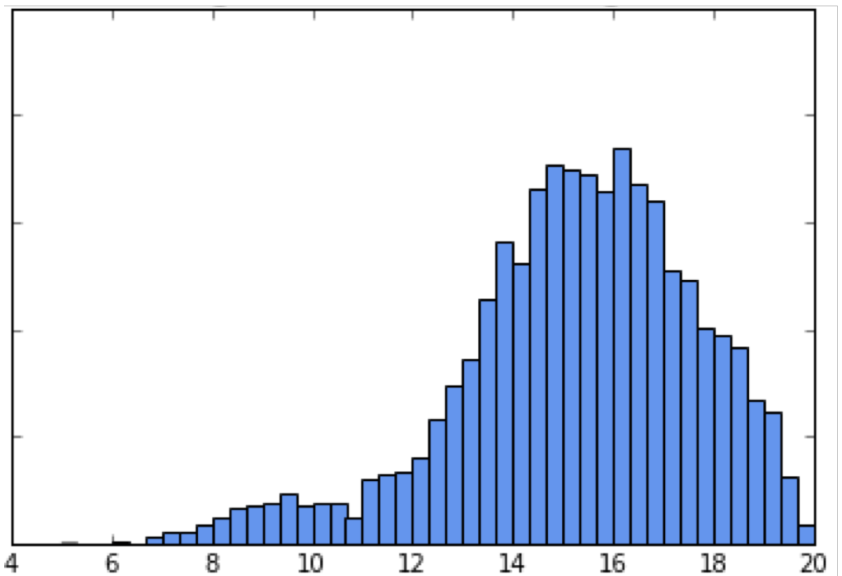


Distance (nm)

Minimum distance = minimum in-trail distance for a given lead-trail pair observed over in-trail time horizon

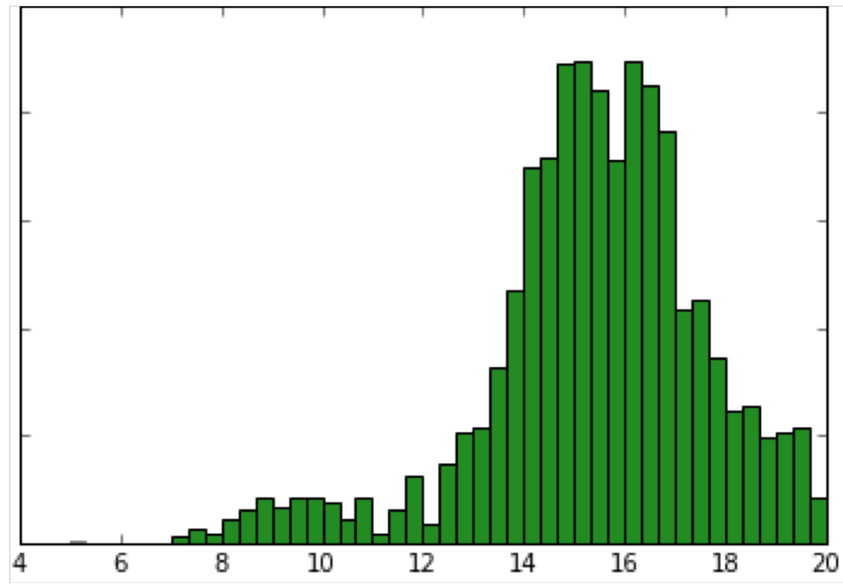
Average Distance Between Pairs

United States



Distance (nm)

Europe

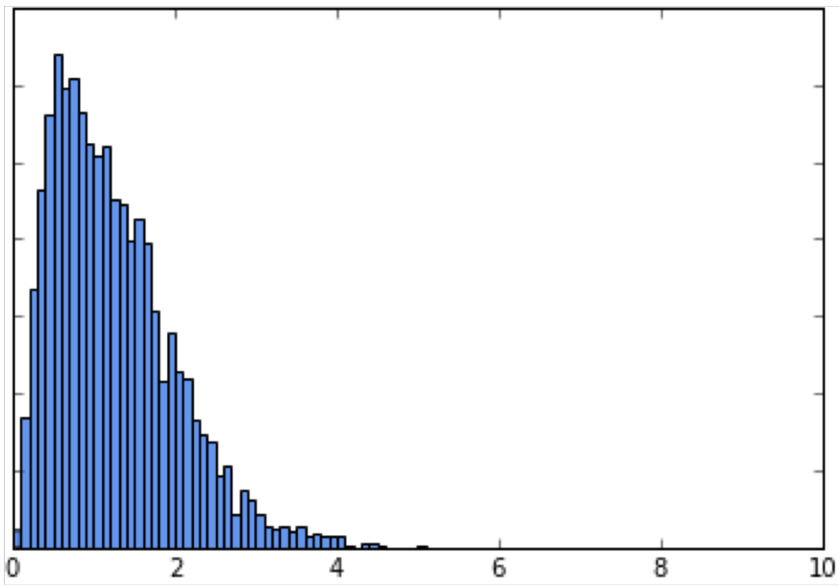


Distance (nm)

