Towards a scientific basis for determining En Route capacity

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Motivation for this work

Importance of en-route capacity (analysis)

Safety, quality of service, delays, efficiency, performance metrics,…

Establishing a scientific base for the envelope of operations

Question 1: reachability in terms of delays

Question 2: reachability in terms of counts or similar metrics

Provide input data for estimating storage capacity

Question 1: how much aircraft / delay can one portion of airspace absorb

Question 2: what is the relation to WITI?

Question 2: space / time definition of capacity

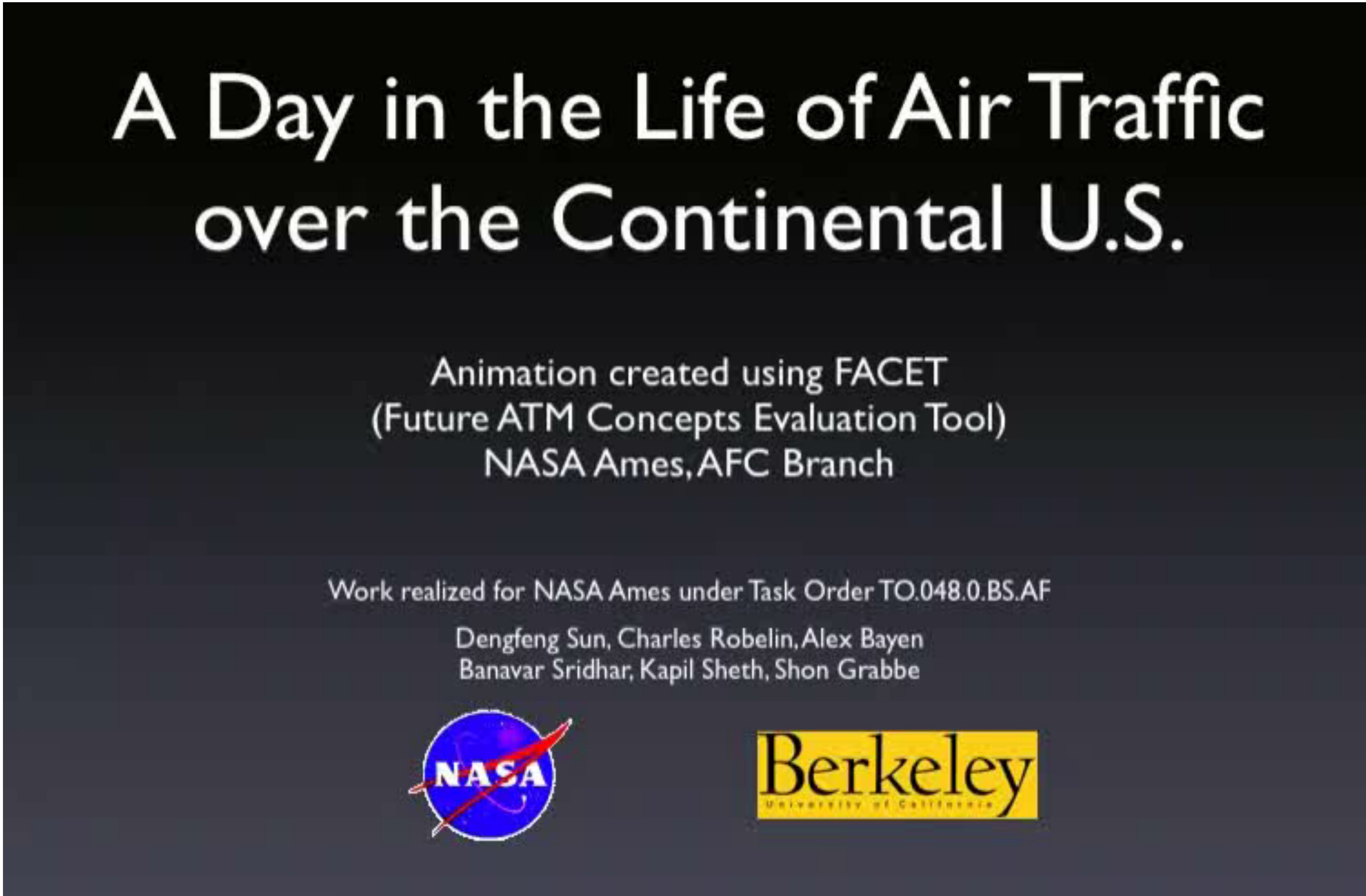
Outline

UC Berkeley

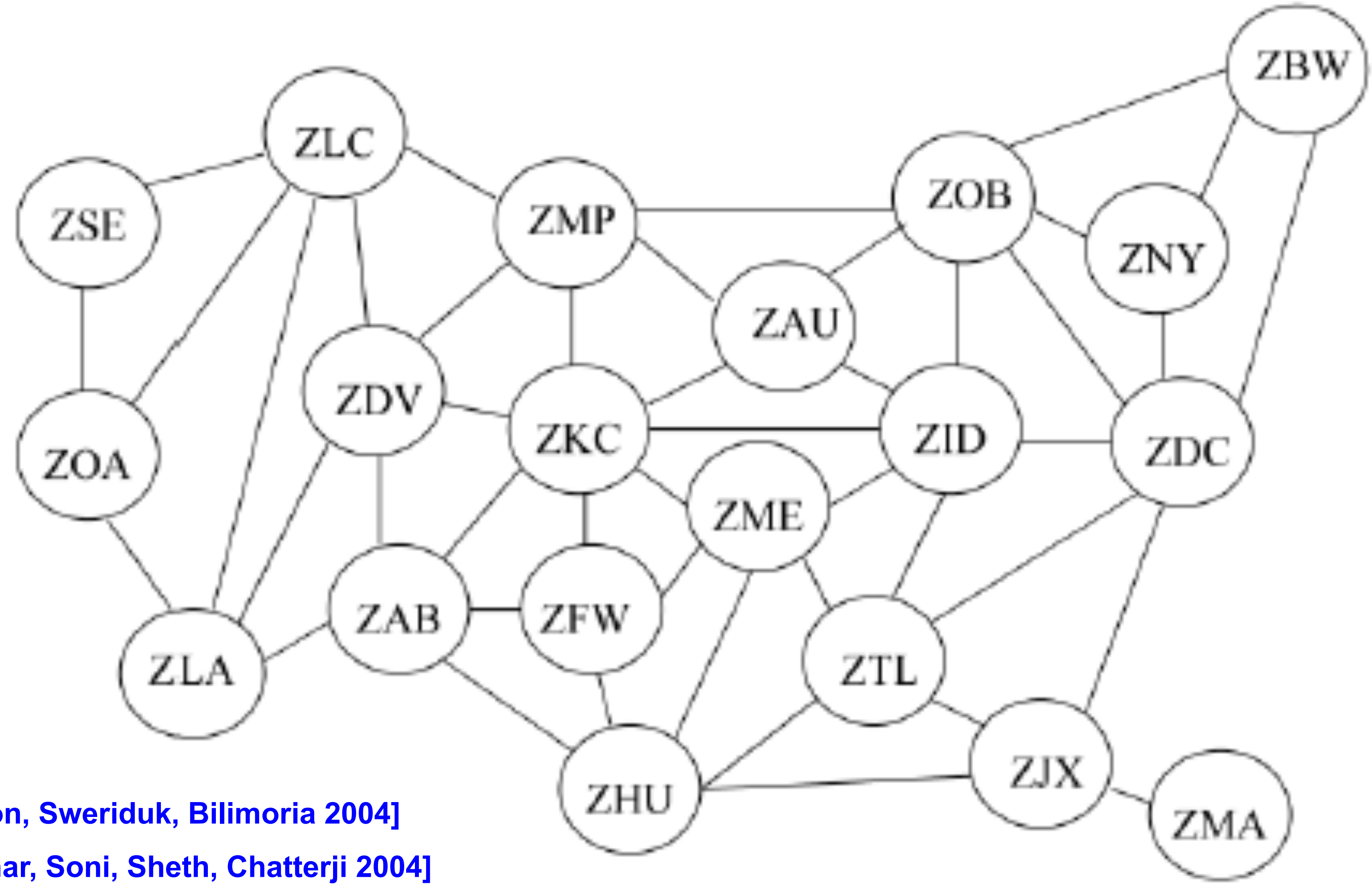
Building blocks towards scientific capacity analysis

