Network-based optimization algorithms for TFM aggregate flow models

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Goals of the project

1. Create a model NAS-wide high altitude traffic
   1. Tool: aggregate description of traffic, which is validated against traffic data (for example ETMS / ASDI)
   2. Make the model accurate, but computationally tractable model

2. Use the model for various TFM applications
   1. Traffic flow management based on airborne delays when demand exceeds capacity. Delay minimization.
   2. Contribute to the analysis of NAS infrastructure for new paradigms (NextGen, NGATS), in the present case: dynamic airspace

3. Practical implementations
   1. Optimization based software environment
   2. Integration into FACET (Metron Aviation)

Outline

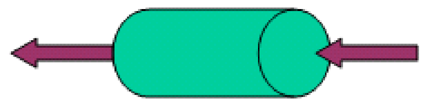
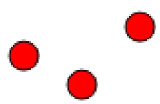
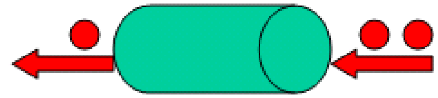
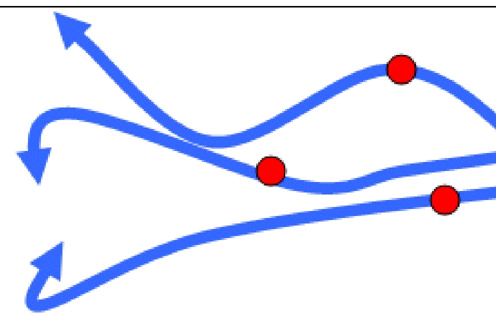
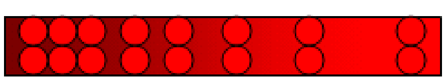
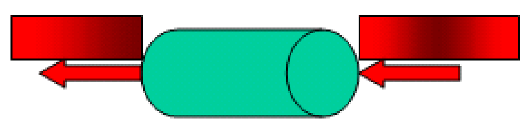
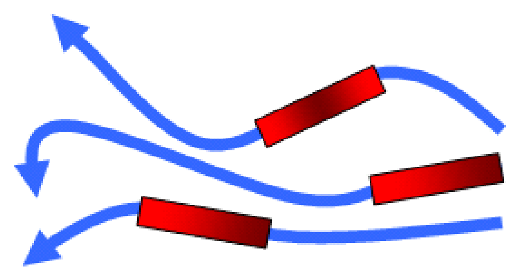
1. Introduction: Eulerian/Langrangian; Micro/Macro-aggregate

2. Multicommodity cell transmission model
   1. Automated graph topology model building
   2. Aggregate travel time estimation

3. LTI models of the NAS
   1. Standard LTI formulation
   2. Constrained optimization formulations
   3. Practical implementation

4. Applications
   1. NAS-wide TFM
   2. Dynamic airspace configuration

Eulerian / Lagrangian; micro / macro (aggregate)

	Eulerian Control volume based	Lagrangian Trajectory based
		
Micro Particles 		
Macro Aggregation of Particles 		

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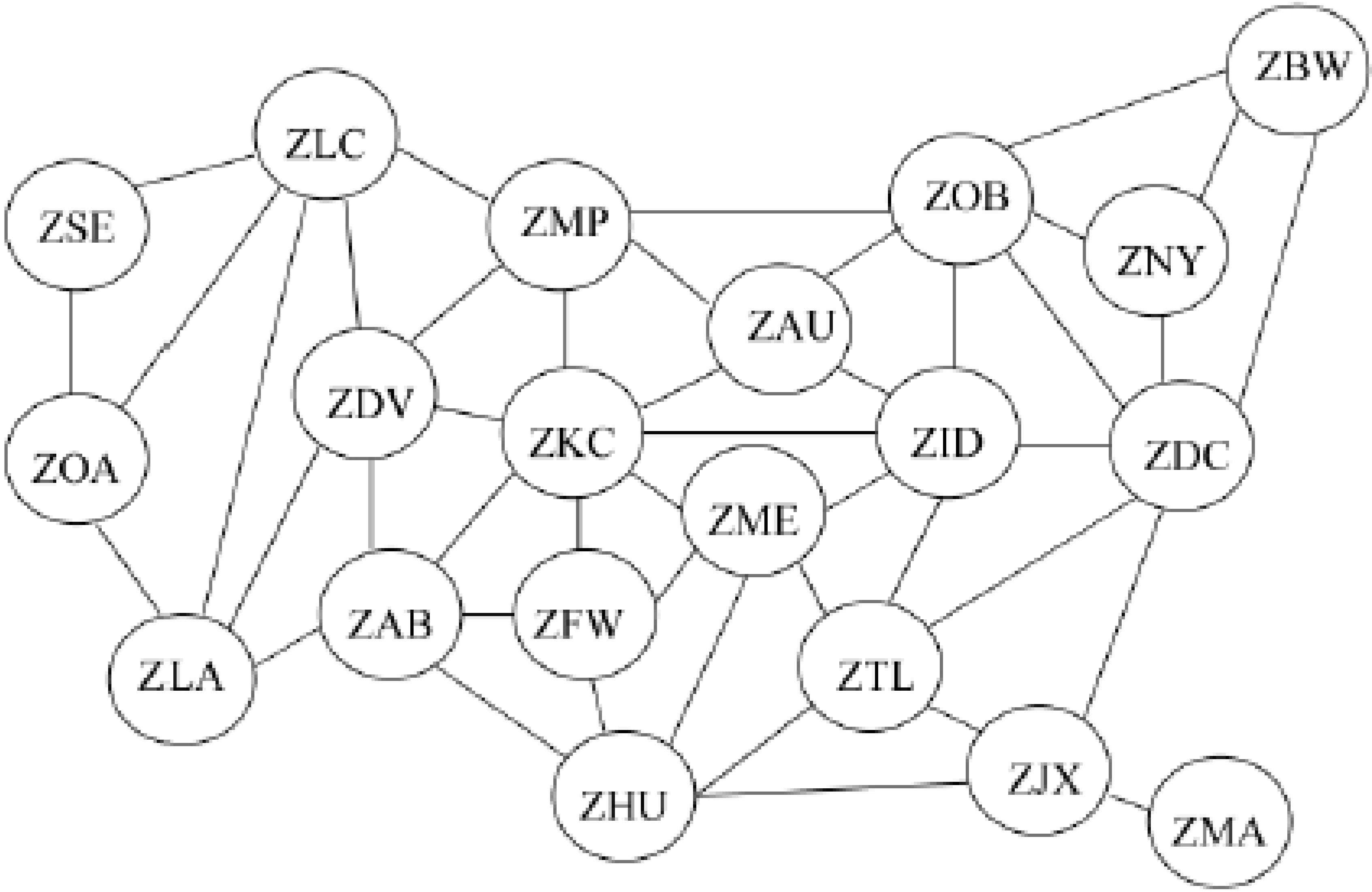
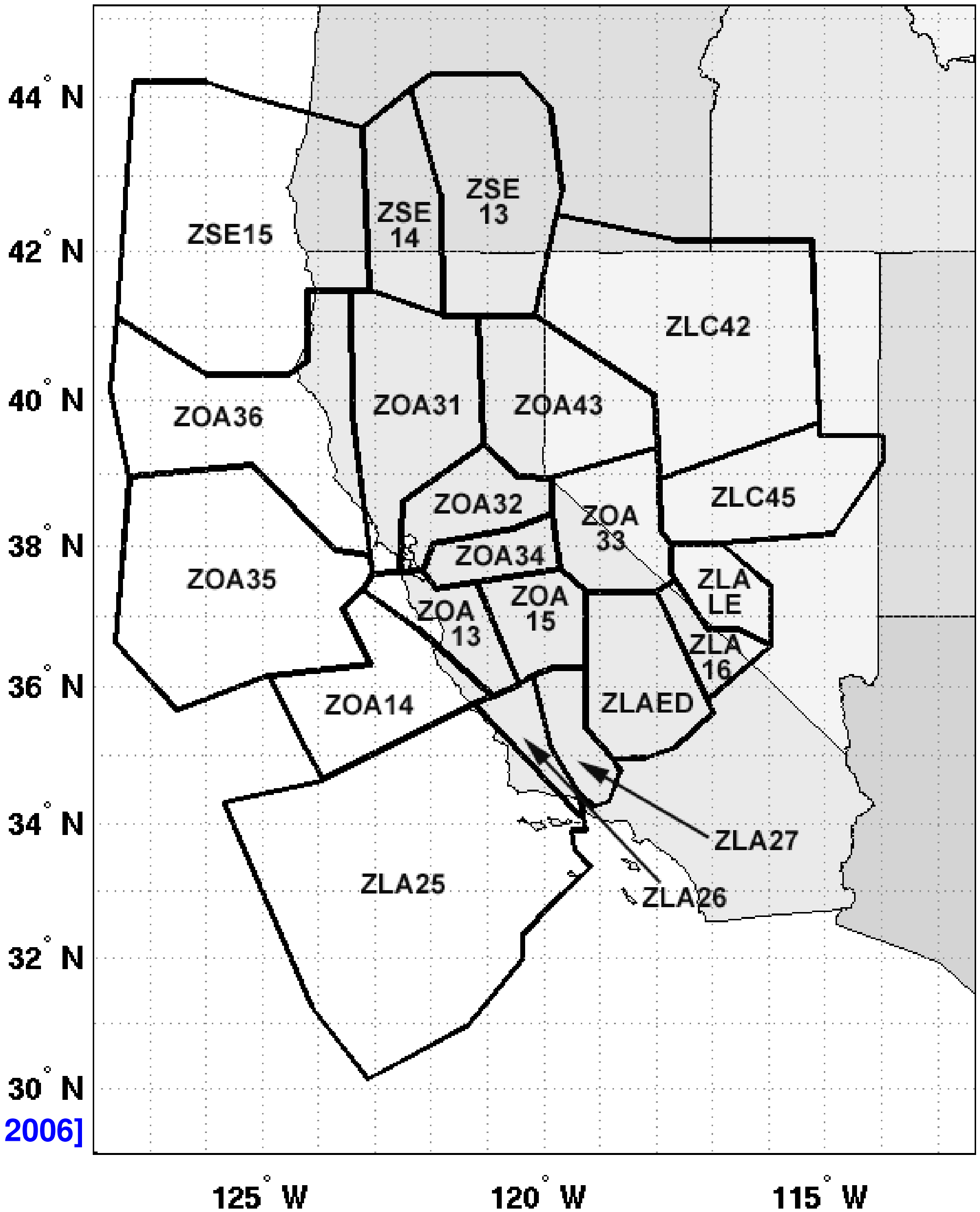
Air Route Traffic Control Centers (ARTCC) in the US



[Sridhar, Soni, Sheth, Chatterji 2004]

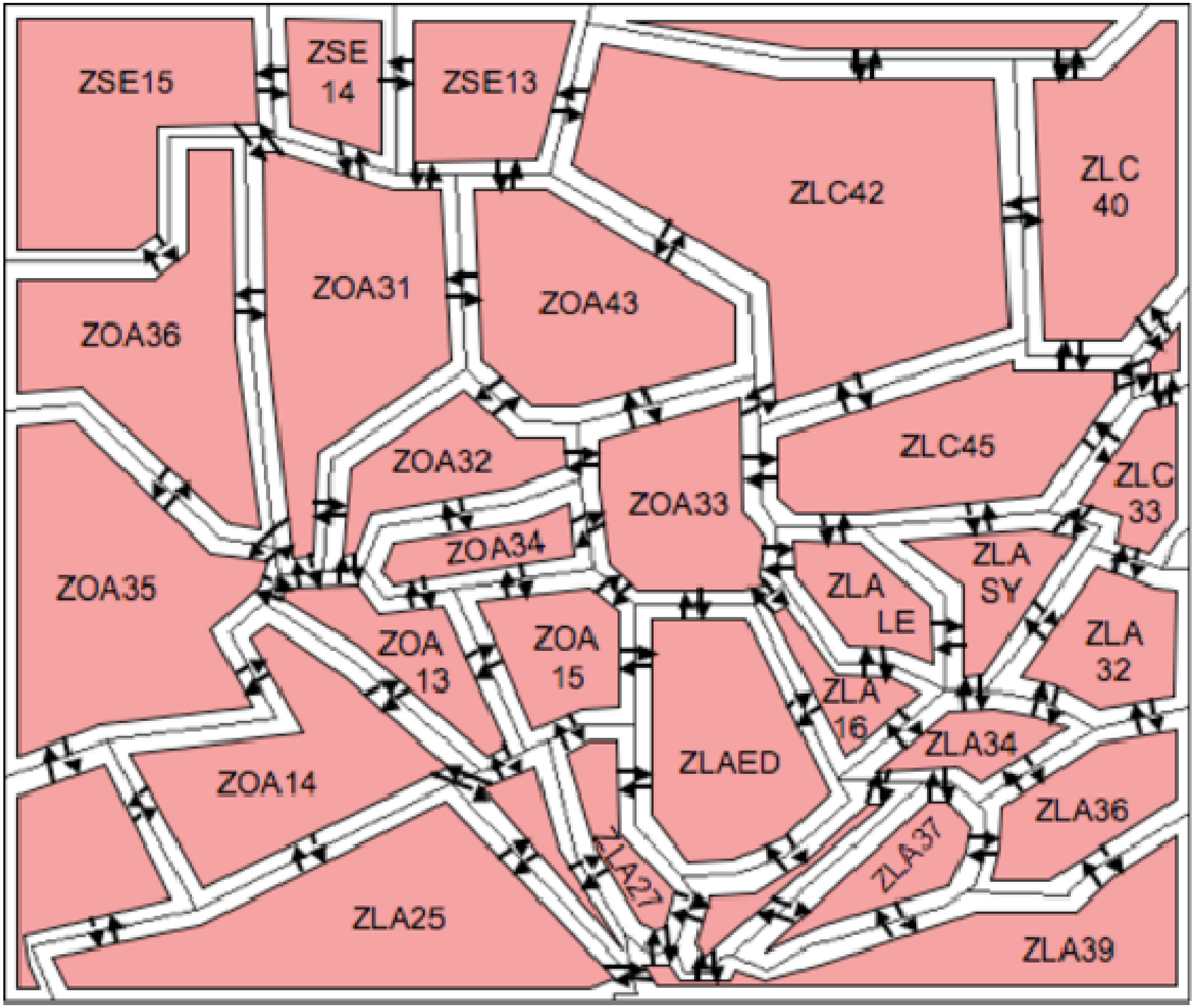
Inside an ARTCC: sectors



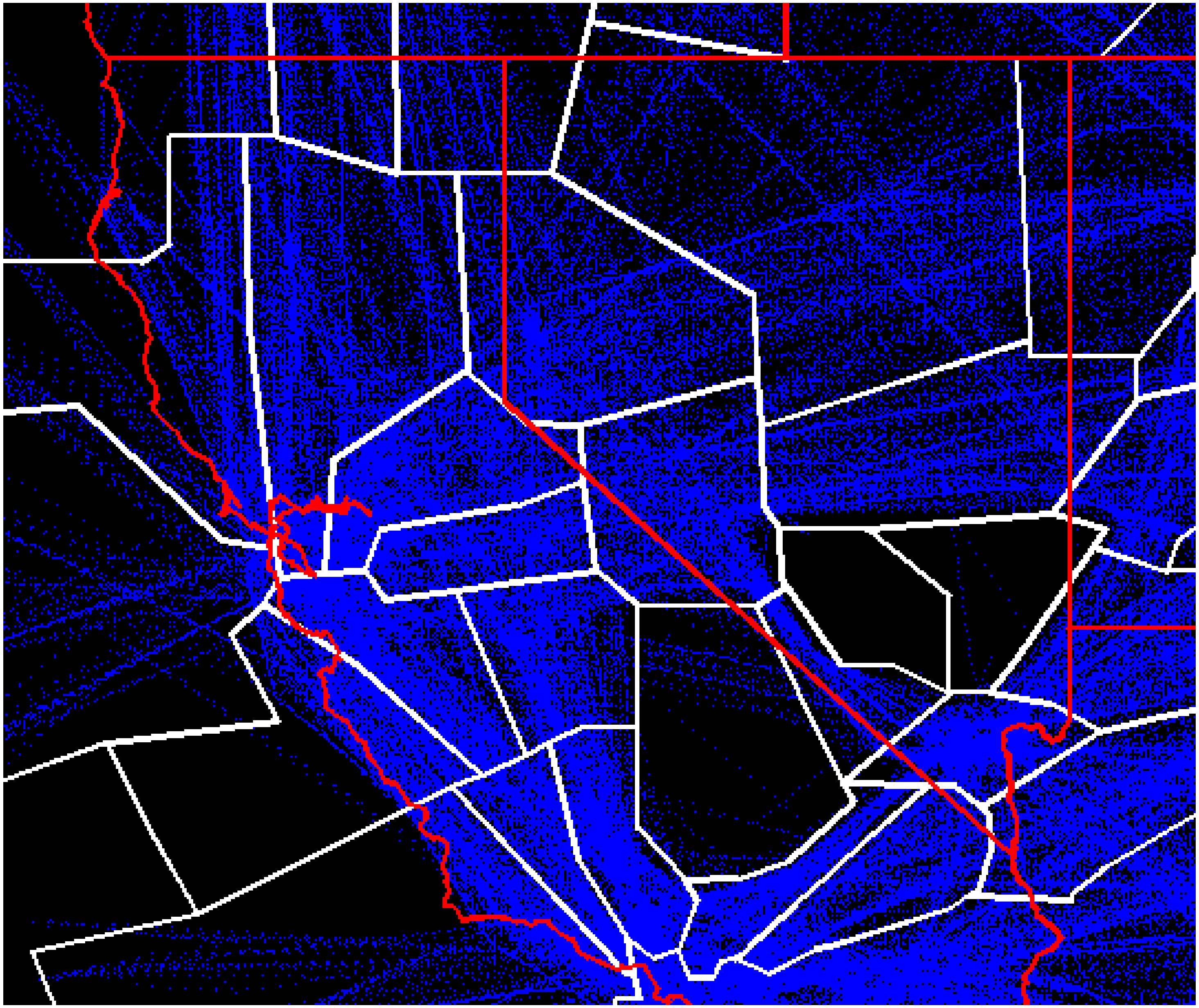
[Robelin, Sun, Wu, Bayen 2006]

[Sun, Bayen 2007]