Average distance = average in-trail distance <u>for a given lead-trail pair</u> observed over in-trail time horizon

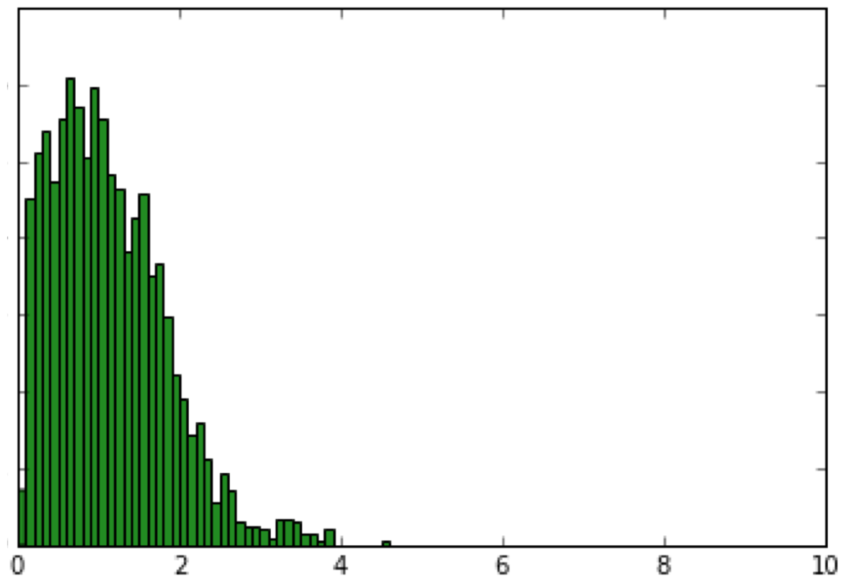
Standard Deviation of Separation

United States



Distance (nm)

Europe

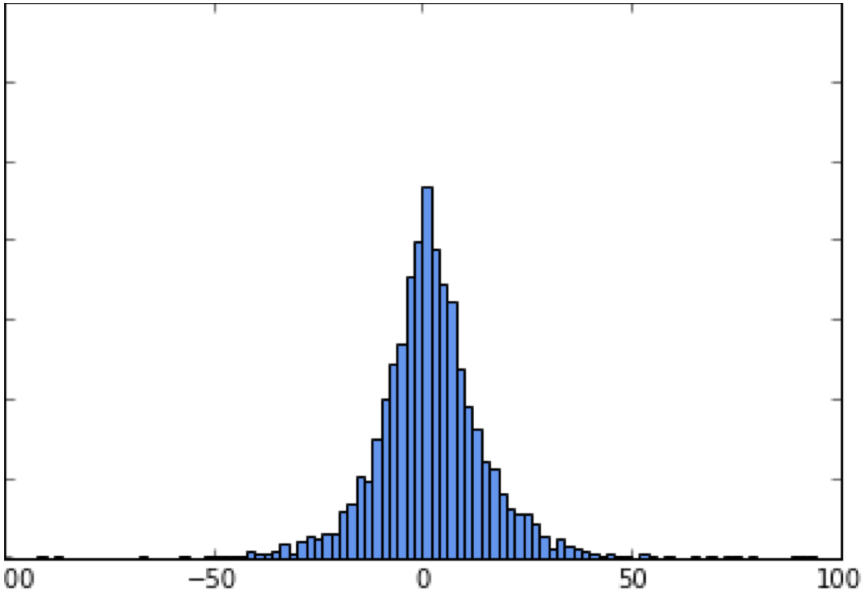


Distance (nm)

Standard deviation = standard deviation of in-trail distance for a given lead-trail pair observed over in-trail time horizon