1. Systematic identification of the topological features of the National Airspace System (graph theoretic)
2. Automated model building (aggregation procedure)
3. Parameter identification (travel time)
4. Model Analysis (storage)
5. Model validation
6. Capacity assessment (in progress)
    1. Delays that can be absorbed
    2. Aircraft that can be stored
    3. Stability of the storage (backpropagation)

The National Airspace System (NAS)



Air Route Traffic Control Centers in the US

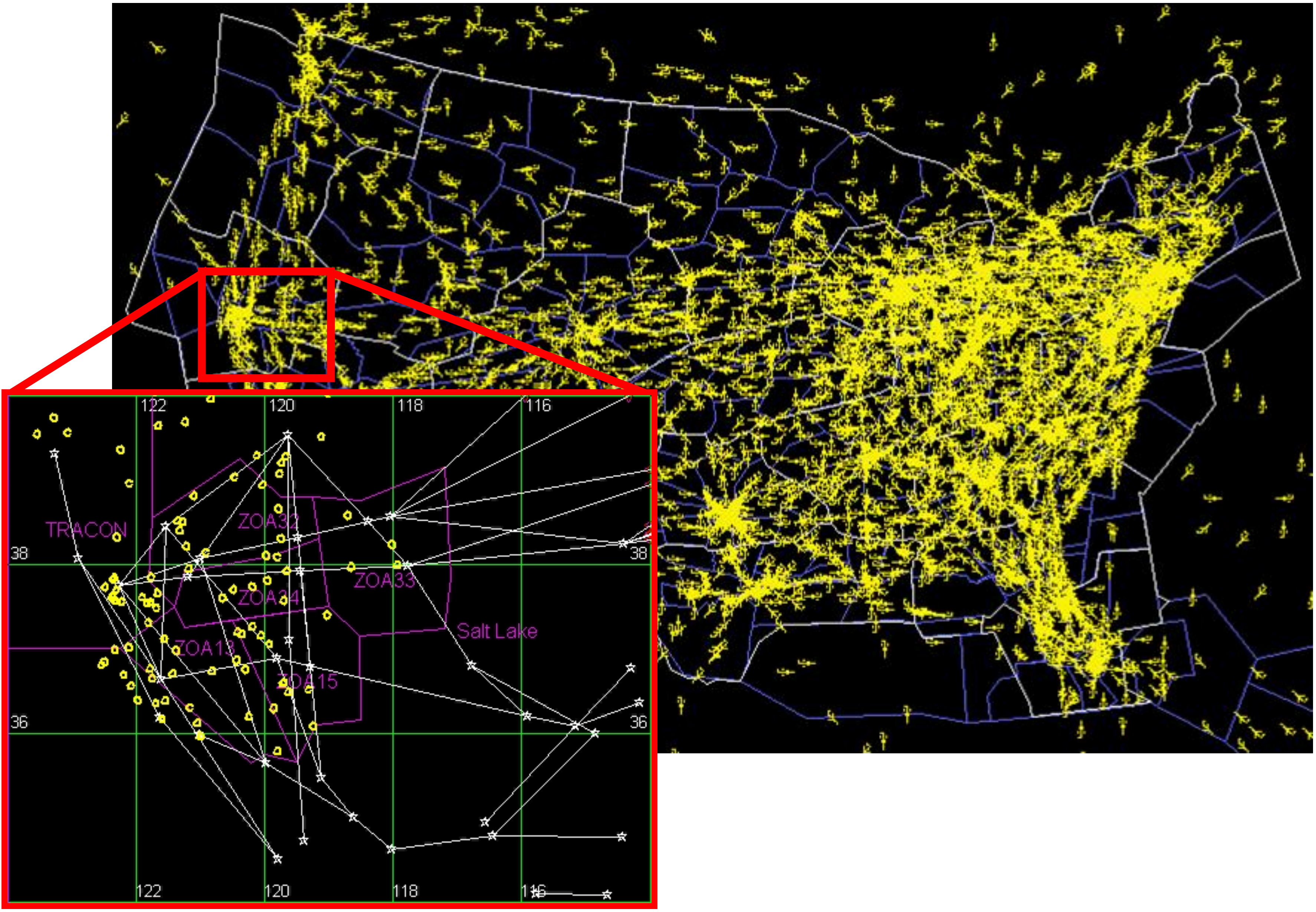


[Menon, Sweriduk, Bilimoria 2004]