Conceptual framework: network flow



Conceptual framework: network flow



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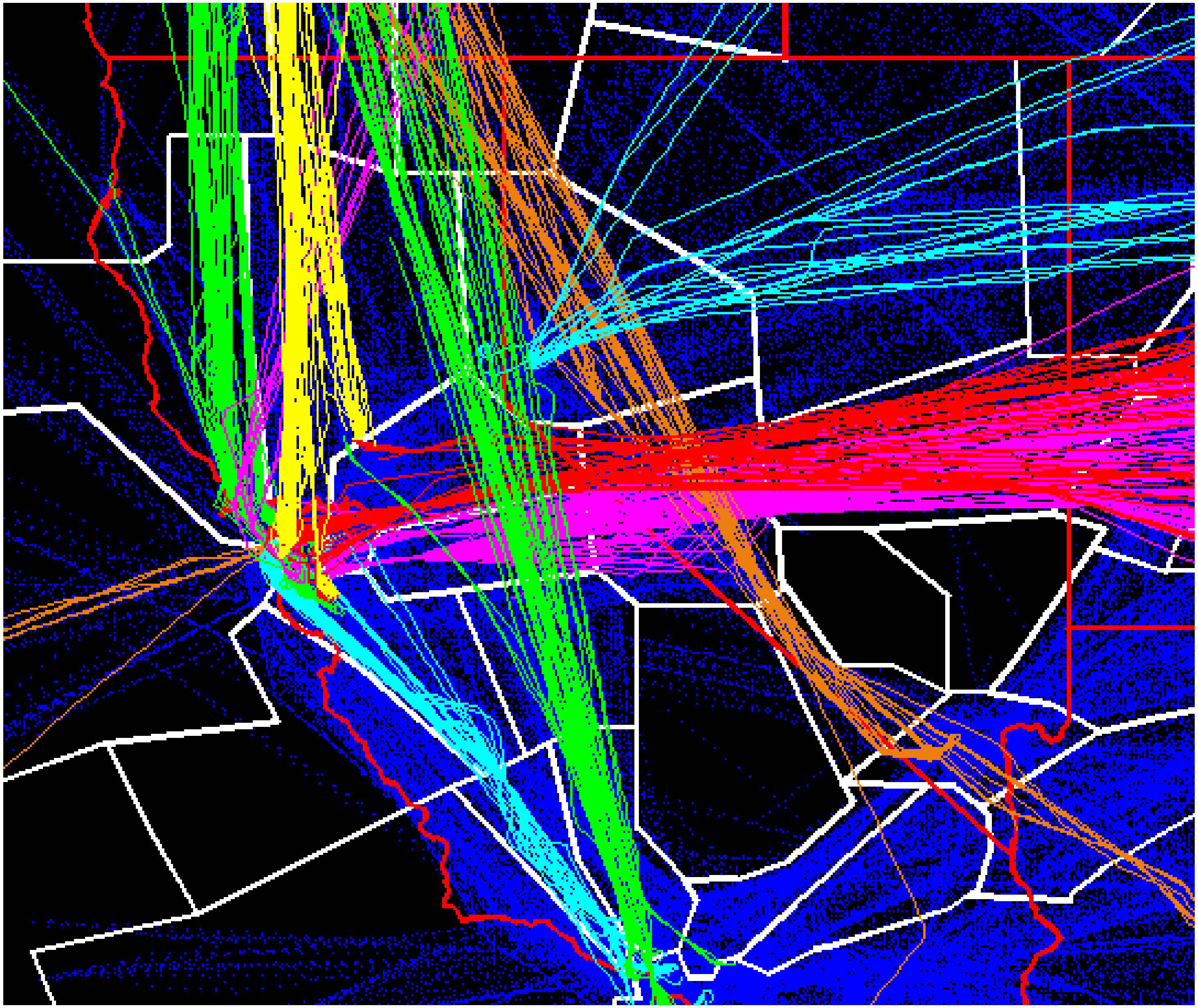

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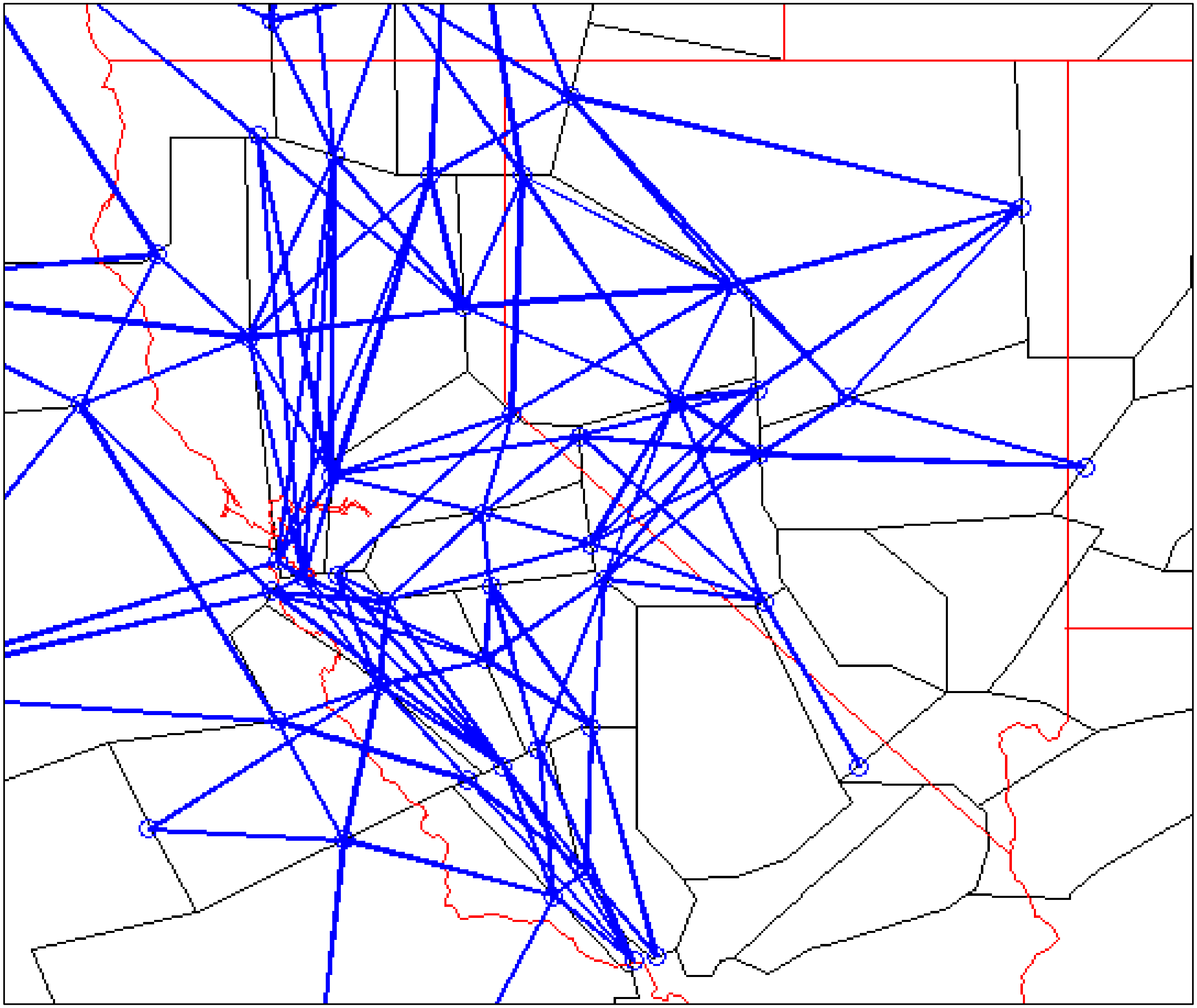
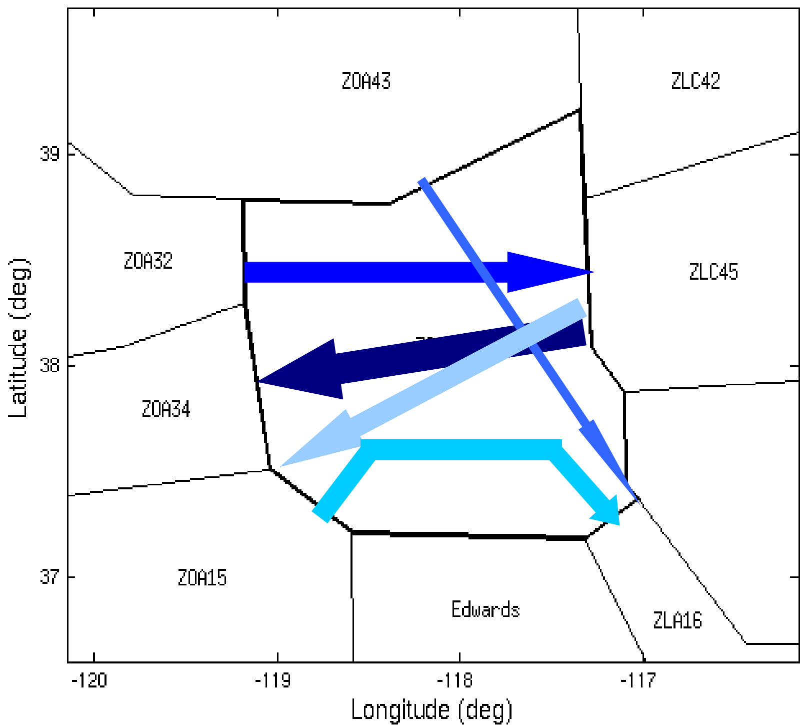
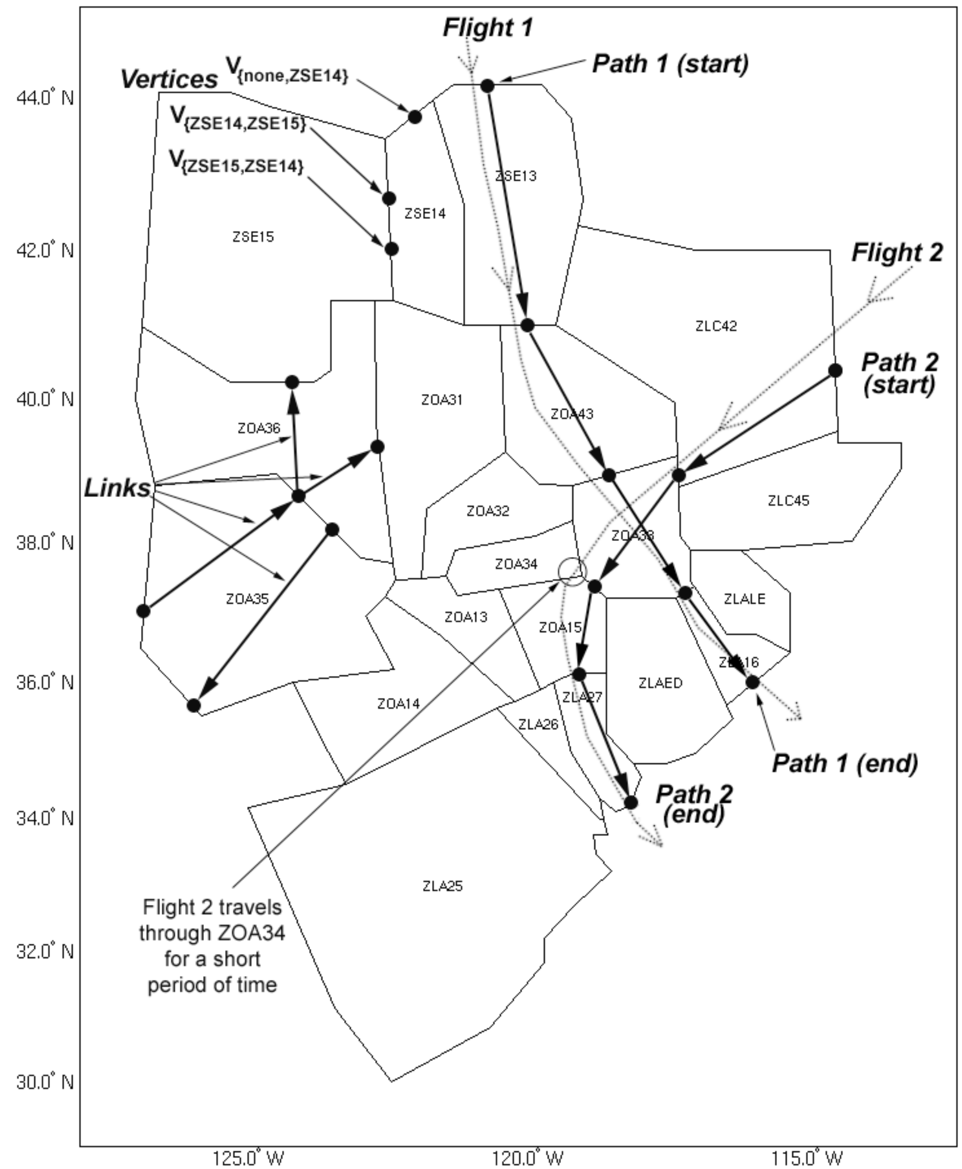


Conceptual goal: graph building (flows)





[Histon, Hansman, 2000]

Model building algorithm

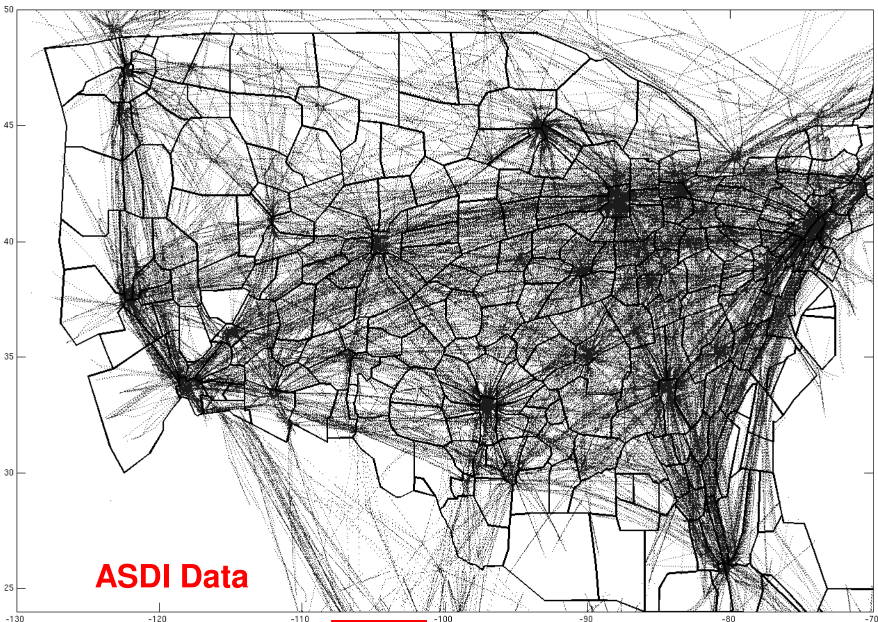
Sequential (automated) algorithm

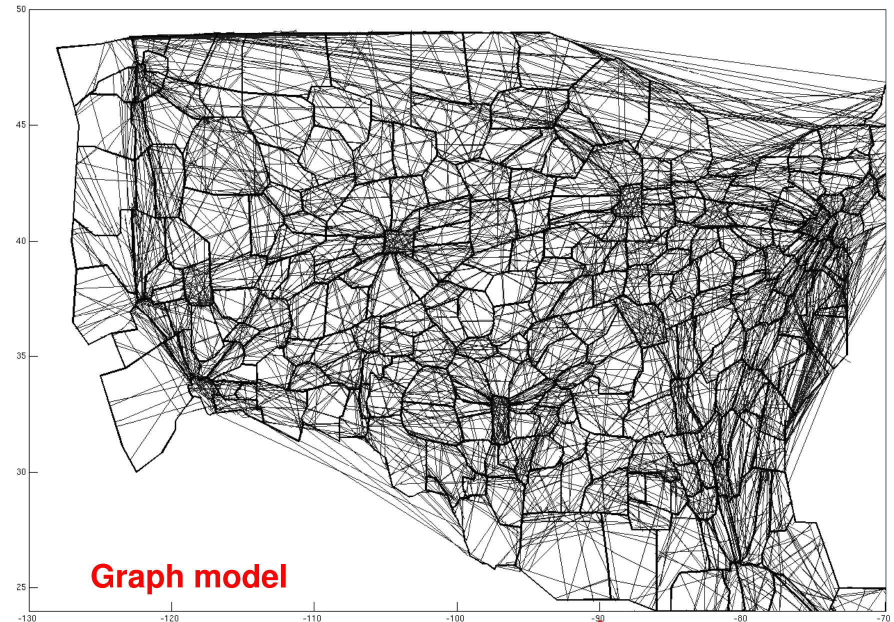
1. Airspace segmentation using sector boundaries
2. Link building using clustering techniques
3. Data aggregation using ASDI/ETMS information (flight plan information)
4. Filtering using LOAs, and observed flow patterns
5. Computation of the aggregate flow pattern features