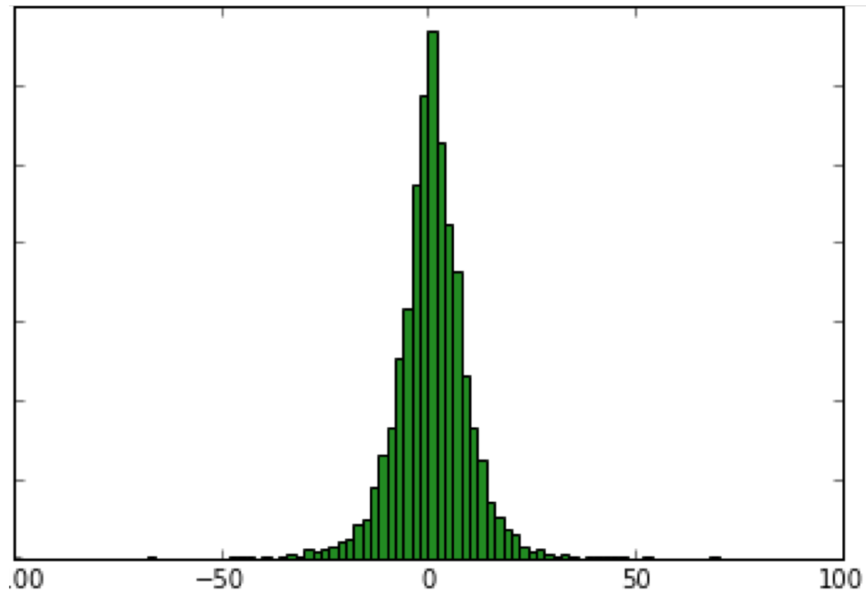
Average Speed Difference

United States



Avg. Speed Difference (knots)

Europe



Avg. Speed Difference (knots)

Average speed difference = average **ground** speed difference between leader and trailer for a given lead-trail pair observed over in-trail time horizon

Aircraft are travelling in approximately same direction, so distribution of air speed differences should be similar

Pair Counts by Wake Category

United States

Number of Pairs

Leading aircraft	Trailing aircraft					
	F	E	D	C	B	A
F			3			
E		10	26	1	1	
D	4	26	2932	92	38	1
C		2	51	53	8	
B		1	28	13	45	1
A					4	1

Europe

Number of Pairs

Leading aircraft	Trailing aircraft					
	F	E	D	C	B	A
F						
E			1			
D	1	1	1645	21	25	4
C			16	10	6	
B			40	5	81	14
A			3		12	14

F: Lower small E: Upper small D: Large
C: Lower heavy B: Upper heavy A: Super

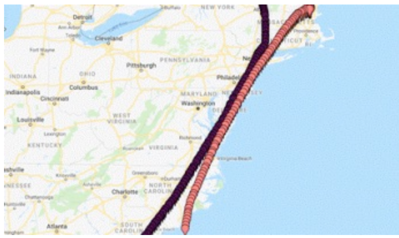
Observations

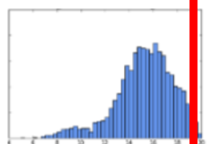
- Large-large trailing pairs most common
- No obvious difference between separation distributions for different pair classes (distributions not shown); most other pair combinations suffer from small sample sizes

Wake categories based on: Wake Turbulence Administration Re-Categorization (RECAT) Basics, Ed Johnson, 63rd Air Safety Forum

Stochastic Aircraft / Wake Model

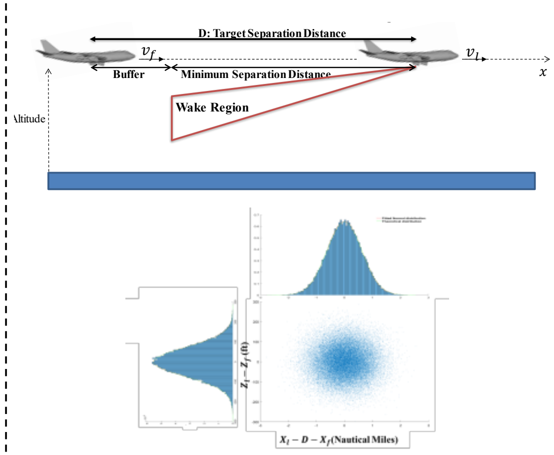
<u>Flight track data for lead/follow pairs</u>





baseline distribution parameters
μ, σ_z, σ_f, etc.

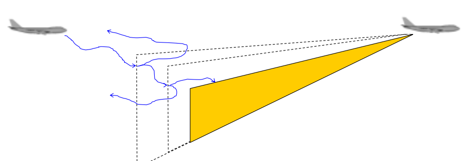
<u>Stochastic en-route aircraft and wake model</u>



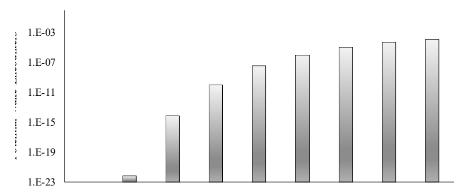
<u>Sensitivity analysis</u>

vary μ, σ_z, σ_f, etc.