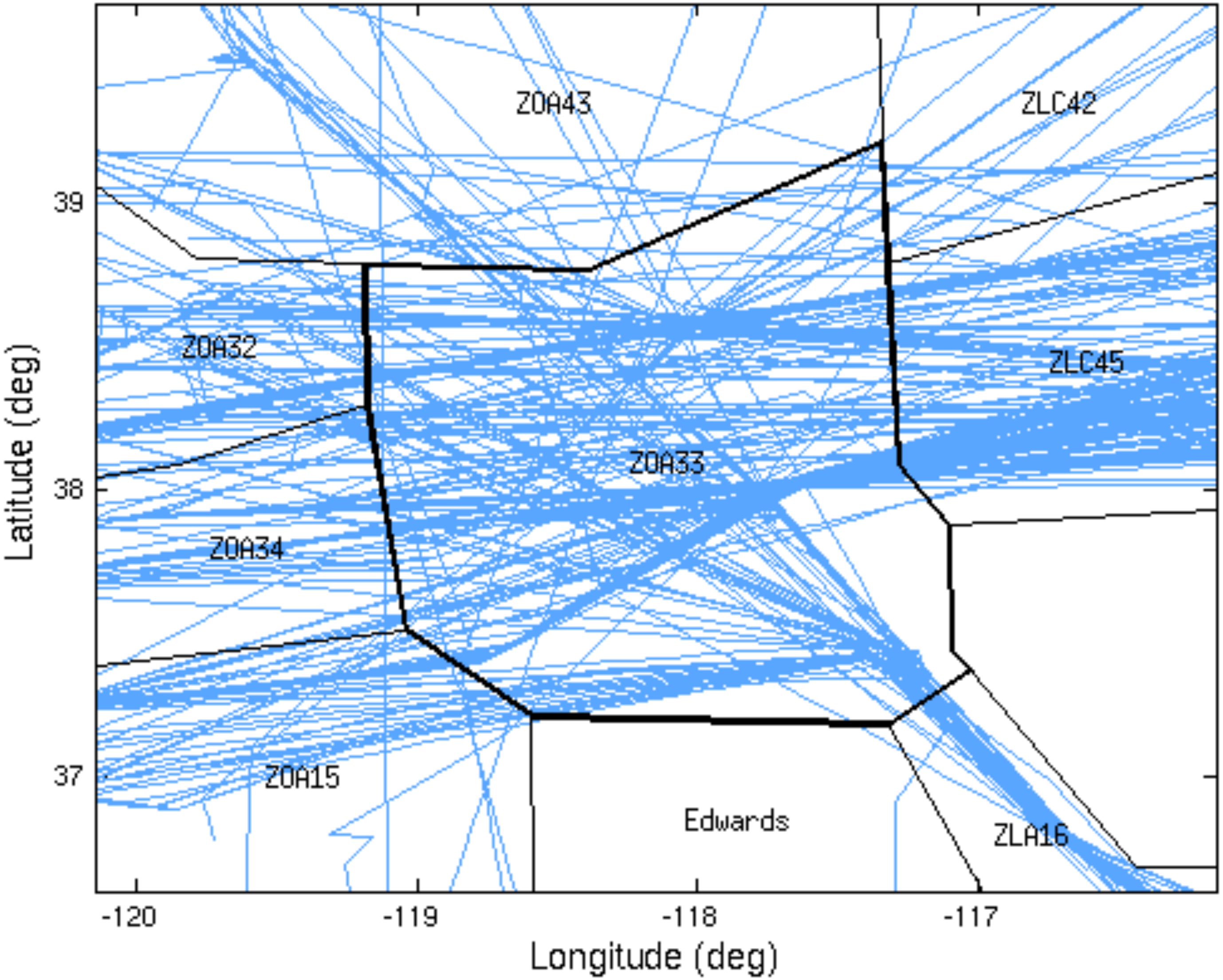
[Sridhar, Soni, Sheth, Chatterji 2004]