Output: topology of the flows

Models: graph model of air traffic flow

- Flight tracks – graph theoretical model



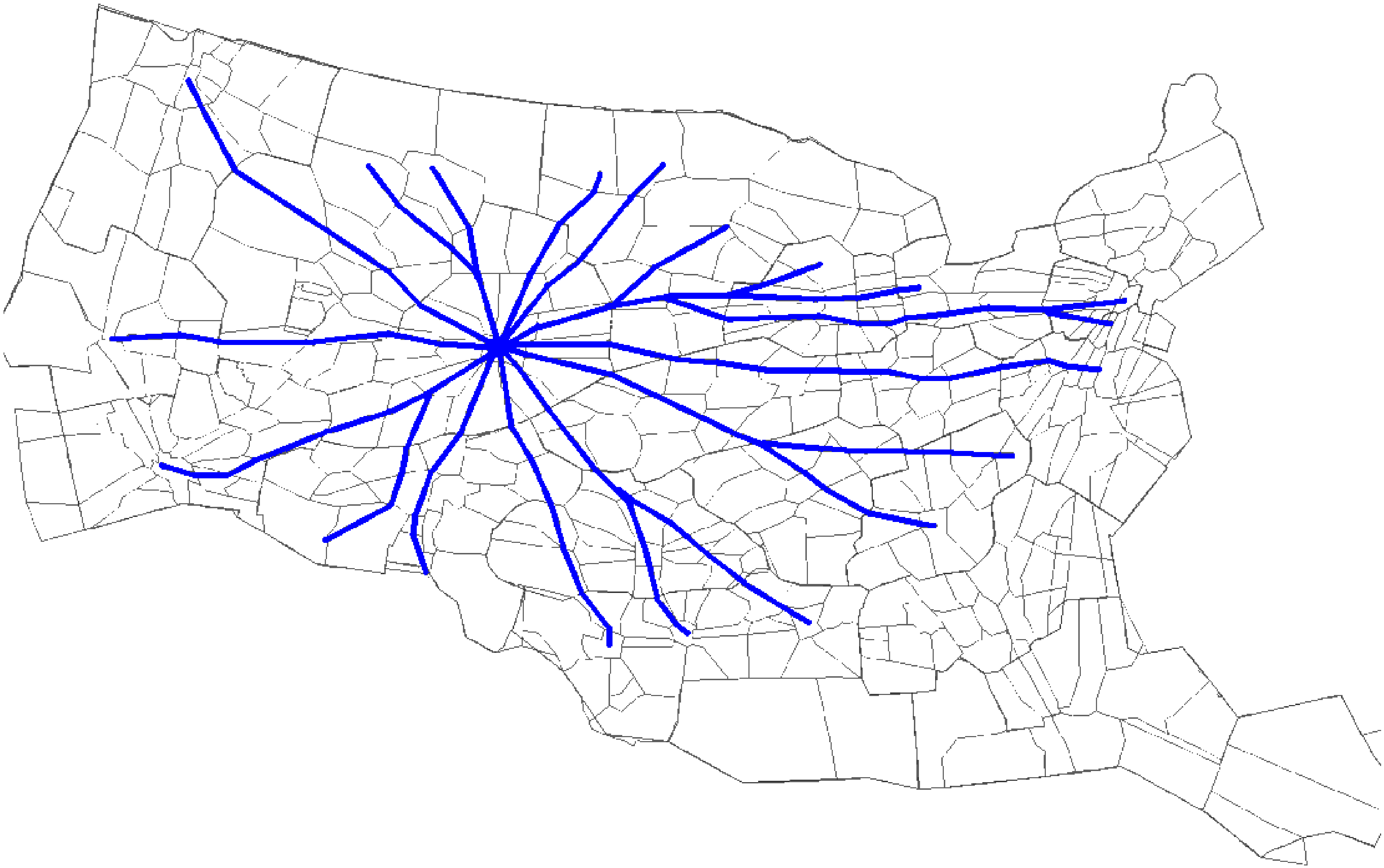


automated identification procedure: clustering; pattern recognition

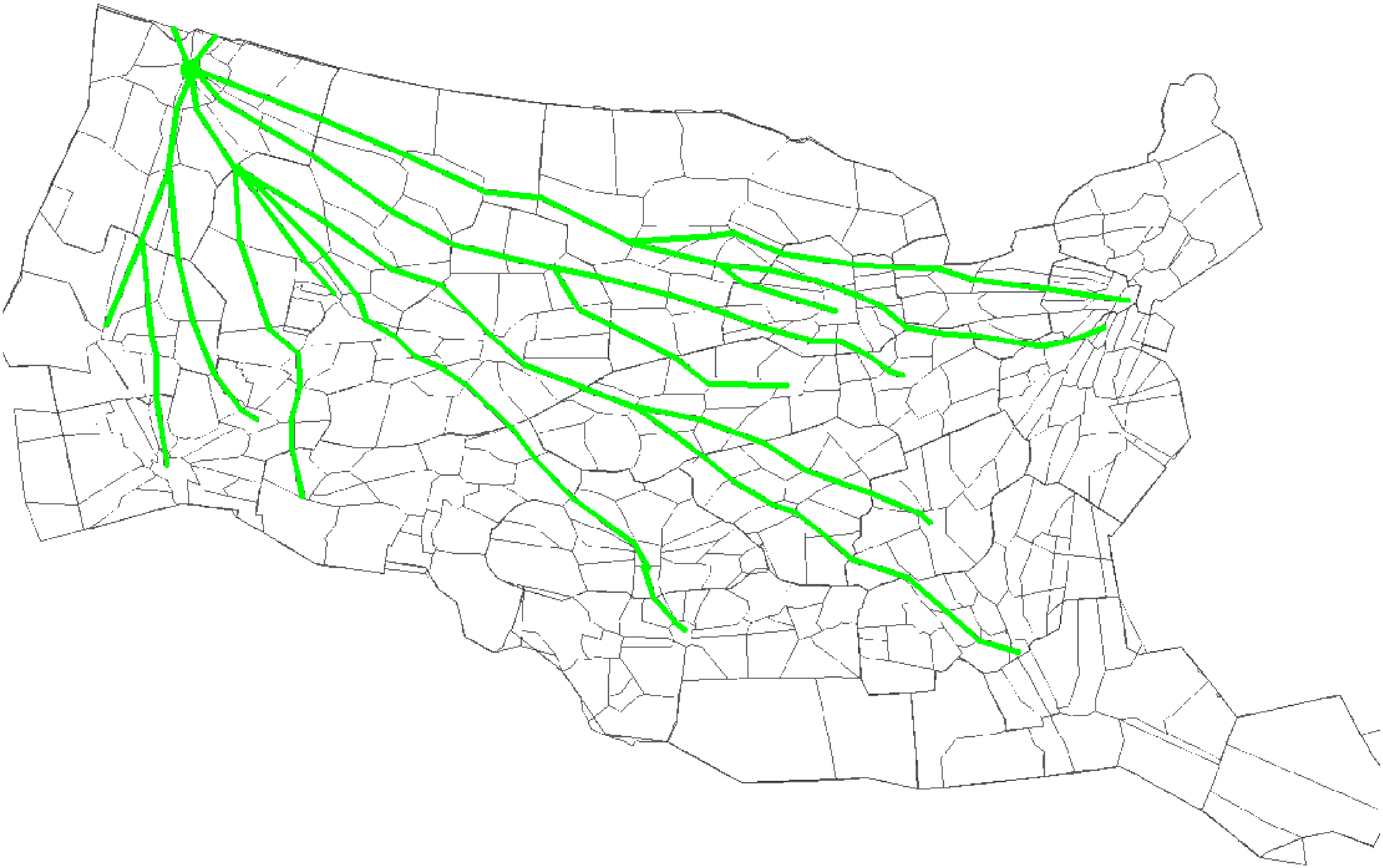
Source of the data: FAA/CNA

[Sun, Strub and Bayen 2007]

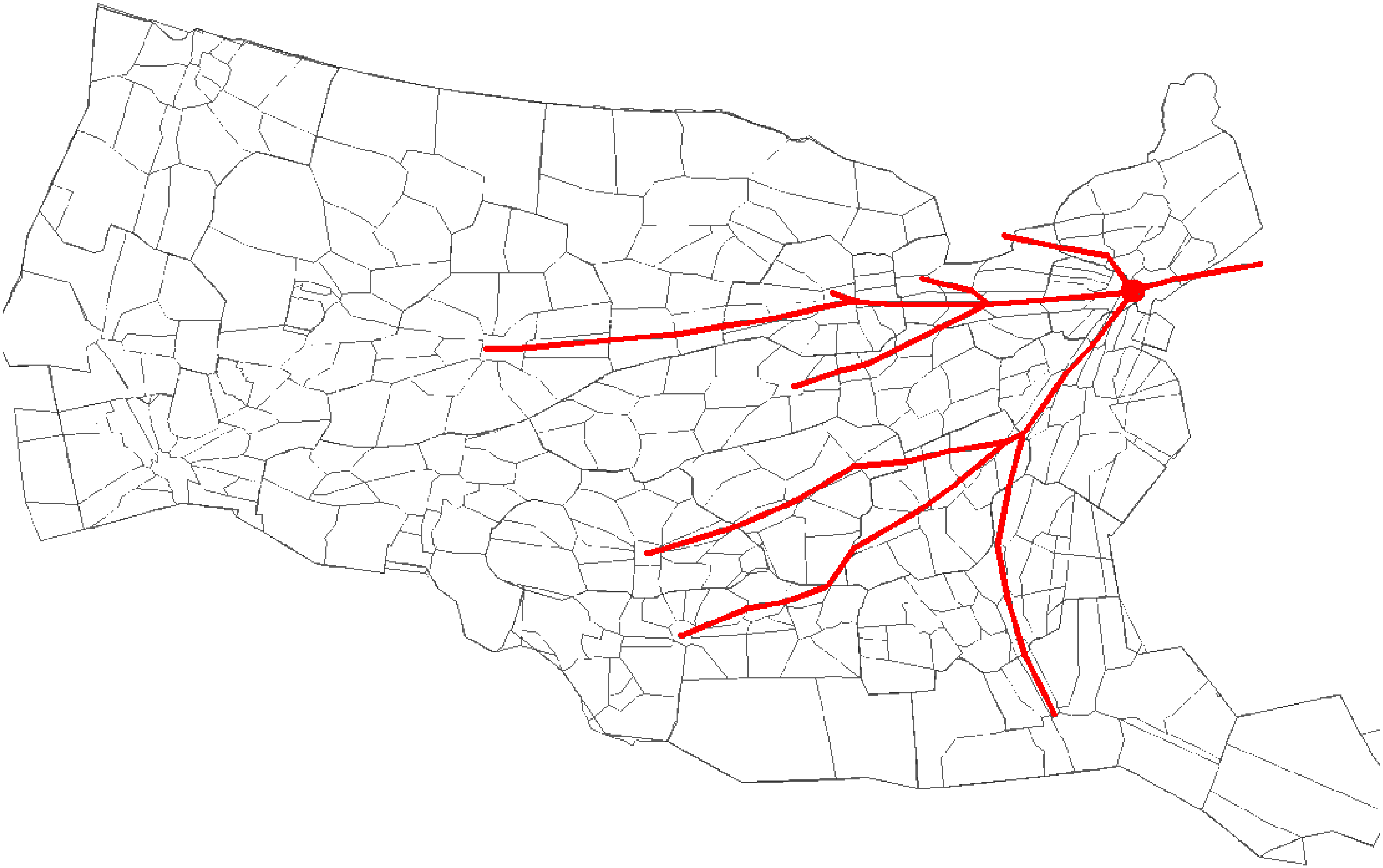
Multicommodity flow - DEN

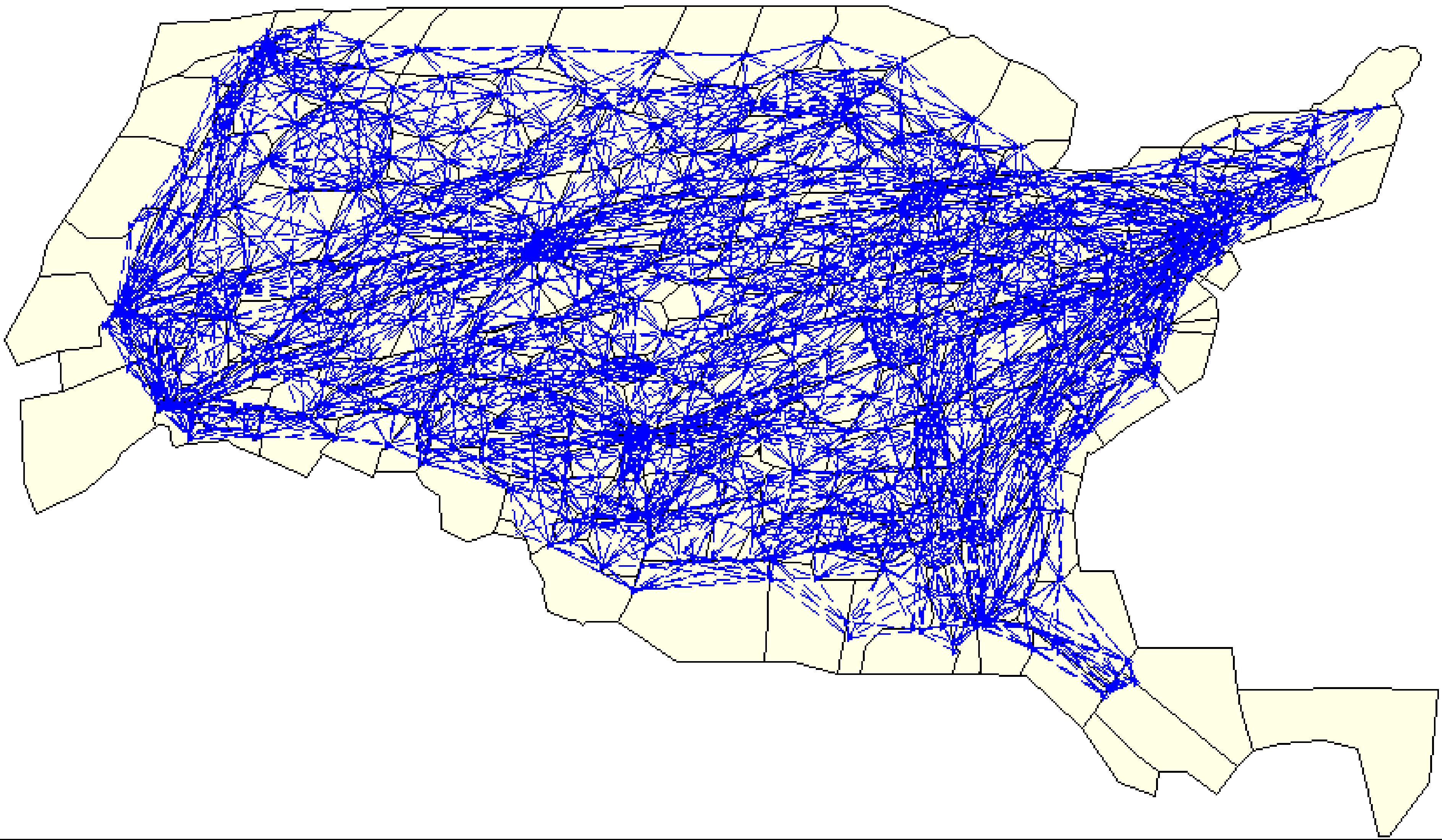


Multicommodity flow - SEA



Multicommodity flow - LGA





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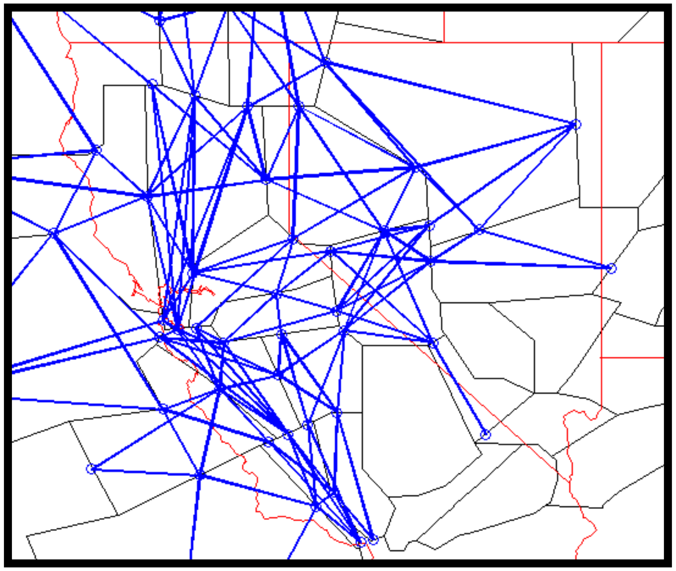
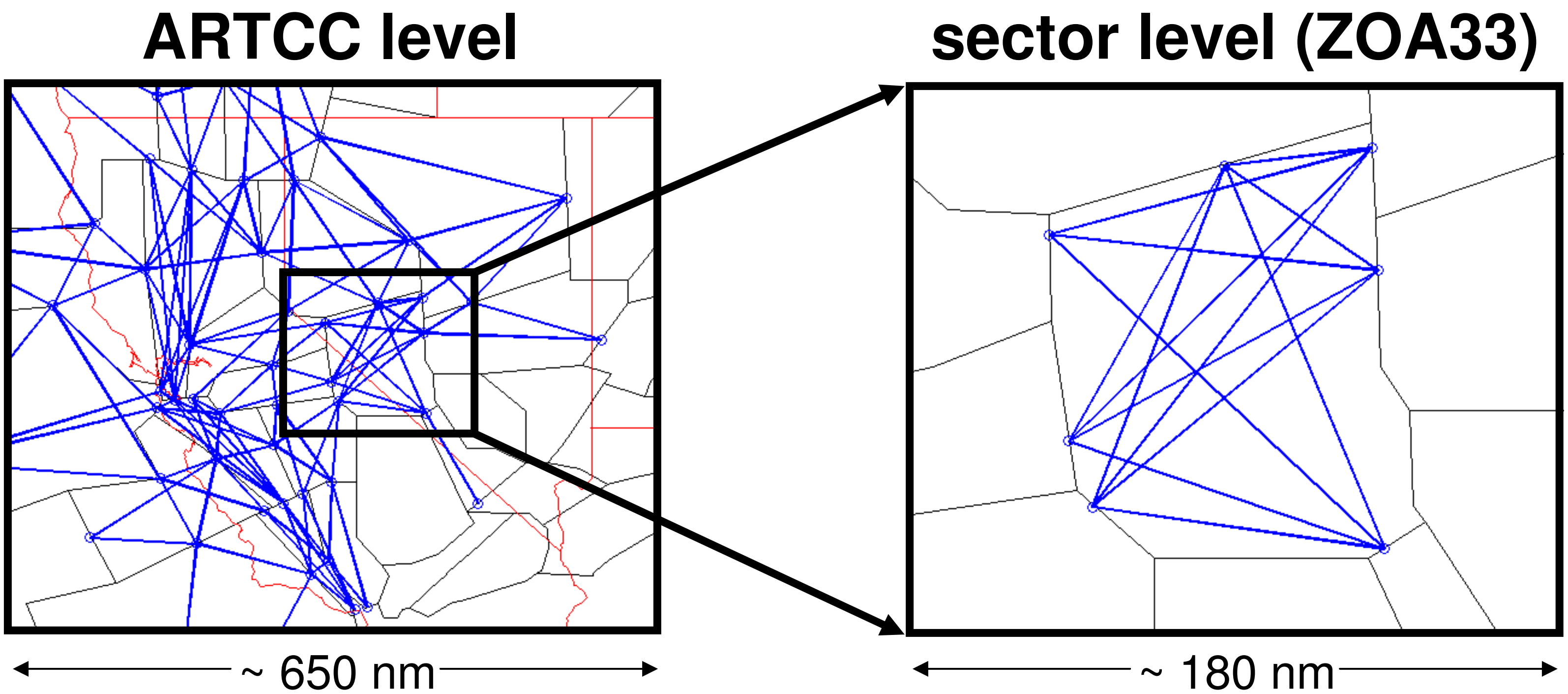
Traffic flow model on a single link