<u>Rare-event simulation engine</u>



Pr{potential wake encounter}

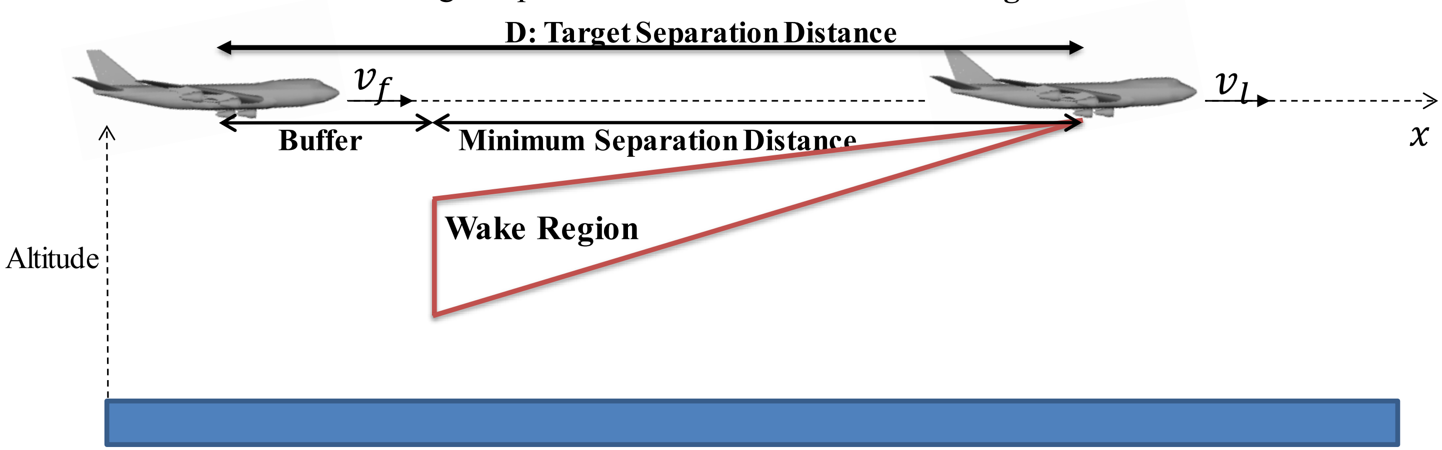


Aircraft Simulation Model

- Leading and trailing aircraft states modeled via stochastic differential equations
- Different control laws for each aircraft

Trailing aircraft: Tries to maintain the target separation

Leading aircraft: Tries to maintain its own target speed



Zare, A., J. Shortle. 2017. Rare event simulation for potential wake encounters. *Proceedings of the 2017 Winter Simulation Conference*.

Along-Track Movements

- Leading aircraft: Maintain target speed

$$dv_l = \underbrace{-\rho(v_l - \mu)dt}_{\textit{Maintain target speed}} + \underbrace{\sigma_l dW_t}_{\textit{Noise}}$$

  - v_l : speed of leading aircraft
  - μ : target velocity
  - σ_l : the volatility parameter
  - ρ : the rate of mean reversion
  - W_t : standard Brownian motion

- Trailing aircraft: Maintain separation with leading aircraft

$$dv_f = \underbrace{k_p(X_l - D - X_f)dt}_{\textit{Maintain target separation distance D}} + \underbrace{k_d(v_l - v_f)dt}_{\textit{Match speed of leading aircraft}} + \underbrace{\sigma_f dW_t}_{\textit{Noise}}$$

Vertical Movements

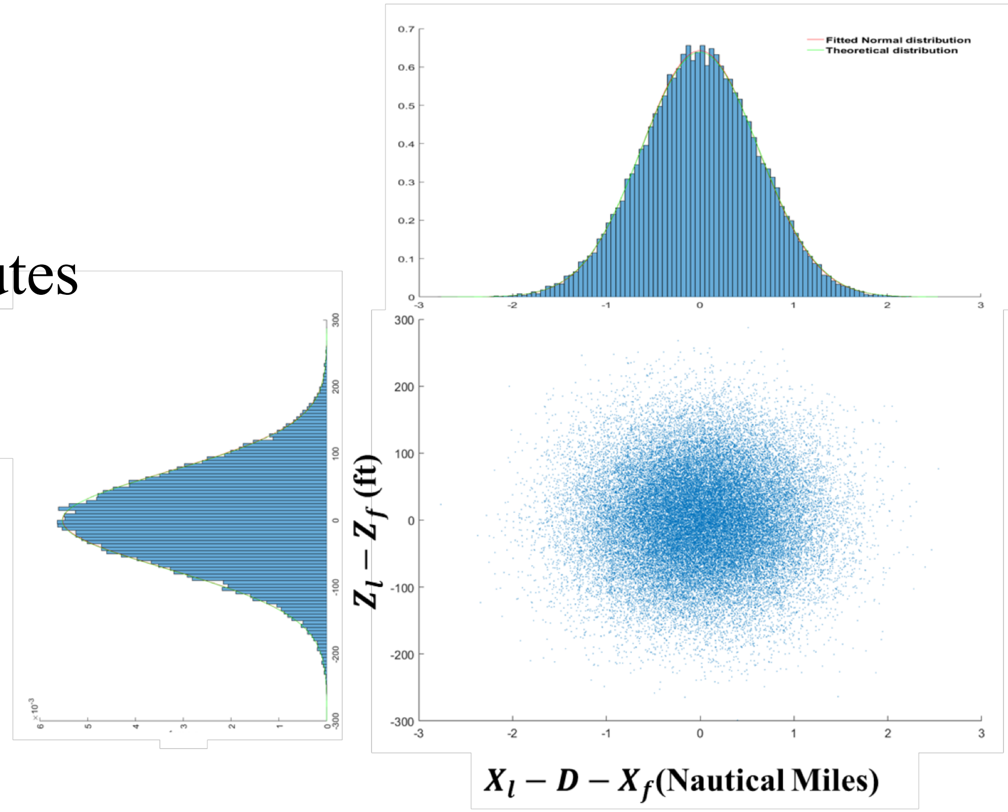
- Both aircraft: Maintain target altitude