[Roy, Sridhar, Verghese 2003], [Devasia, Meyer 2002]

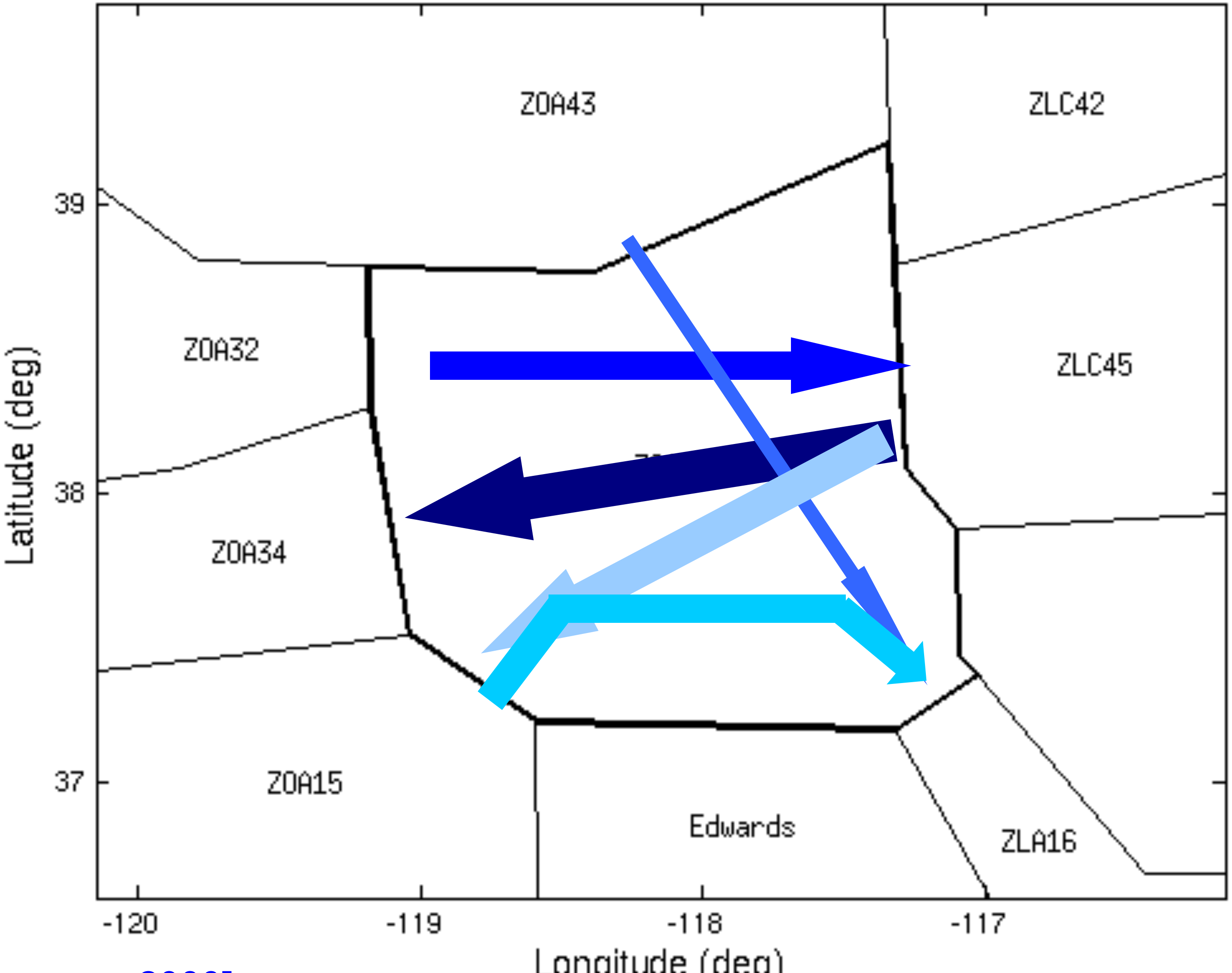
Air Route Traffic Control Center (Oakland)