ARTCC level

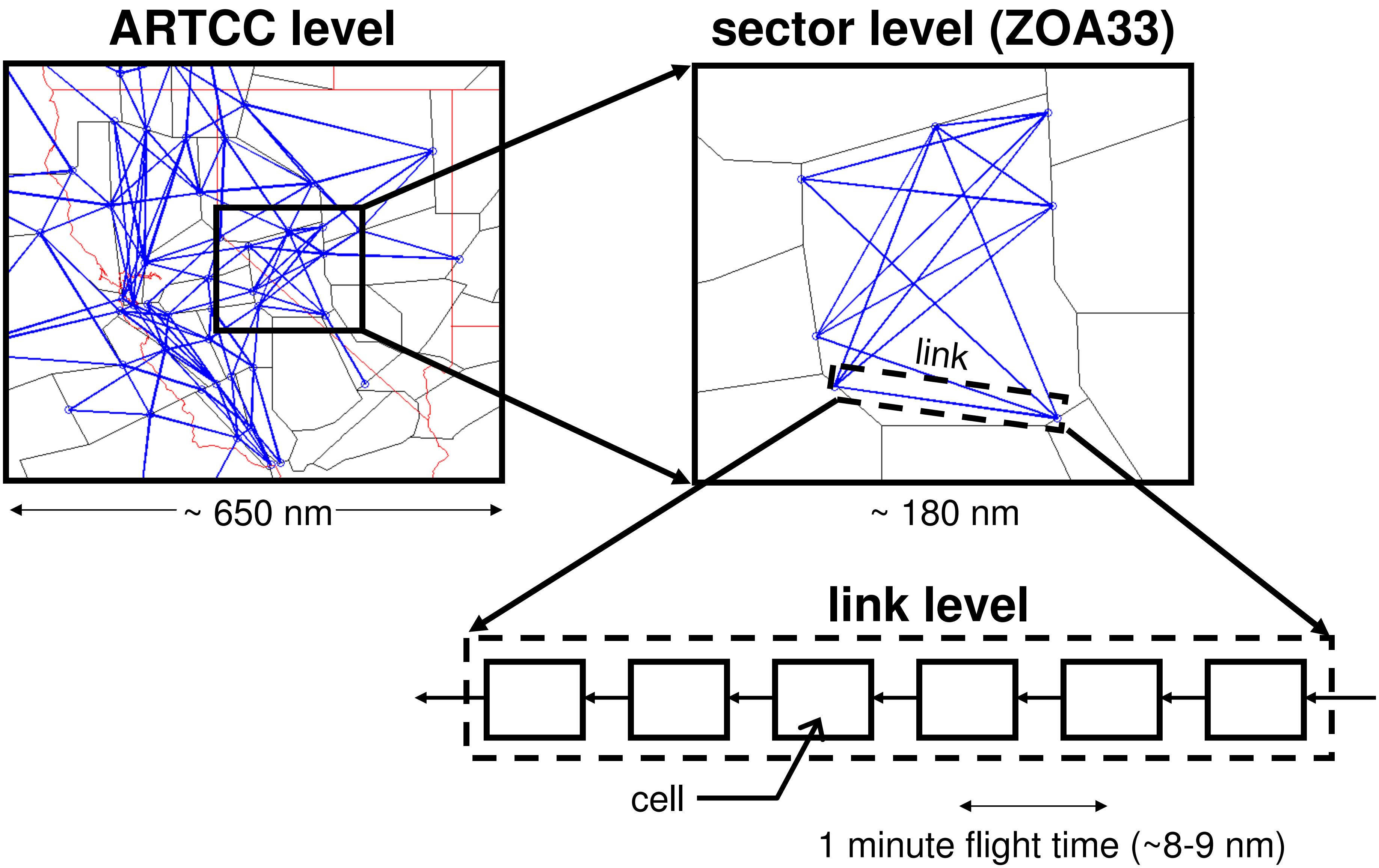


~ 650 nm

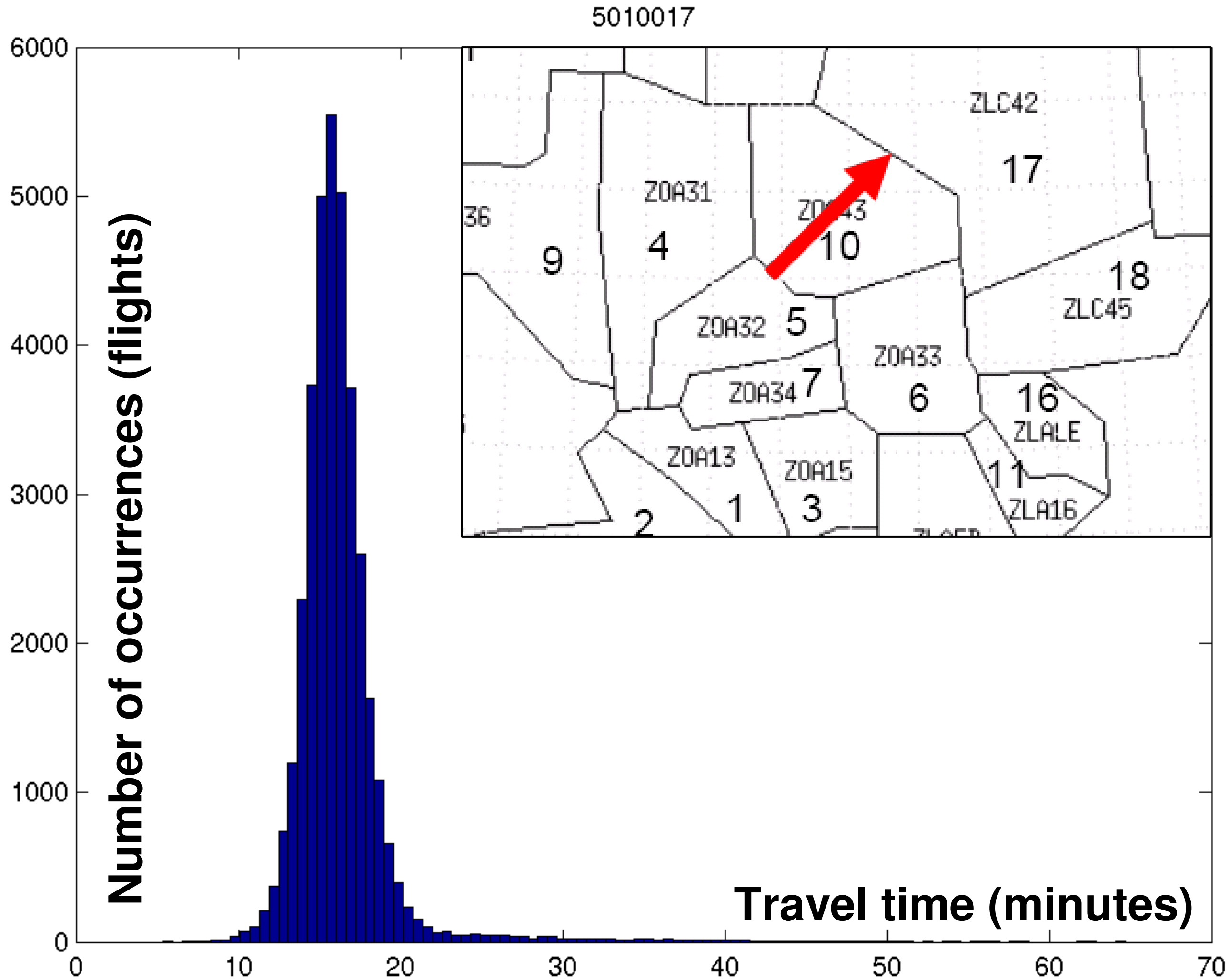
Traffic flow model on a single link



Traffic flow model on a single link



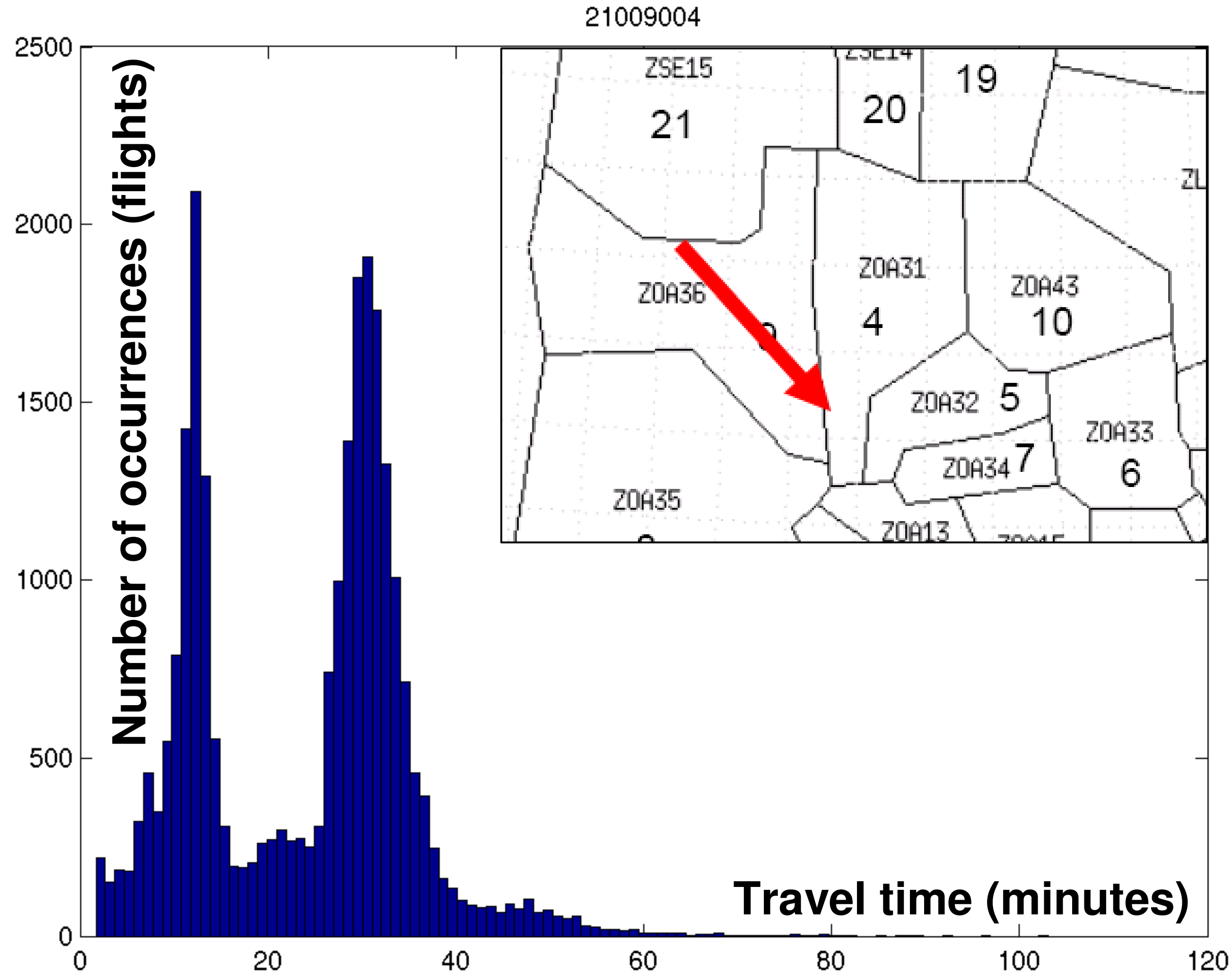
Distribution of travel time



Travel times through the link ZOA32-ZOA43-ZLC42

Departure flights from the bay area.

Occurrence of a second peak

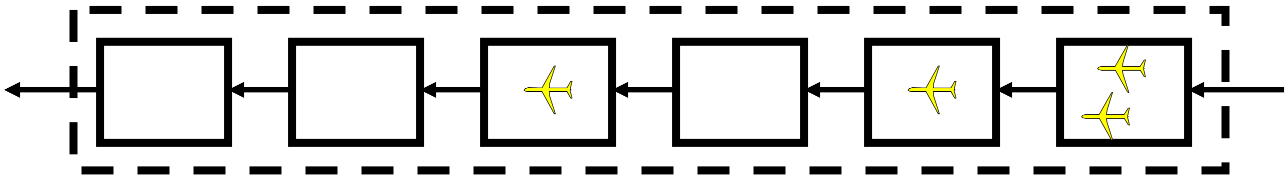


Travel times through the link ZSE15-ZOA26-ZOA31

Possibility: one descending bay area, one passing by

Eulerian dynamics on a link

delay system at the link level



state: cell counts

$$x(1) = \begin{bmatrix} x_6(1) \\ x_5(1) \\ x_4(1) \\ x_3(1) \\ x_2(1) \\ x_1(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

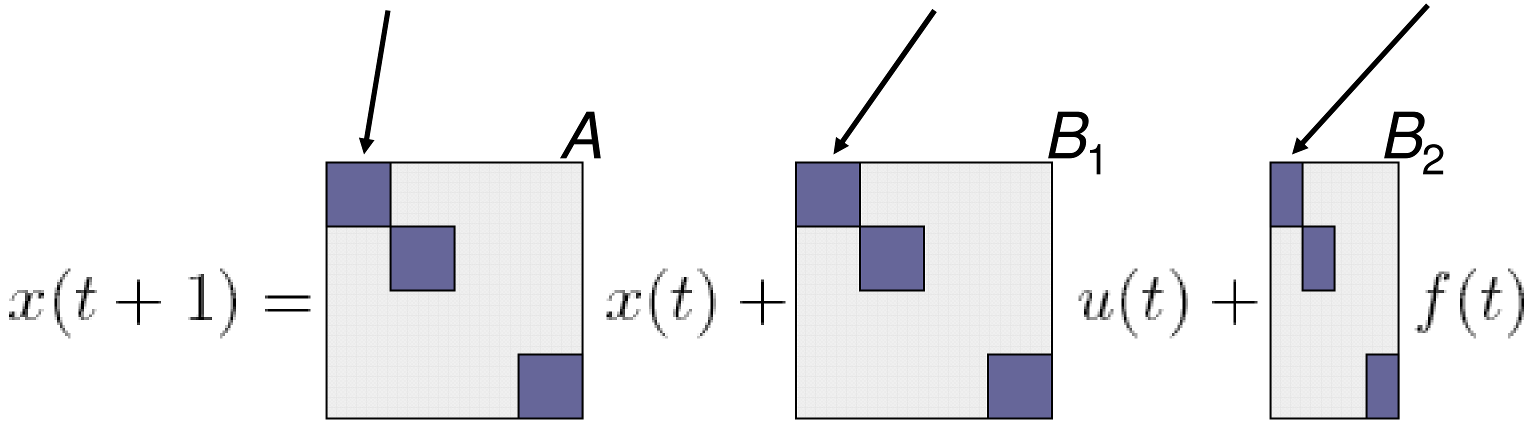
[Robelin, Sun, Wu, Bayen 2006]

[Sun, Bayen 2007]

ARTCC level Eulerian model

link level

$$\begin{bmatrix} x_6(t+1) \\ x_5(t+1) \\ x_4(t+1) \\ x_3(t+1) \\ x_2(t+1) \\ x_1(t+1) \end{bmatrix} = \overset{A_i}{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}} \begin{bmatrix} x_6(t) \\ x_5(t) \\ x_4(t) \\ x_3(t) \\ x_2(t) \\ x_1(t) \end{bmatrix} + \overset{B_{1,i}}{\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} u_6(t) \\ u_5(t) \\ u_4(t) \\ u_3(t) \\ u_2(t) \\ u_1(t) \end{bmatrix} + \overset{B_{2,i}}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}} f(t)$$



Air Route Traffic Control Center (ARTCC) level

Sparse LTI dynamical system:
blocks are nilpotent or upper diagonal matrices

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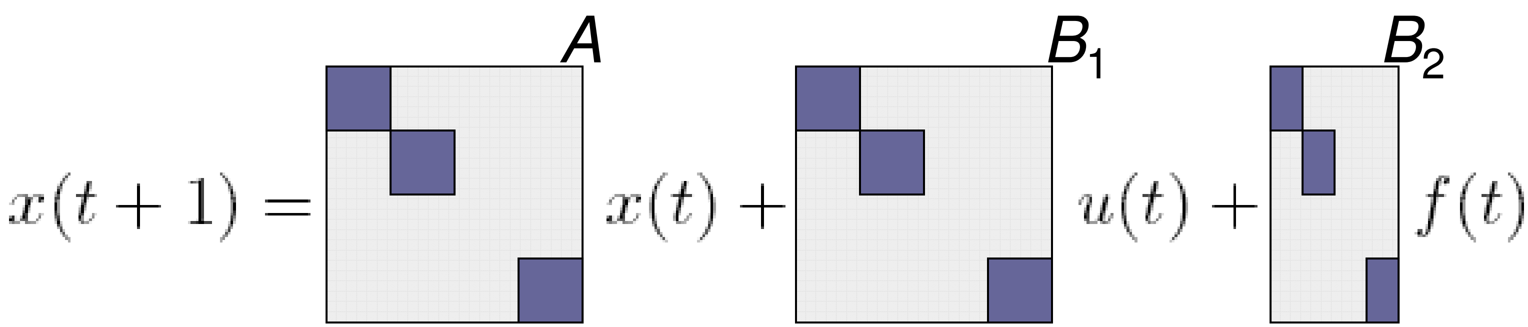
Analysis of control theoretic properties

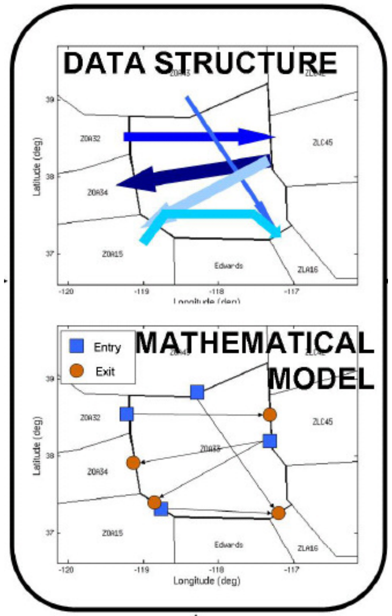
System controllable (unconstrained)

Case with constraints of nonnegativity and integrality of *x*, *u* and *f*: problem becomes NP-hard



Validation of predictive capabilities

Model: based on a full year of ETMS/ASDI data: Sep 04 → Sep 05



Validation of predictive capabilities

Input: filed flight plans for a particular day not in Sep 04 → Sep 05 (perfect OD knowledge, requested schedules)


```
AD324075551KZDVTZ AWE778/006 424 330 3725N/11016W
AD424075551KZDVTZ COA1803/353 472 290 3725N/10500W
AD524075557KZDVTZ NKS788/063 429 310 3822N/10744W
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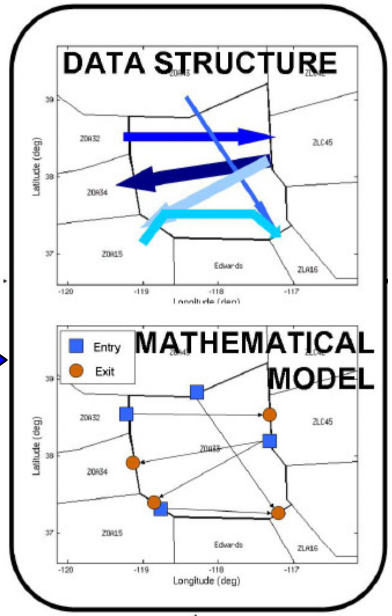

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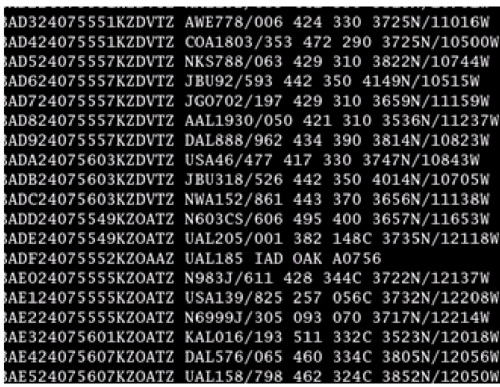

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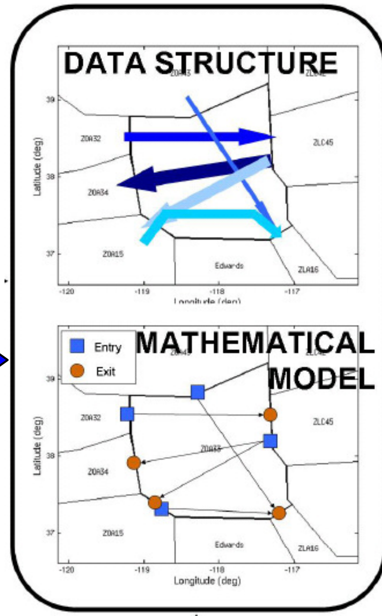
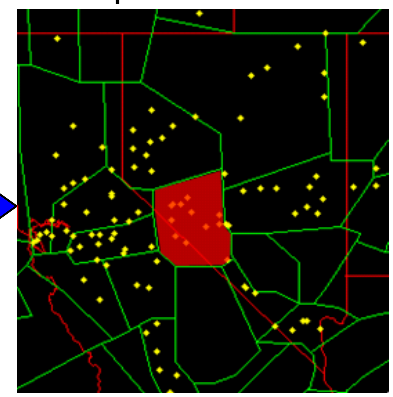


Output: aggregate flow predictions



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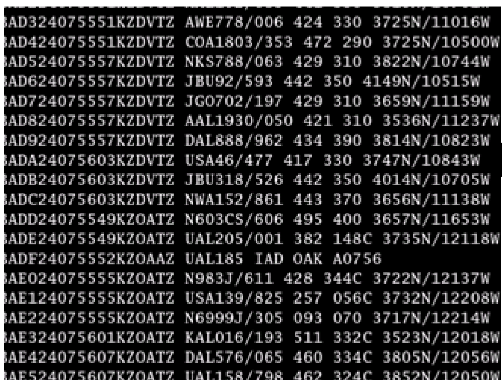

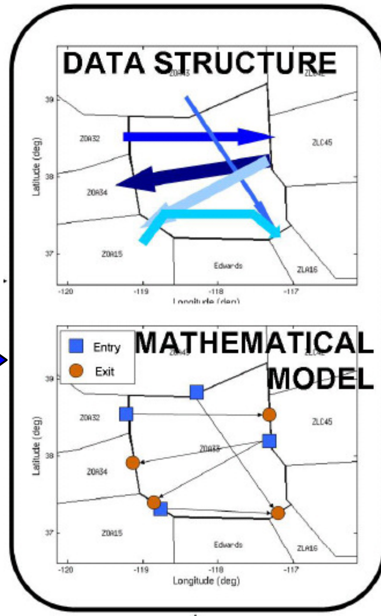

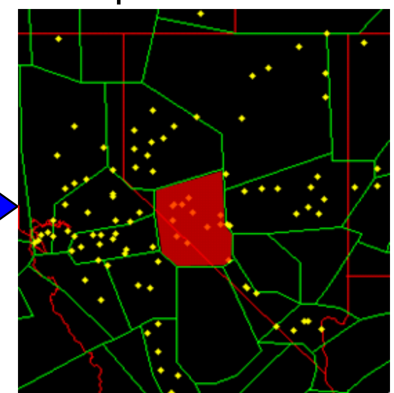

Comparison data:
For the same day:
true sector counts,
(not used by the model)

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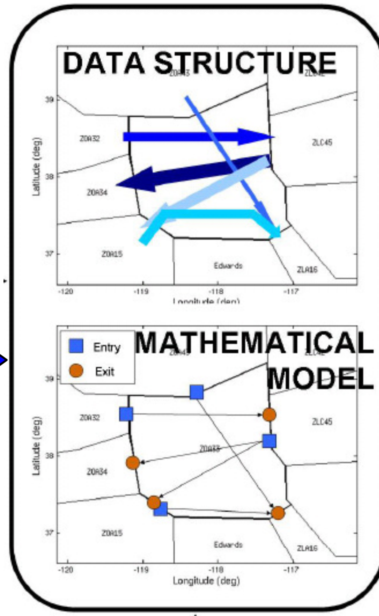

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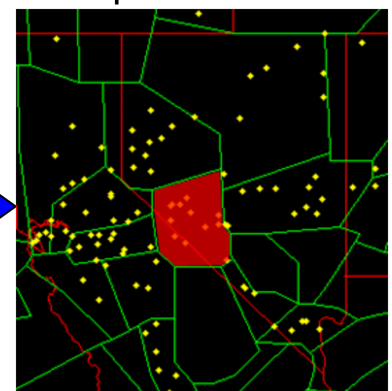

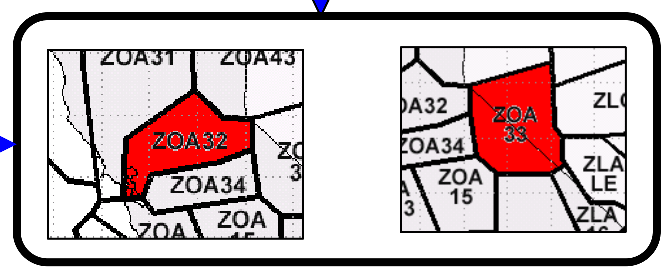

Comparison data:
For the same day:
true sector counts,
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Comparison metrics:
for example sector counts

Validation of predictive capabilities

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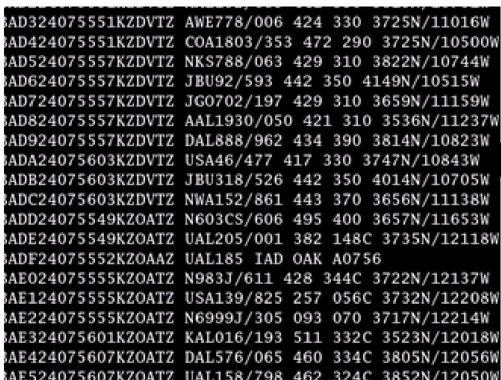

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ADE24075549KZOATZ UAL205/001 382 148C 3735N/12118W
ADF24075552KZOAAZ UAL185 IAD OAK A0756
AE024075555KZOATZ N983J/611 428 344C 3722N/12137W
AE124075555KZOATZ USA139/825 257 056C 3732N/12208W
AE224075555KZOATZ N6999J/305 093 070 3717N/12214W
AE324075601KZOATZ KAL016/193 511 332C 3523N/12018W
AE424075607KZOATZ DAL576/065 460 334C 3805N/12056W
AE524075607KZOATZ UAL158/798 462 324C 3852N/12050W
```
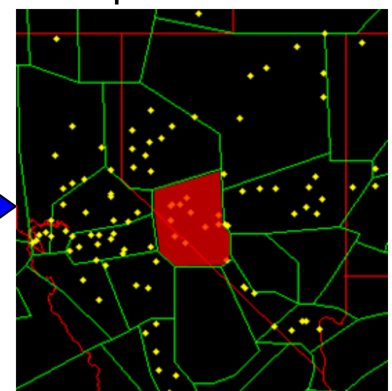
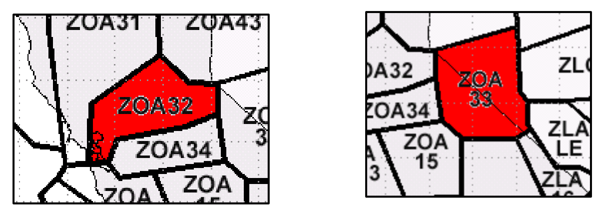