$$dz = \underbrace{-\rho_z(z - \mu_z)dt}_{\textit{Maintaining target altitude}} + \underbrace{\sigma_z dW_t}_{\textit{Noise}}$$

  - z : altitude
  - μ_z : target altitude
  - σ_z : the volatility parameter
  - ρ_z : the rate of mean reversion
  - W_t : standard Brownian motion

Along-track / Vertical Distributions

- 50,000 simulated pairs of aircraft, 30 simulated minutes for each pair
- Each point shows relative position at the end of the simulation
- Fitted probability density functions match theory



Relative vertical position

Along-track separation

Possible to choose model parameters to yield a pre-specified normal distribution

Estimating Model Parameters

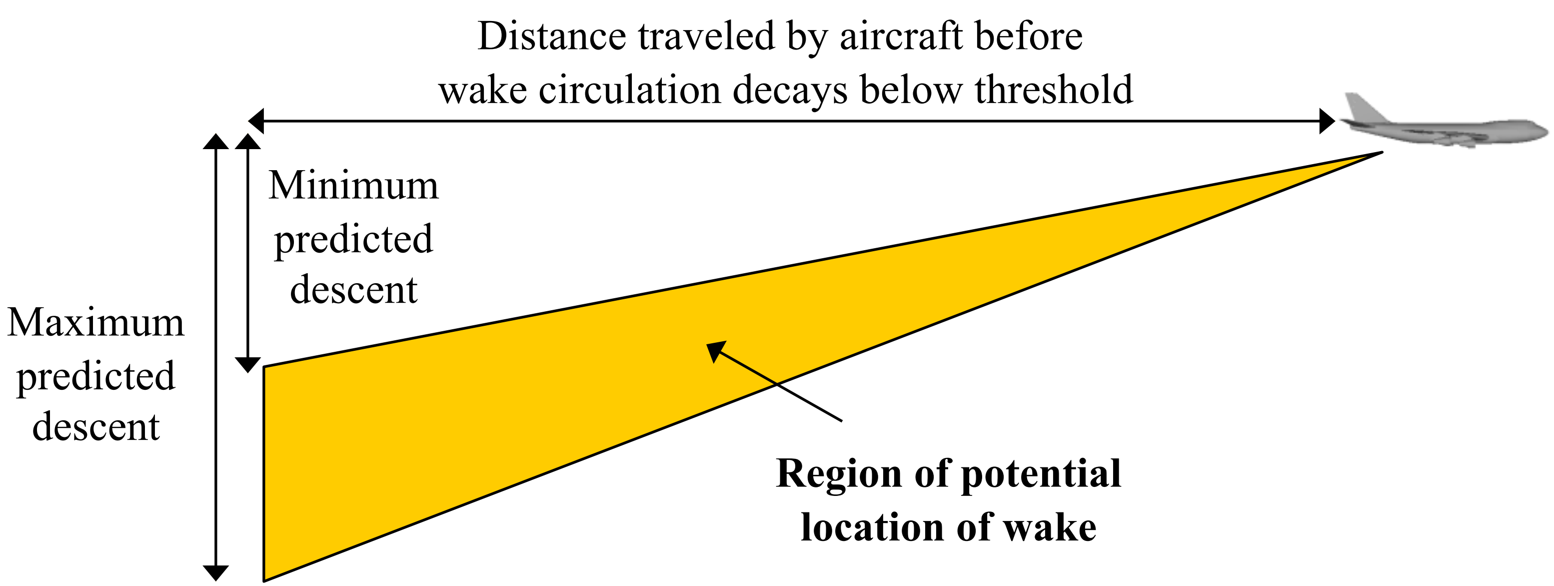
- Restrict data to trailing pairs where both leading and trailing aircraft are one of:
  - Airbus: A318, A319, A320, A321
  - Boeing: B737-600, B737-700, B737-800, B737-900
- Number of trailing pairs in U.S. – 1,653
- Number of trailing pairs in Europe – 1,510