Model: based on a full year of ETMS/ASDI data: Sep 04 → Sep 05

Your model here

Output: aggregate flow predictions



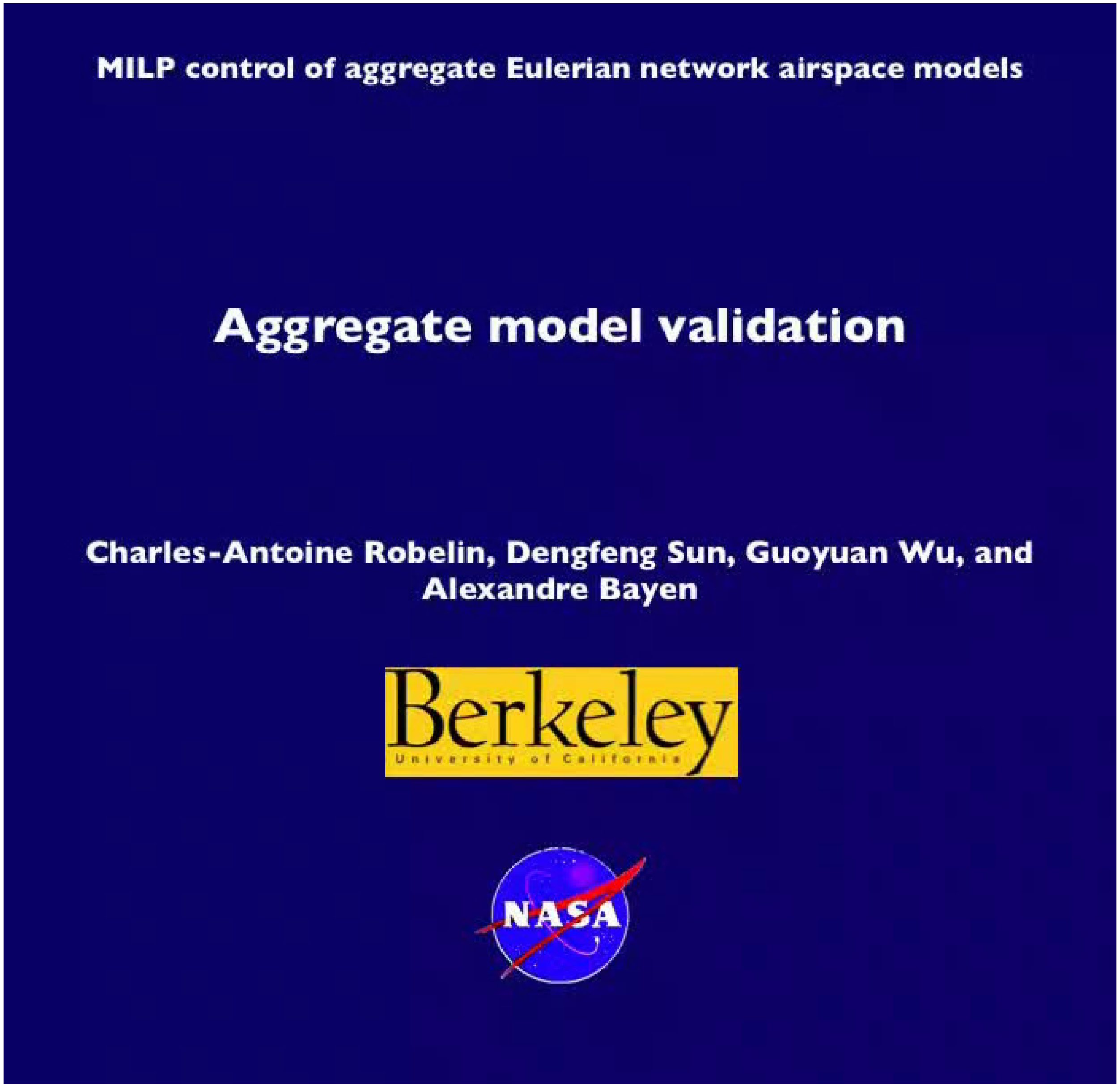


Comparison data:
For the same day:
true sector counts,
(not used by the model)

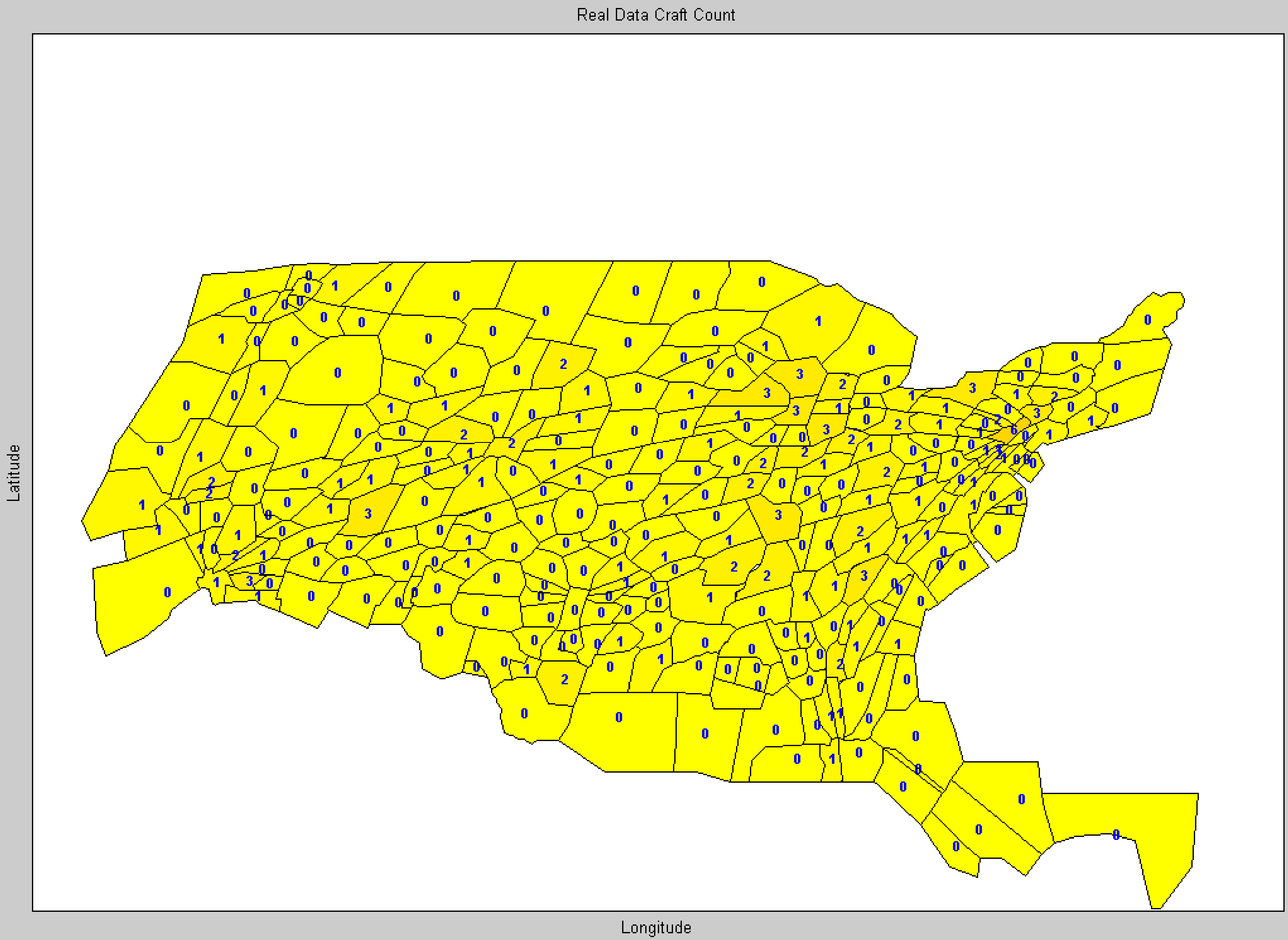
Comparison metrics:
for example sector counts

Aggregate model validation

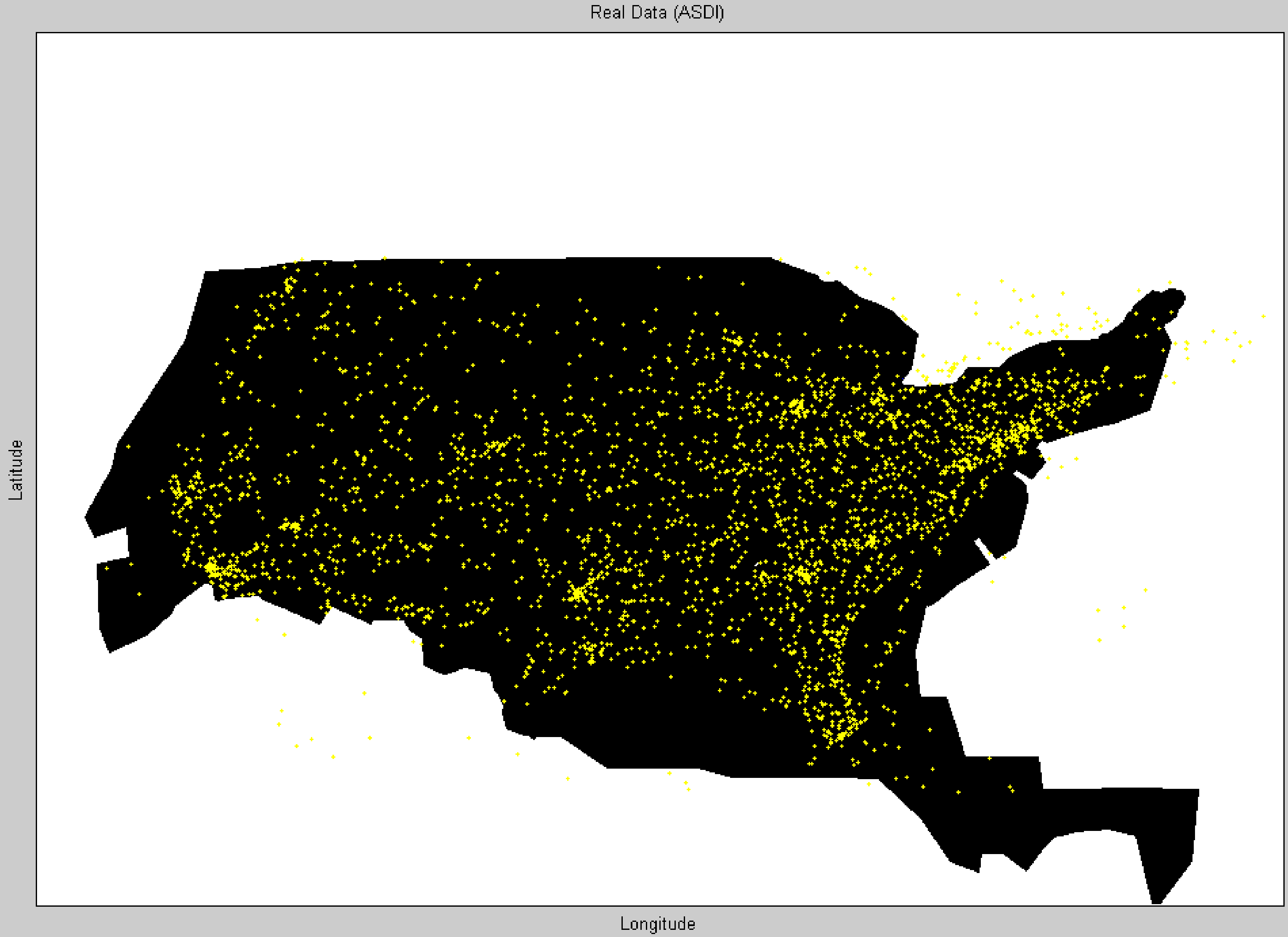
UC Berkeley



Example: sector load forecast model



Based on LTI dynamical systems forecast sim



Simulation capabilities

1. Software environment written in C/C++ with a Matlab interface, in which we can
    1. Implement traffic simulations (with four traffic flow models so far)
    2. Implement Traffic Flow Management optimization algorithms
    3. Input/output results from/to FACET

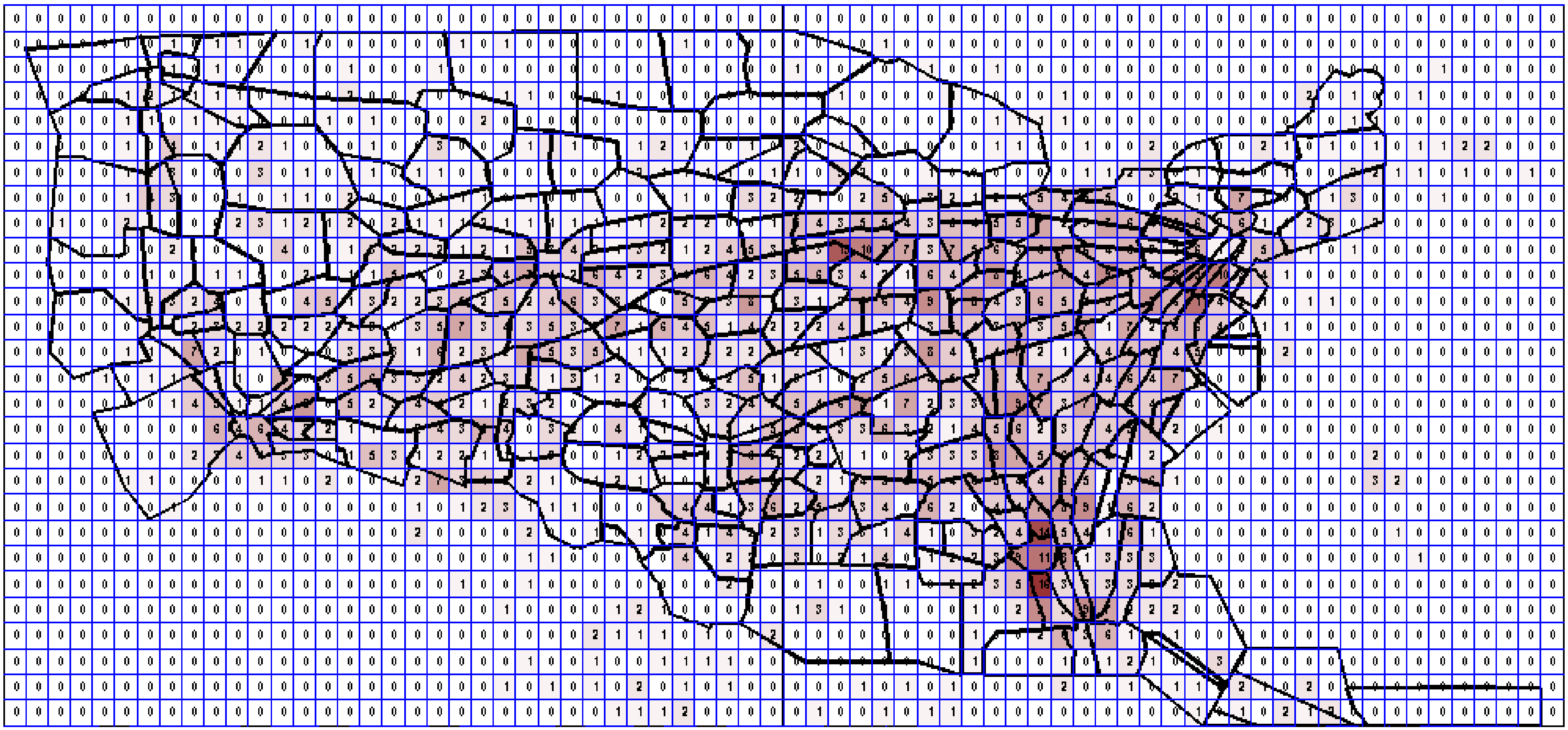
2. Once completed will be an open source software, online for download
    1. Provided with a Matlab interface
    2. In which the user can input their own model or run preprogrammed models
    3. With which the user can run optimization software such as CPLEX for TFM algorithms