Estimated Model Parameters

Observable Parameter	Associated Model Parameters	Estimated Value
Average (target) separation distance	D	15.2 nm
Standard deviation of separation distance	$\frac{\sigma_f}{\sqrt{2k_d k_p}}$	1.14 nm
Target speed for leading aircraft	μ	435 kts
Standard deviation of speed for leading aircraft	$\sigma_l/\sqrt{2\rho}$	19.9 kts
Standard deviation of speed for following aircraft	$\sigma_f/\sqrt{2k_d}$	19.9 kts
Standard deviation of altitude	$\sigma_z/\sqrt{2\rho_z}$	10 ft

Potential Wake Encounter Region

- Dimensions of the wake zone are approximated from APA model with selected parameters
- The region defines an area that is likely to contain the wake (potential wake encounters)

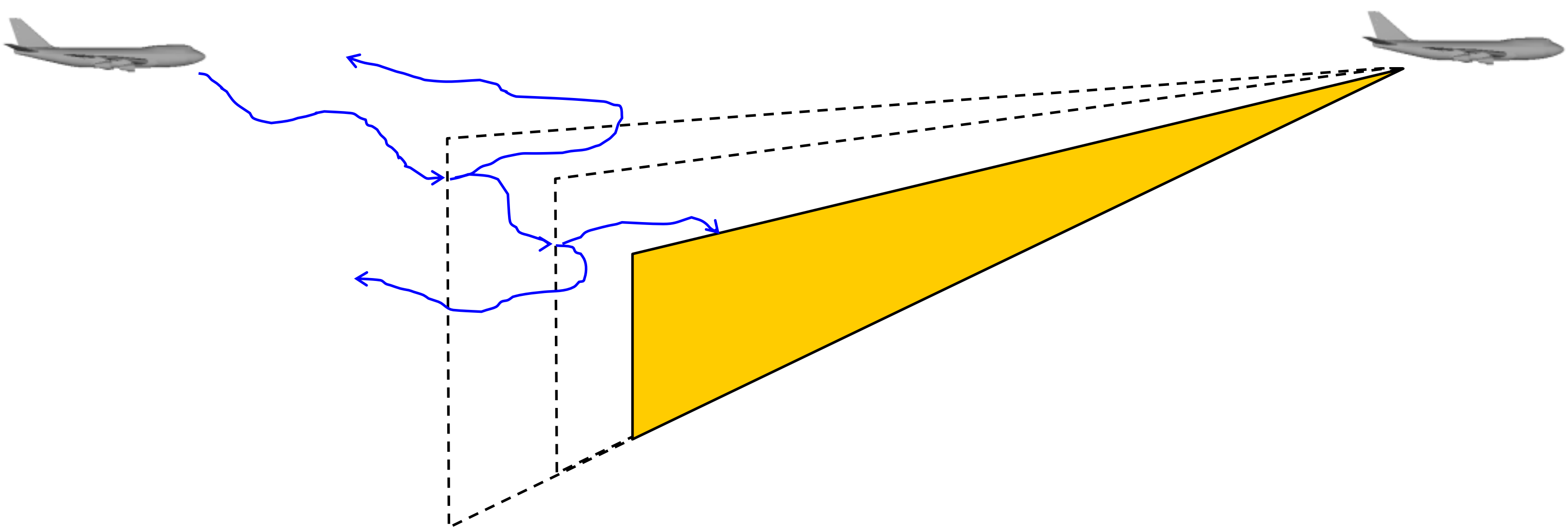


Caveats

- Lateral transport not considered
  - Assume wake zone is laterally aligned with flight path of trailing aircraft
- Results represent instances when aircraft penetrates the wake zone (area where wake is likely to be), which is not necessarily an instance of hitting the wake
- No climbing / descending scenarios

Rare-Event Splitting Technique

- Challenge: Low probability events computationally expensive to simulate
- Rare-event simulation approach: Split (clone) trajectories that get near the rare event



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En-route Aircraft / Wake Model

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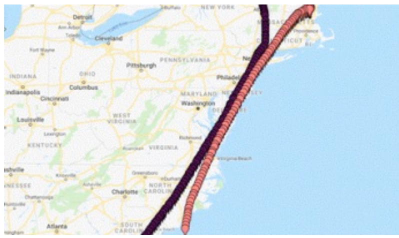

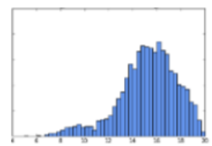

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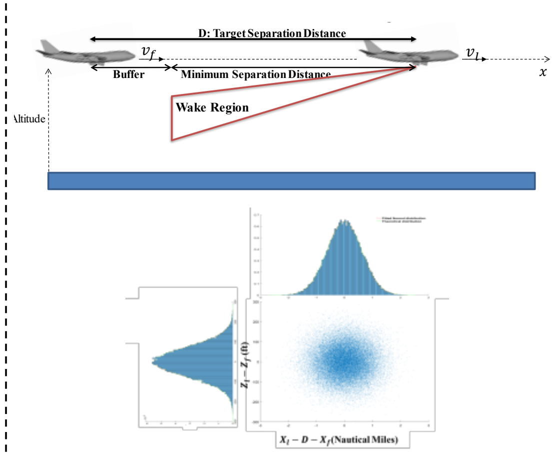
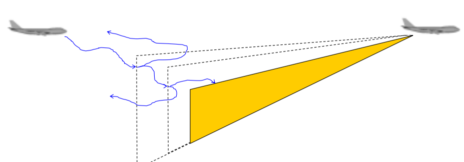


Sensitivity analysis

vary μ, σ_z, σ_f, etc.

Rare-event simulation engine



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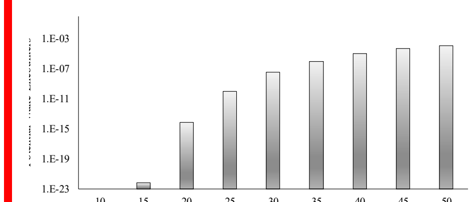
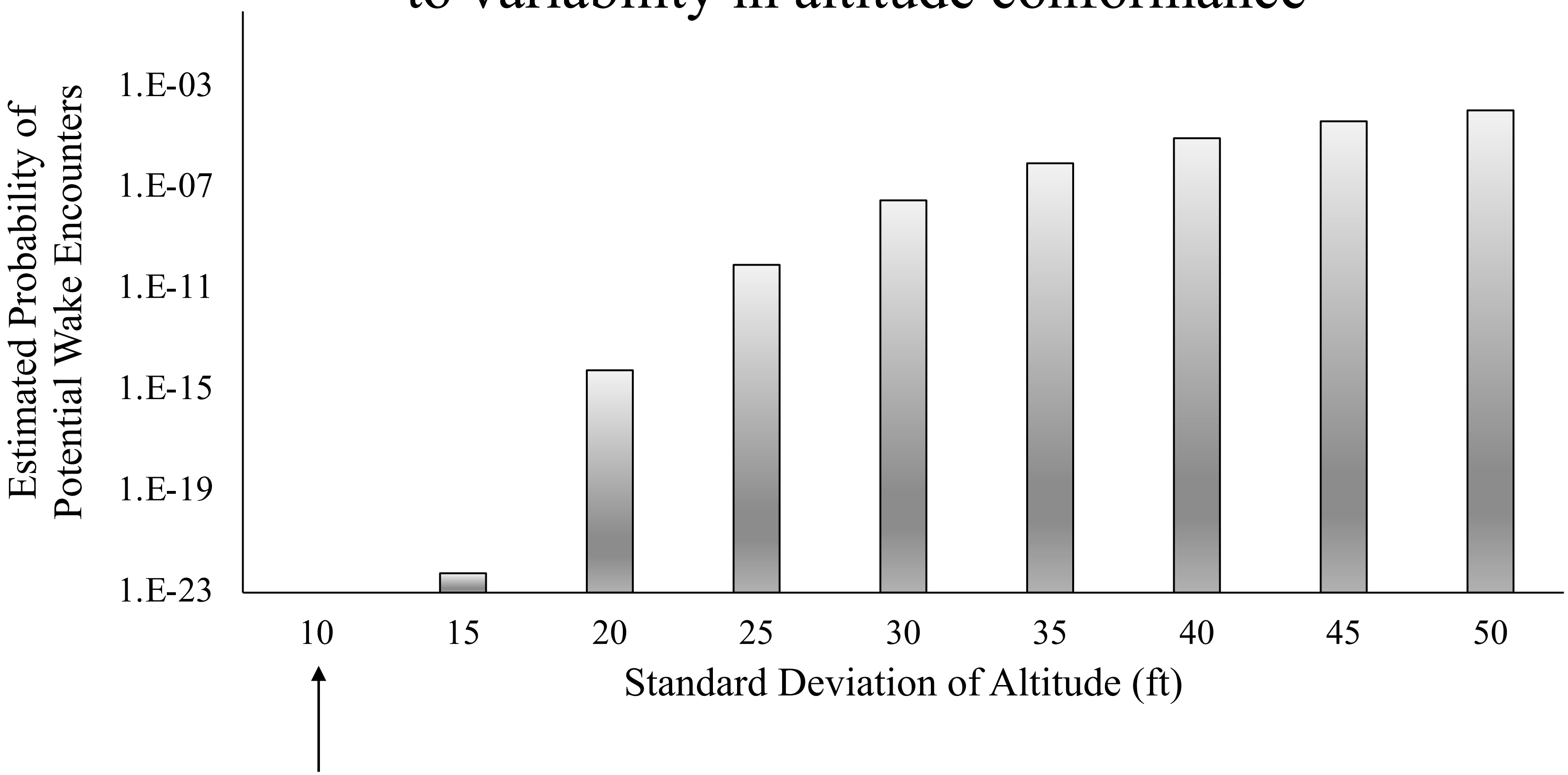