3. Currently includes 4 flow models
    1. The Stanford continous PDE flow model (Bayen, Raffard, Waslander, Tomlin)
    2. The Multicommodity cell transmission model (Sun, Robelin, Bayen)
    3. The 1D Menon Model (Menon, Sweriduk, Bilimoria)
    4. The 2D Menon Model (Menon, Sweriduk, Lam, Diaz, Bilimoria)

4. Its current functionalities include
    1. Validation of the predictive capabilities of the different models
    2. Optimal flow routing algorithms using: LP, MILP, adjoint-based optimization

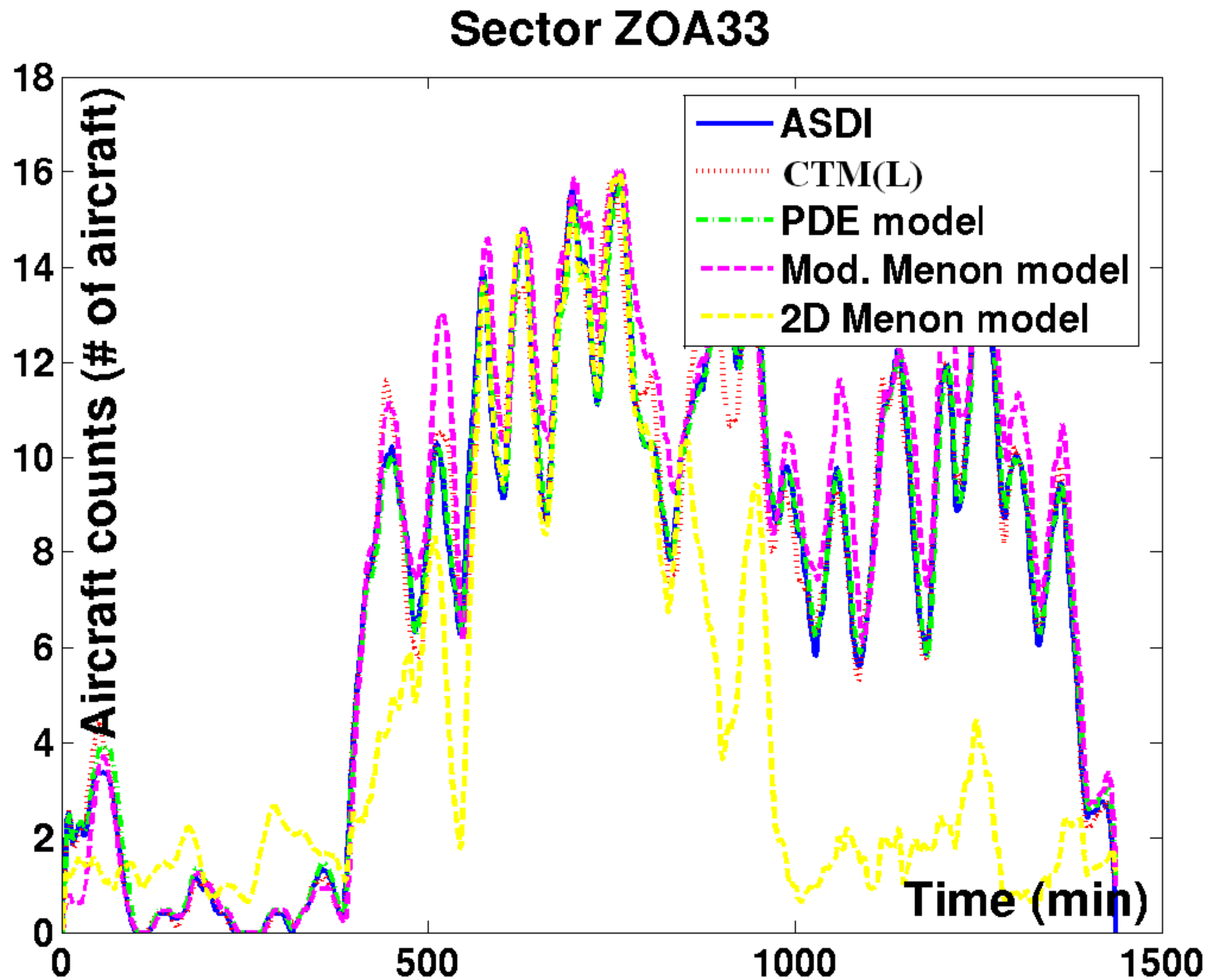
Other simulation capabilities

Optimal synthesis model



[Sun, Strub, Bayen 2007]

"Eulerian quadrilogy" work…



[Sun, Strub, Bayen 2007]

Outline

1. Introduction: Eulerian/Langrangian; Micro/Macro-aggregate

2. Multicommodity cell transmission model
    1. Automated graph topology model building
    2. Aggregate travel time estimation

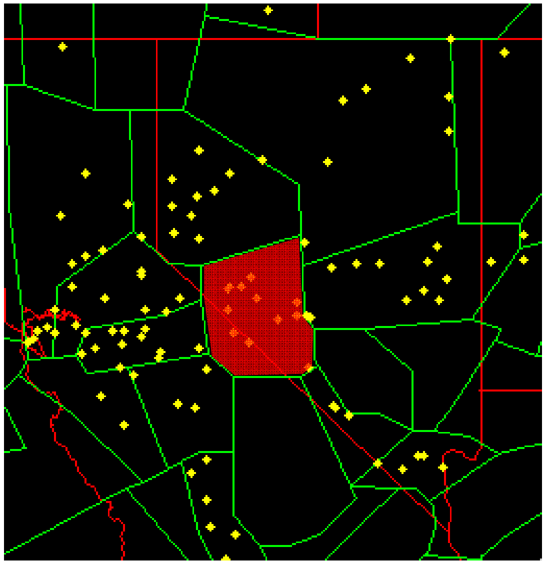
3. LTI models of the NAS
    1. Standard LTI formulation
    2. Constrained optimization formulations
    3. Practical implementation

4. Applications
    1. NAS-wide TFM
    2. Dynamic airspace configuration

Example 1: control application

Operational problem

- Hard sector count enforcement
- Objective function: minimization of overall delay



Formulation

- MILP formulation of delay mitigation
- Practical CPLEX implementation, LP relaxation

$$\textbf{min:} \quad \sum_{k=0}^{N} c^T x_k$$

subject to:

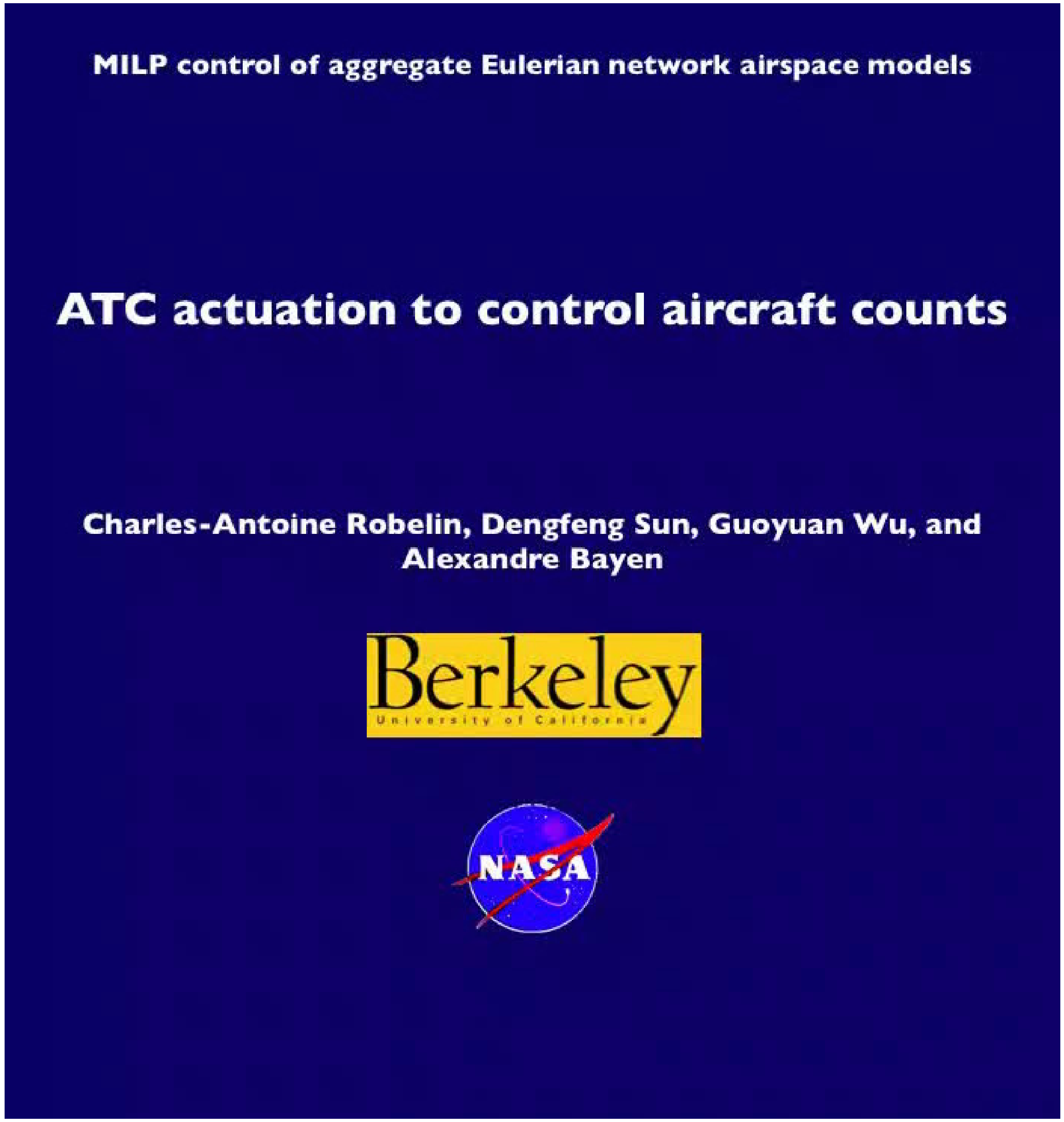
$$x_0 = B_2 f_0$$
$$x_{k+1} = A x_k + B_1 u_k + B_2 f_k, \; k \in \{0, \cdots, N-1\}$$
$$E x_k + L u_k \leq M, \; k \in \{0, \cdots, N-1\}$$
$$x_N \in \chi_f$$

N: number of time steps, *c*: vector of 1's

E, L, M: implement user-specified constraints (capacity, nonnegativity, etc)

χ_f: set of feasible final states, *x*, *f*, *u*, *A*, B_1, B_2: as defined earlier

Overload control



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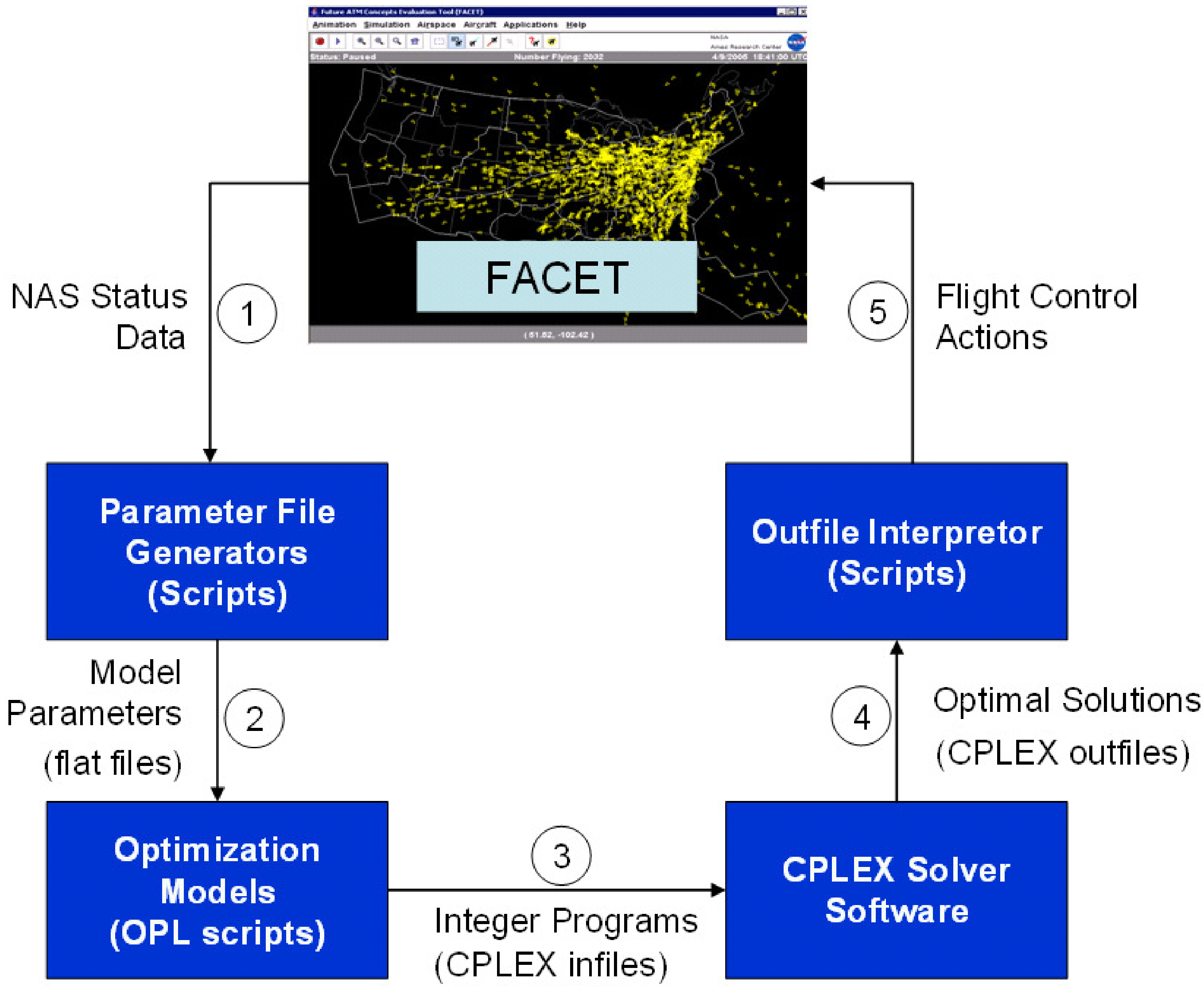
Future steps



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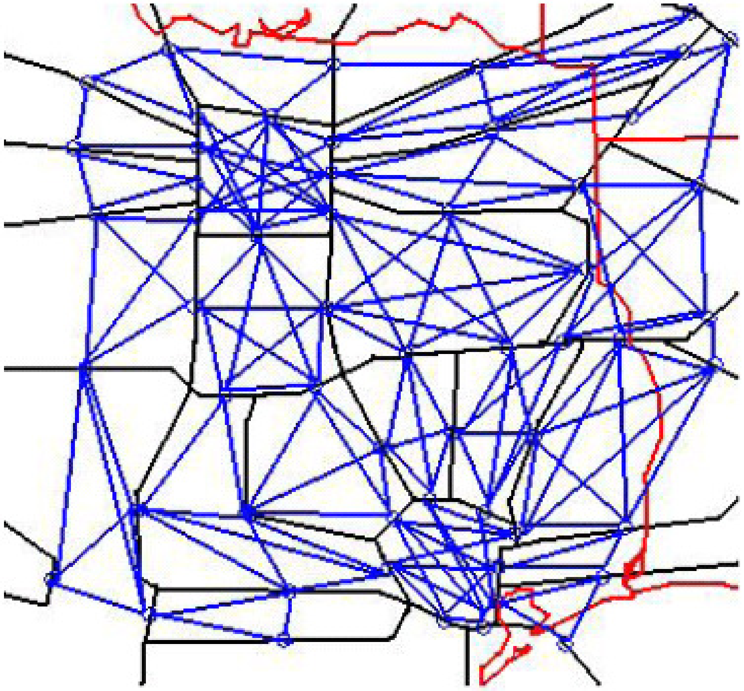
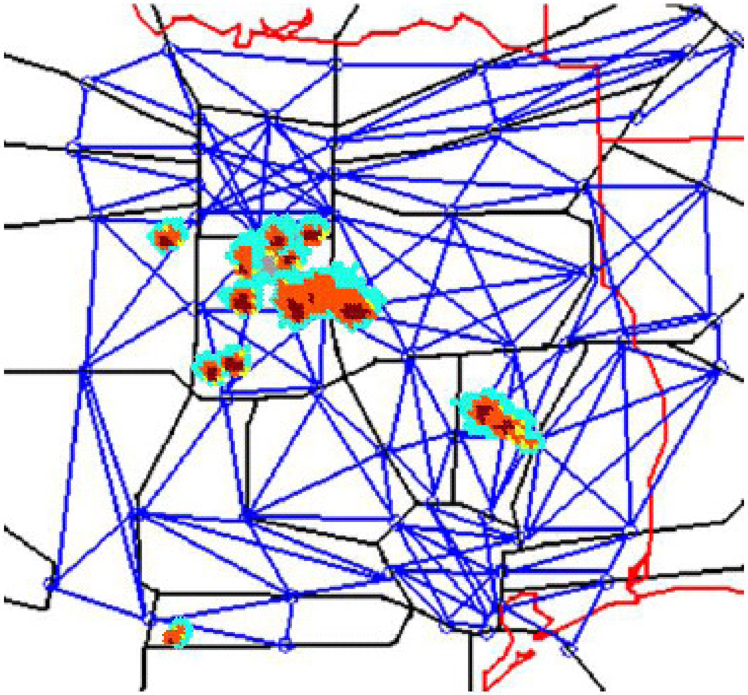
Application 1: NAS-wide TFM

Impact of weather on capacity

→ Delays

→ Optimal reroutes / playbooks





Control of sector count (2-hour TFM)