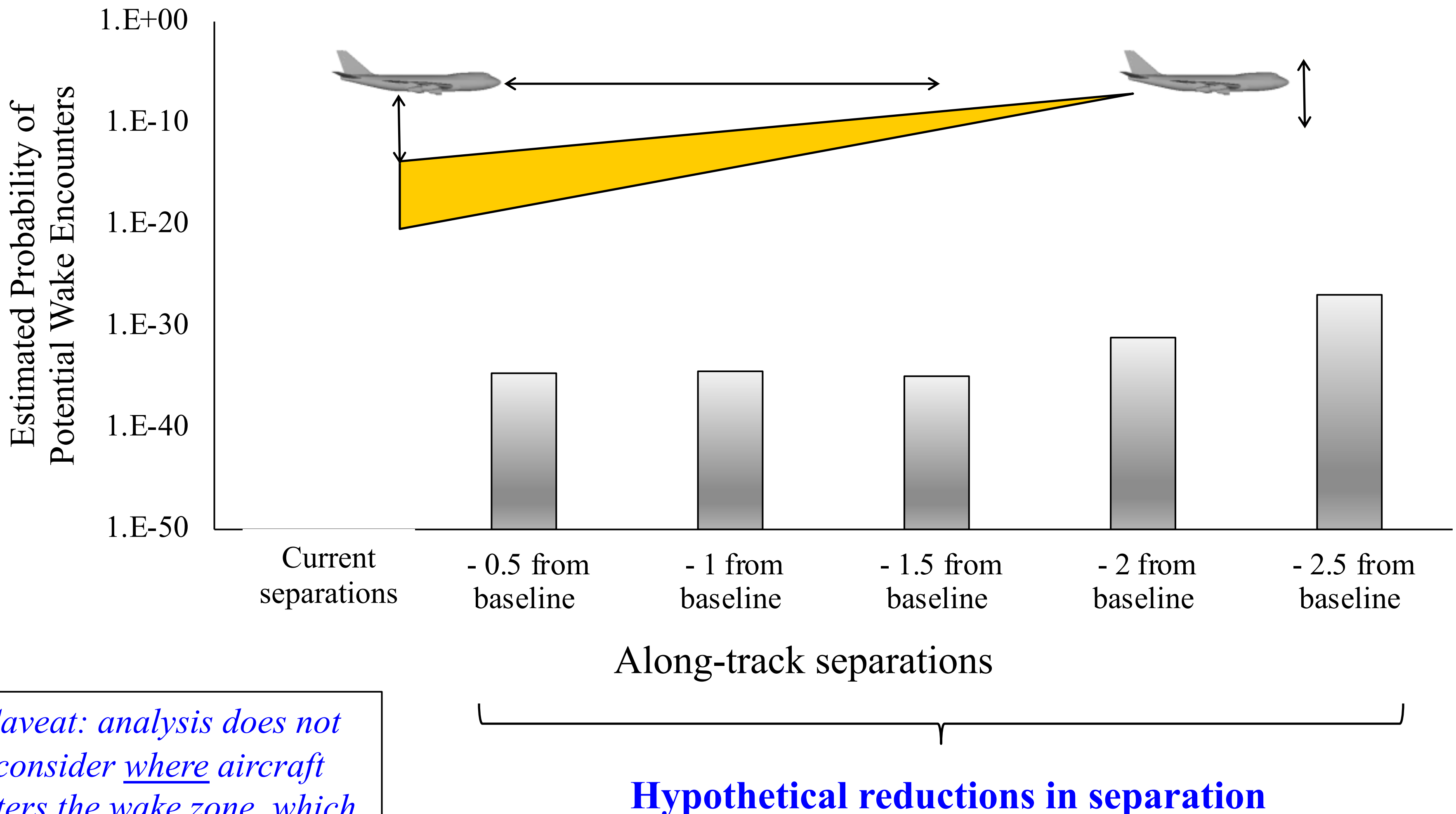
Sensitivity: Altitude Conformance

Potential wake encounter probability is <u>sensitive</u> to variability in altitude conformance



Value estimated from flight tracks

Sensitivity: Separation Distance



Caveat: analysis does not consider where aircraft enters the wake zone, which may impact wake severity

Summary and Conclusions

- Developed toolset for potential en-route wake encounters
  - Stochastic aircraft model, wake zone, rare-event simulation engine
- Analysis of track data to establish baseline parameters of model
  - Stochastic distributions for separation, altitude, speed, etc.
- Simulated potential encounter probabilities are very small, but sensitive to altitude conformance
- Ongoing work: Climbing / descending aircraft