MILP formulation

min: $$\sum_{k=0}^{N} c^T x_k$$

subject to:

$$x_0 = B_2 f_0$$

$$x_{k+1} = A x_k + B_1 u_k + B_2 f_k, \ k \in \{0, \cdots, N-1\}$$

$$E x_k + L u_k \leq M, \ k \in \{0, \cdots, N-1\}$$

$$x_N \in \chi_f$$

Challenges

ARTCC level

- **>1M variables,**
- **>1M constraints**

NAS-wide

- **> 5 B variables,**
- **> 10 B constraints**

N: number of time steps

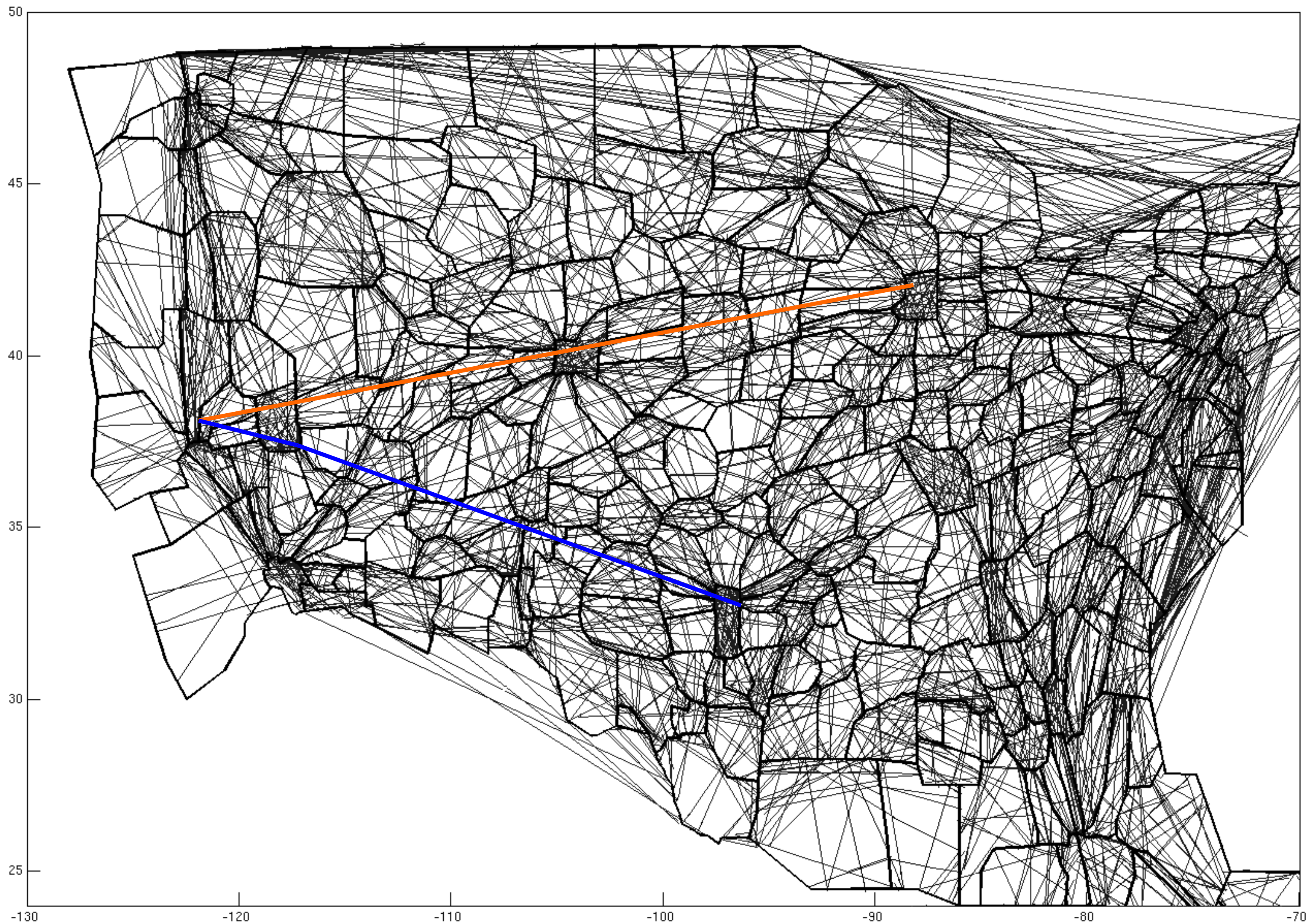
c: vector of 1's

E, L, M: implement user-specified constraints (capacity, nonnegativity, etc)

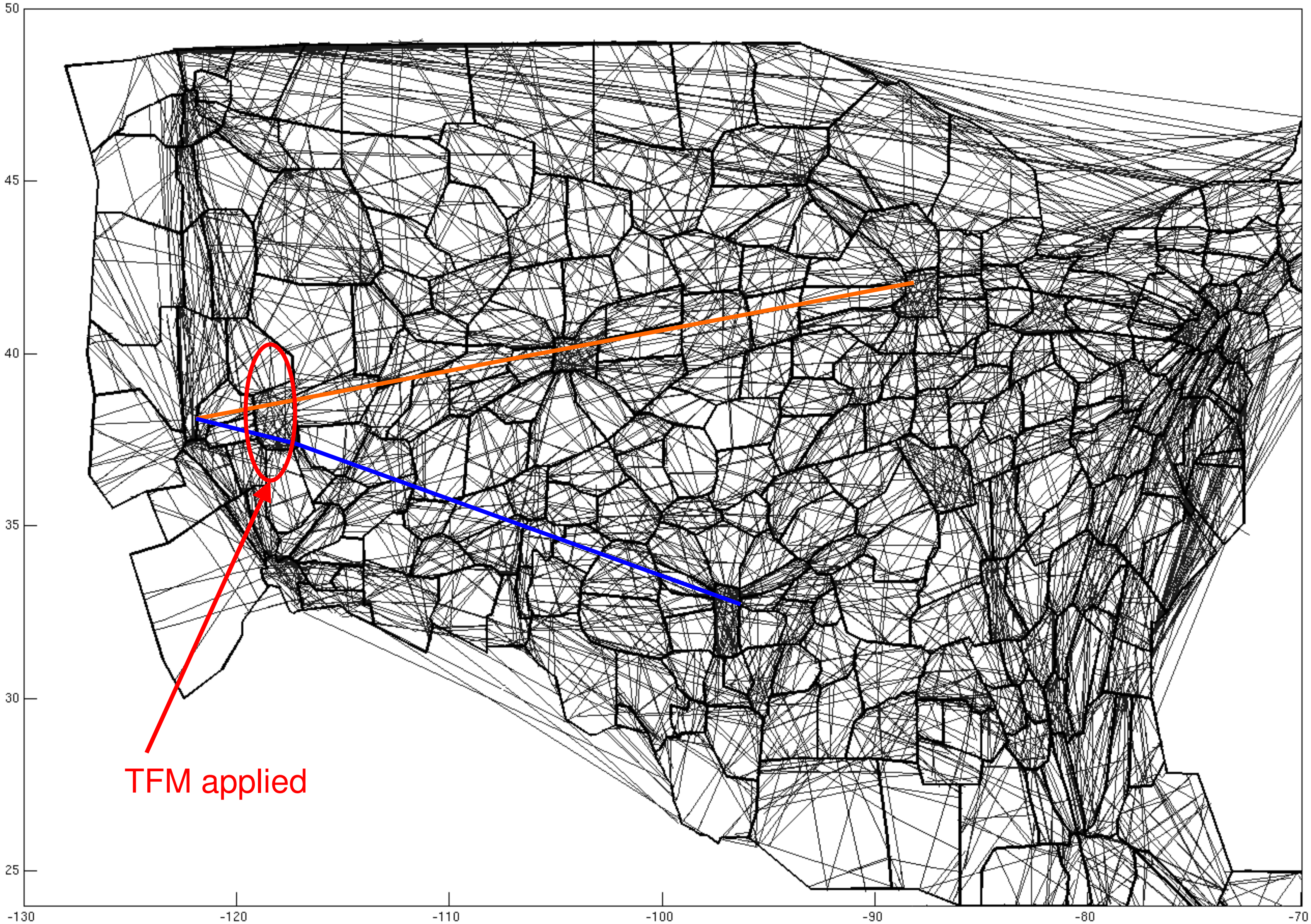
χ_f: set of feasible final states

x, *f*, *u*, *A*, B_1, B_2: as defined earlier

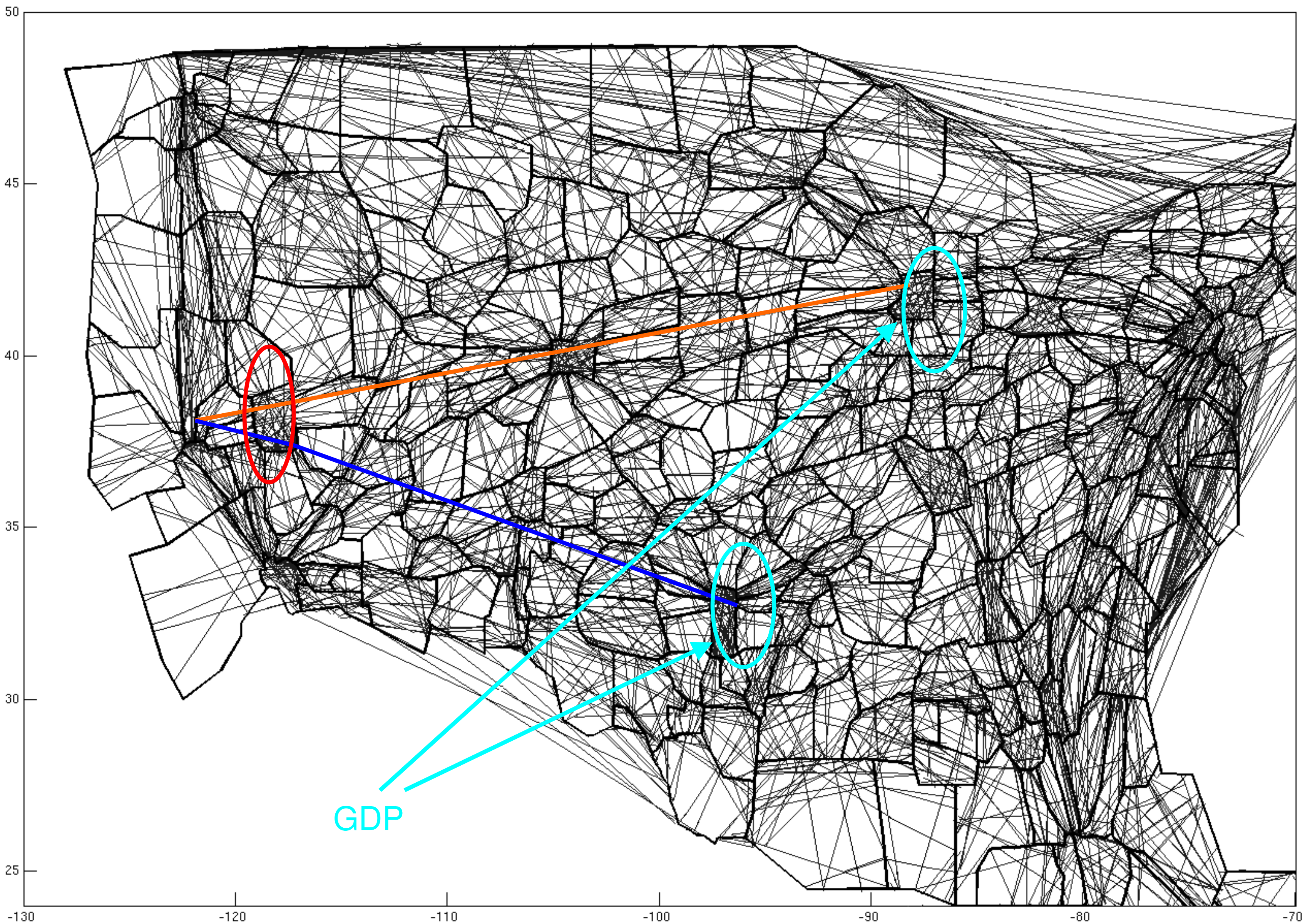
Dual decomposition for NAS-wide TFM



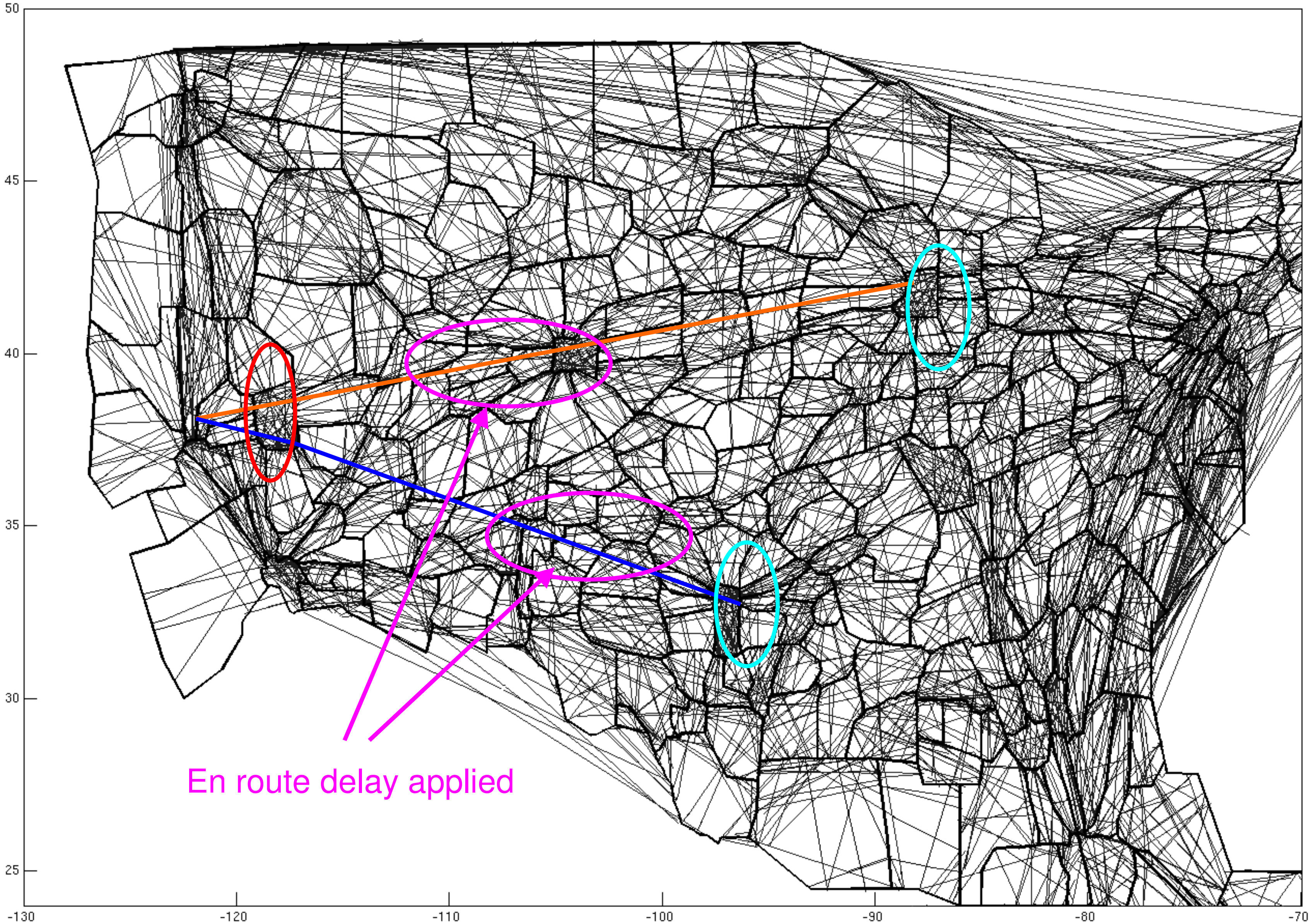
Dual decomposition for NAS-wide TFM



Dual decomposition for NAS-wide TFM

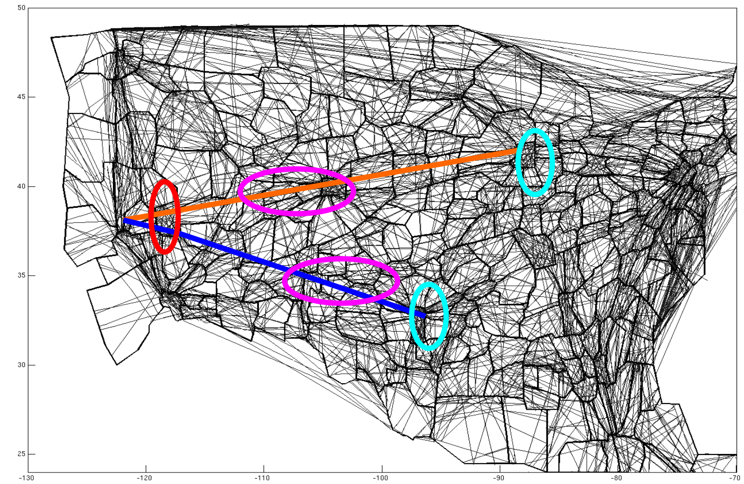


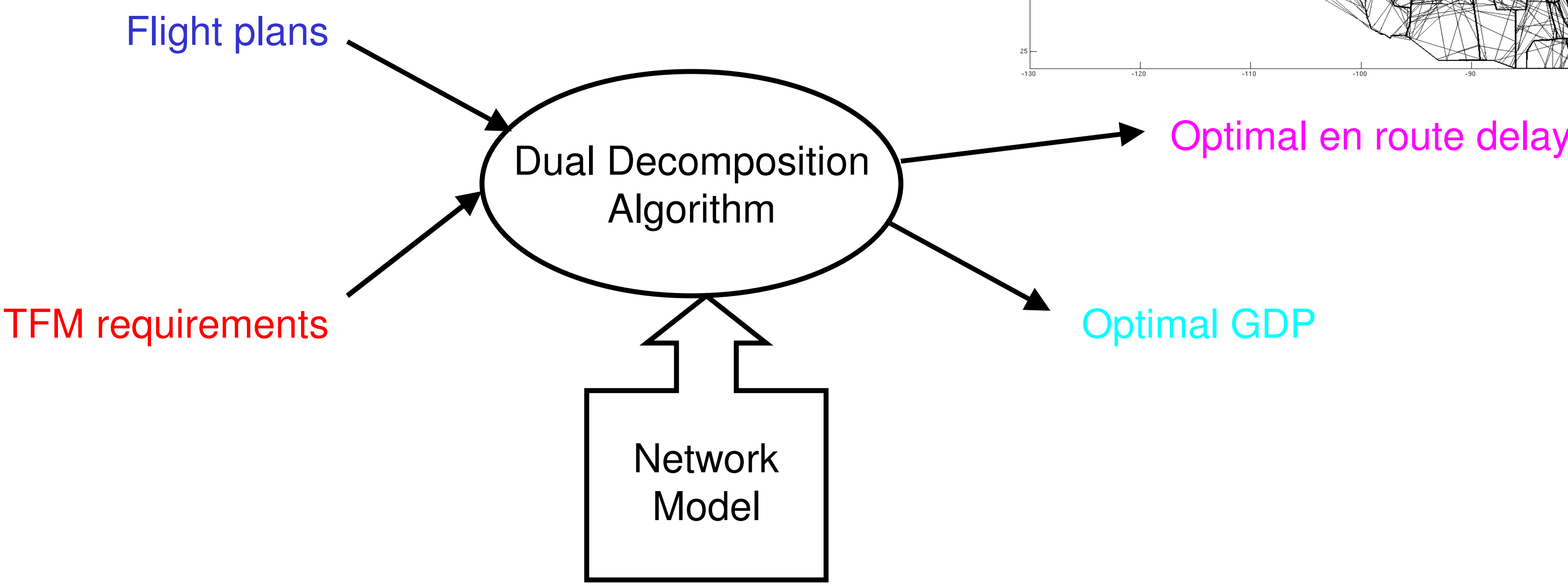
Dual decomposition for NAS-wide TFM



Dual decomposition for NAS-wide TFM

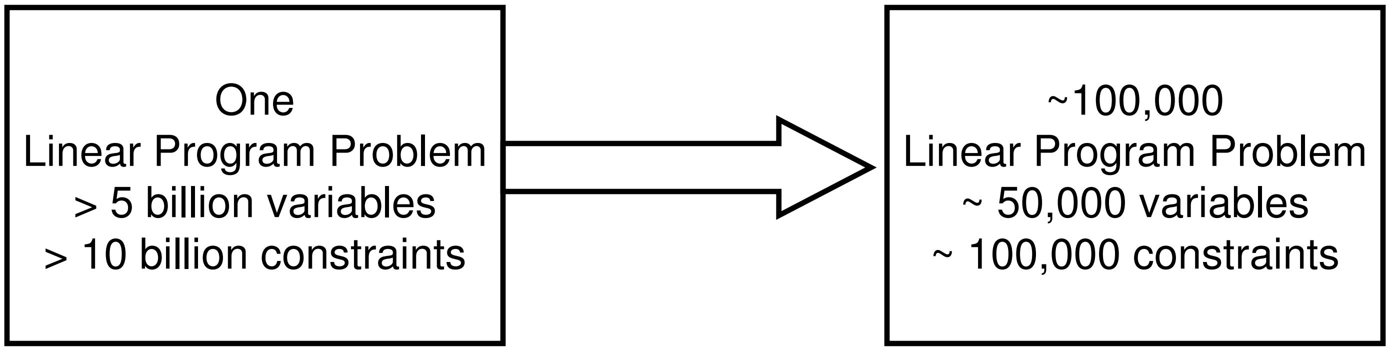
- More than 5 Billion variables
- More than 10 Billion constraints





Dual decomposition

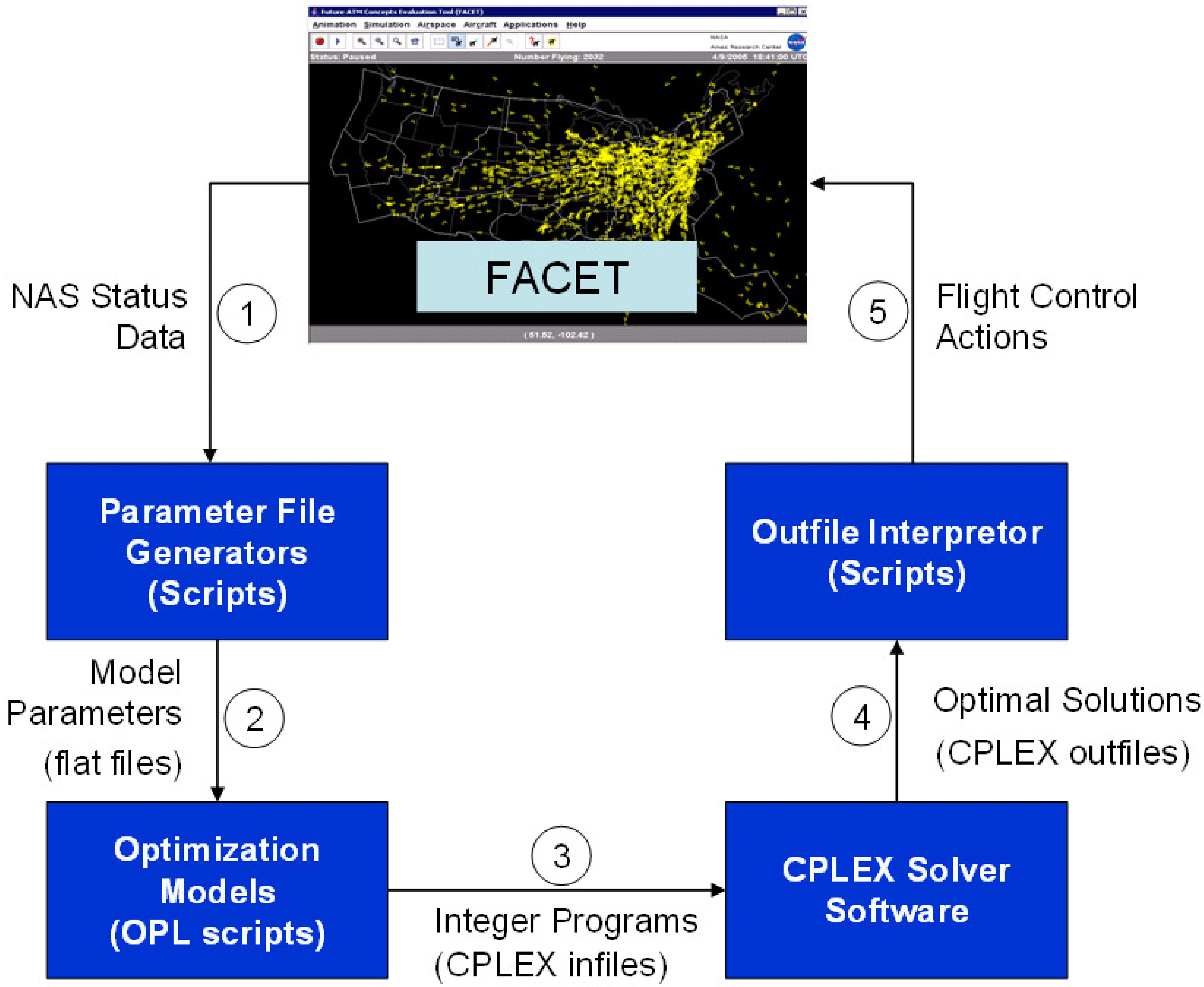
Why is dual decomposition useful?



Not solvable!!!

Solvable in real time!!!

Future steps



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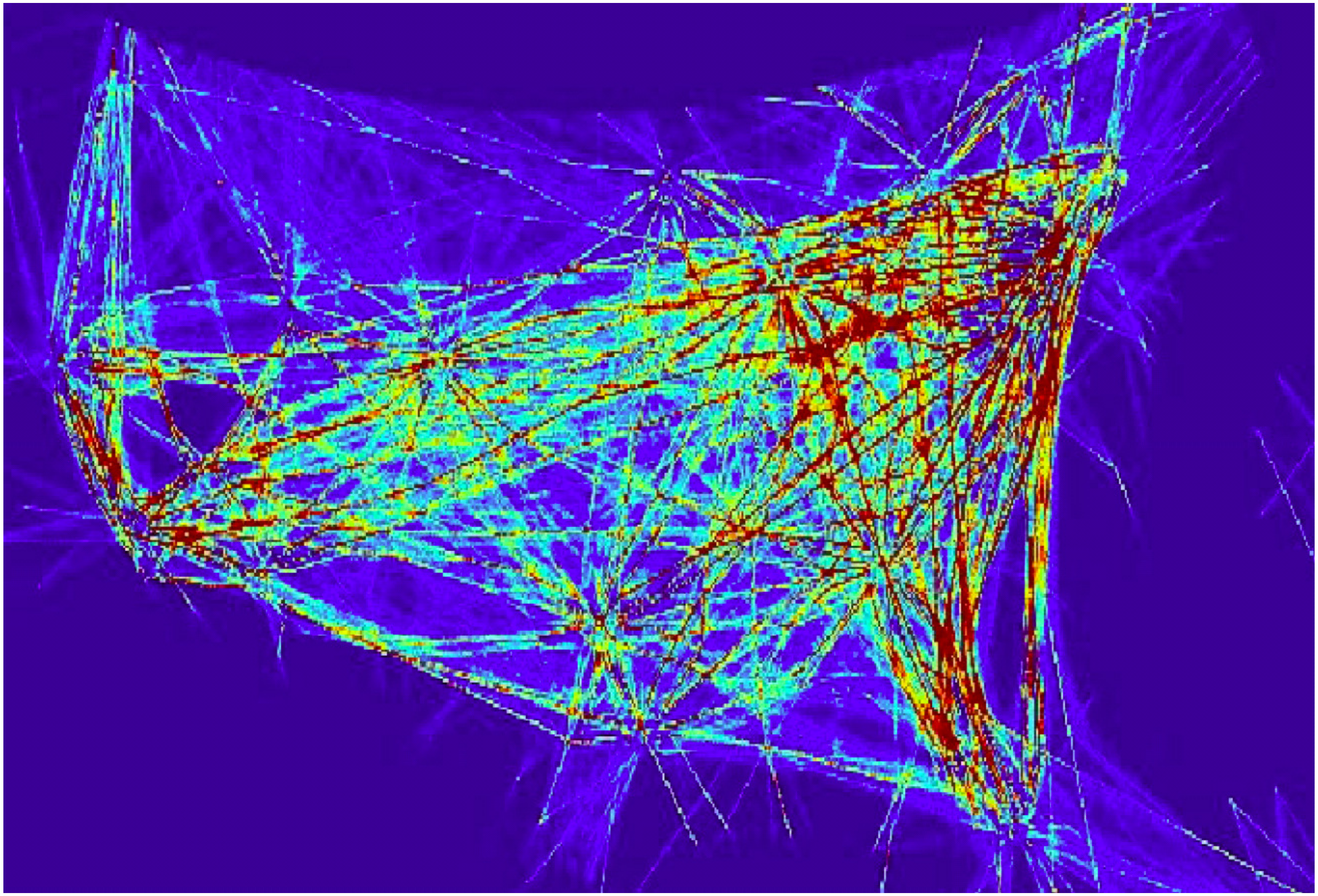
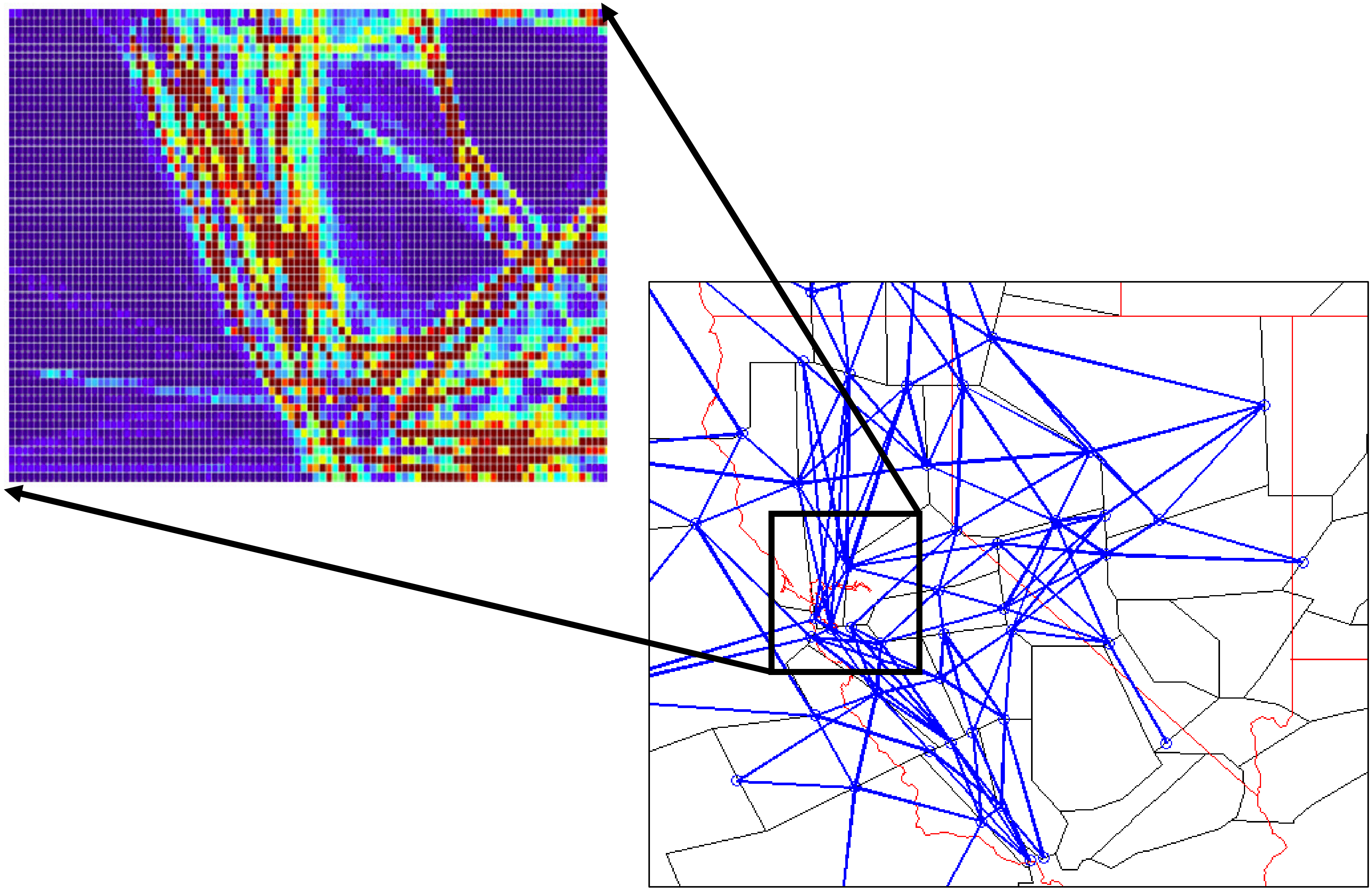
Application 2: dynamic sctorization

Forecast model gives forecast of demand

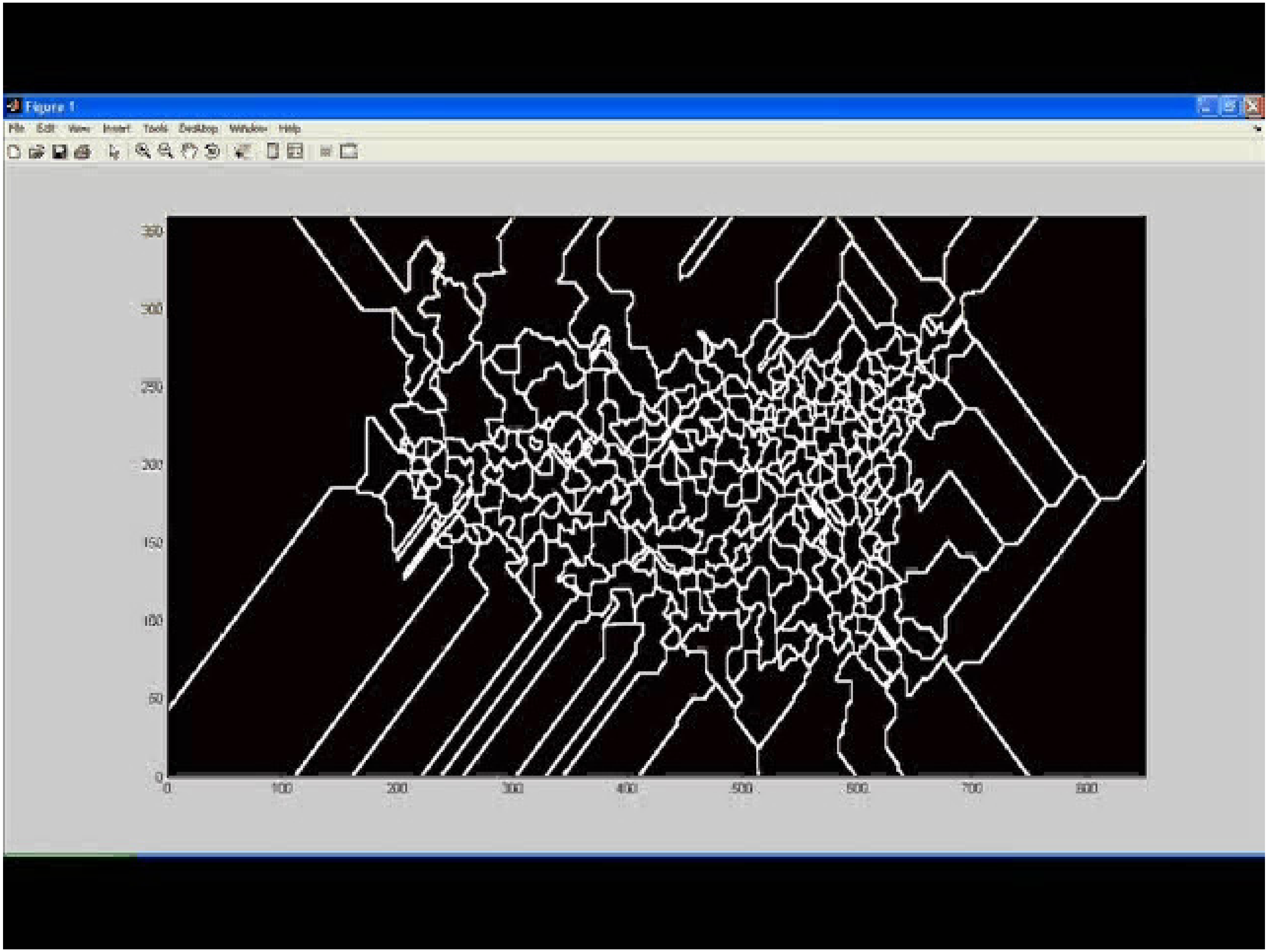


Typical granularity of traffic forecast

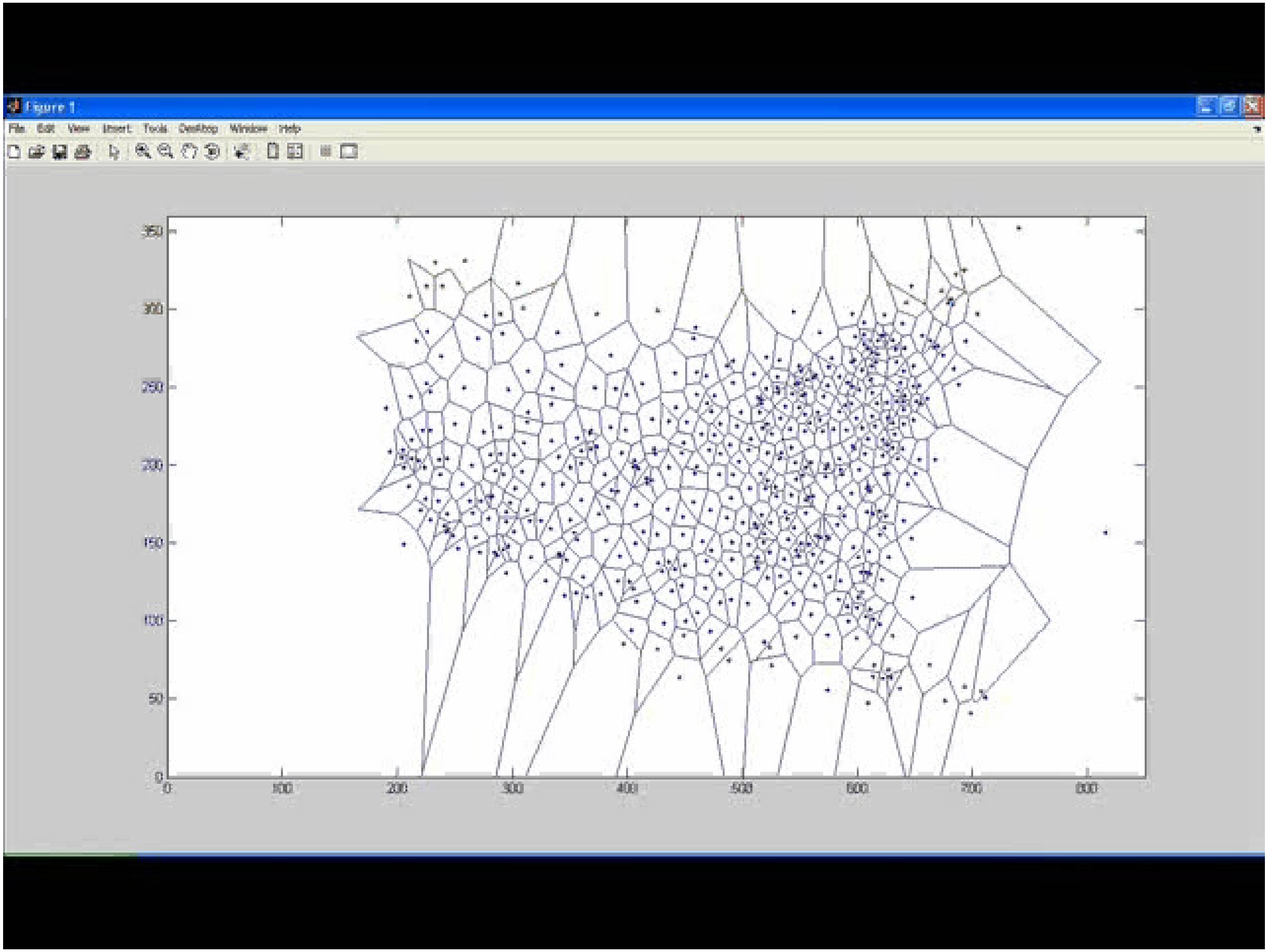


Example: dynamic sectorization (Oakland ARTCC)

NAS-wide Dynamic sectorization



NAS-wide Dynamic sectorization



Future steps

Impact of disturbances on traffic flow in the NAS

- Impact of weather on congestion and delays
- Imbalance between high demand and low capacity
- Towards a scientific definition of capacity
- Analysis of 2D/3D bottlenecks of the NAS

Traffic flow management

- Flow-based routing: lower complexity: performance metrics
- Flow-based CD&R
- Tube traffic (airborne hubs)

Infrastructure and operations

- Dynamic airspace configuration
- Tube / flow traffic

Acknowledgments / Questions

UC Berkeley: Mark Hansen

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FAA: Dave Knorr

CNA: Doug Williamson

Metron: Bob Hoffman