Flight trajectories

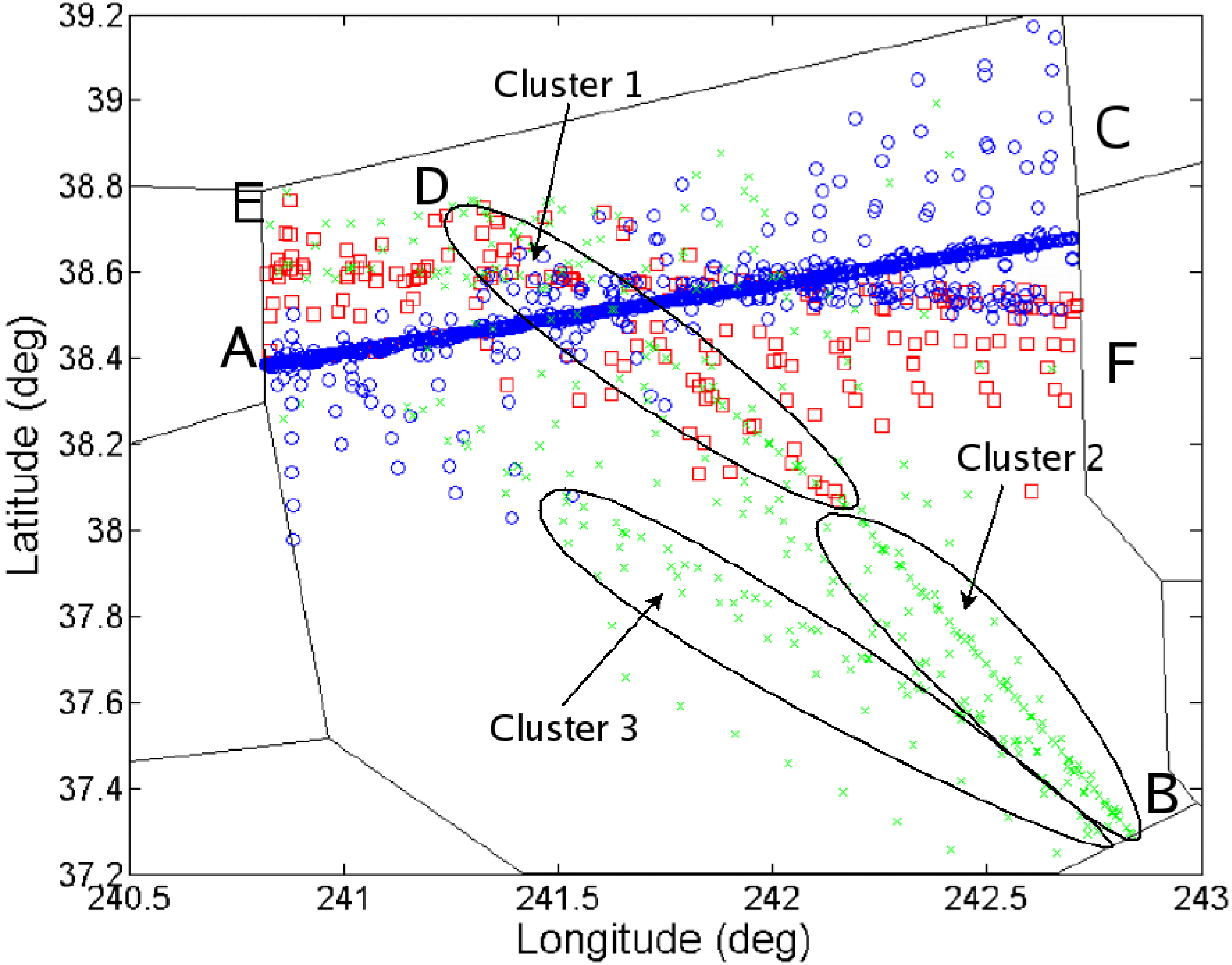


Conceptual goal: graph building

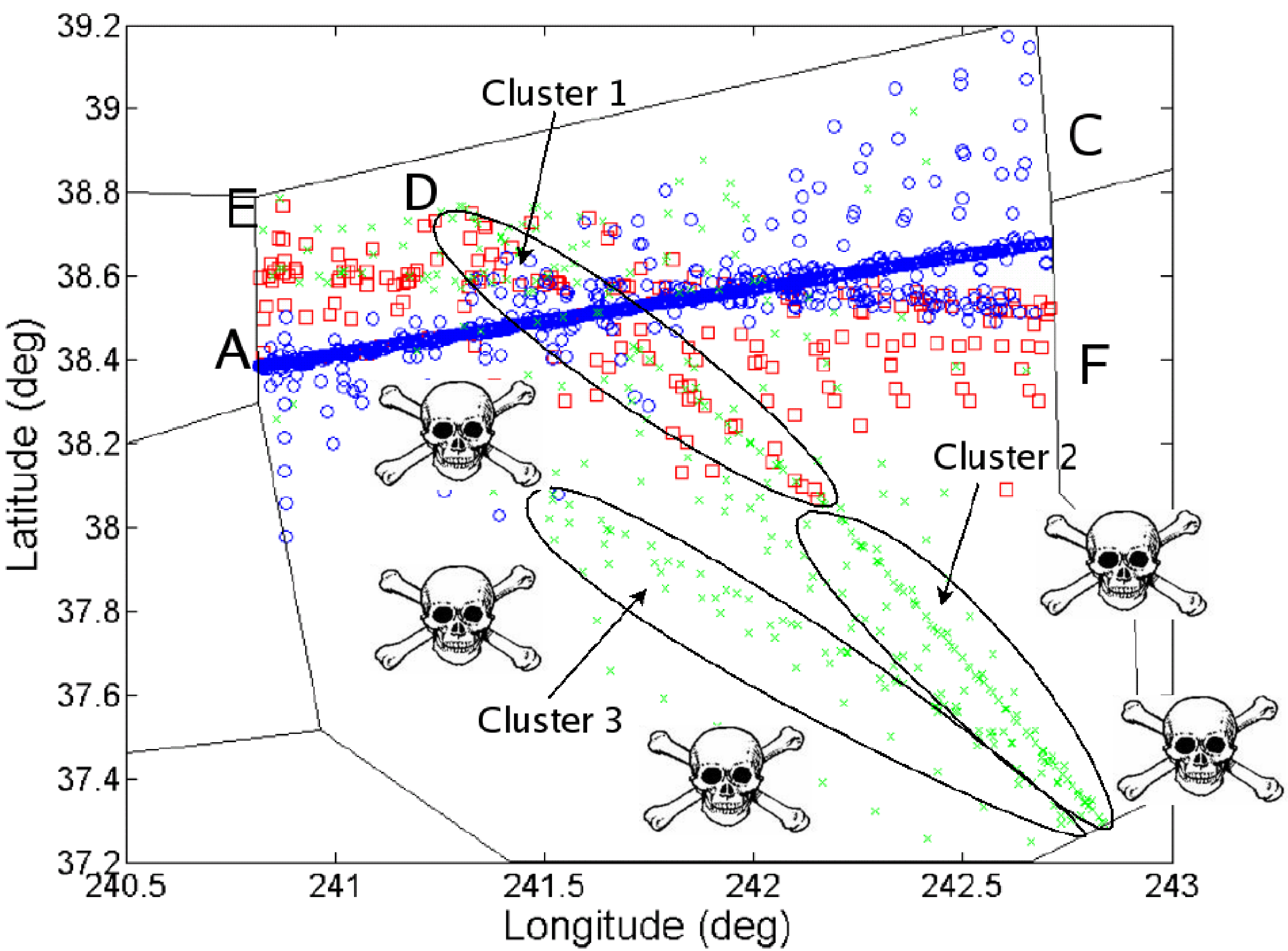


[Histon, Hansman, 2000]

A state of the art clustering algorithm



A state of the art clustering algorithm



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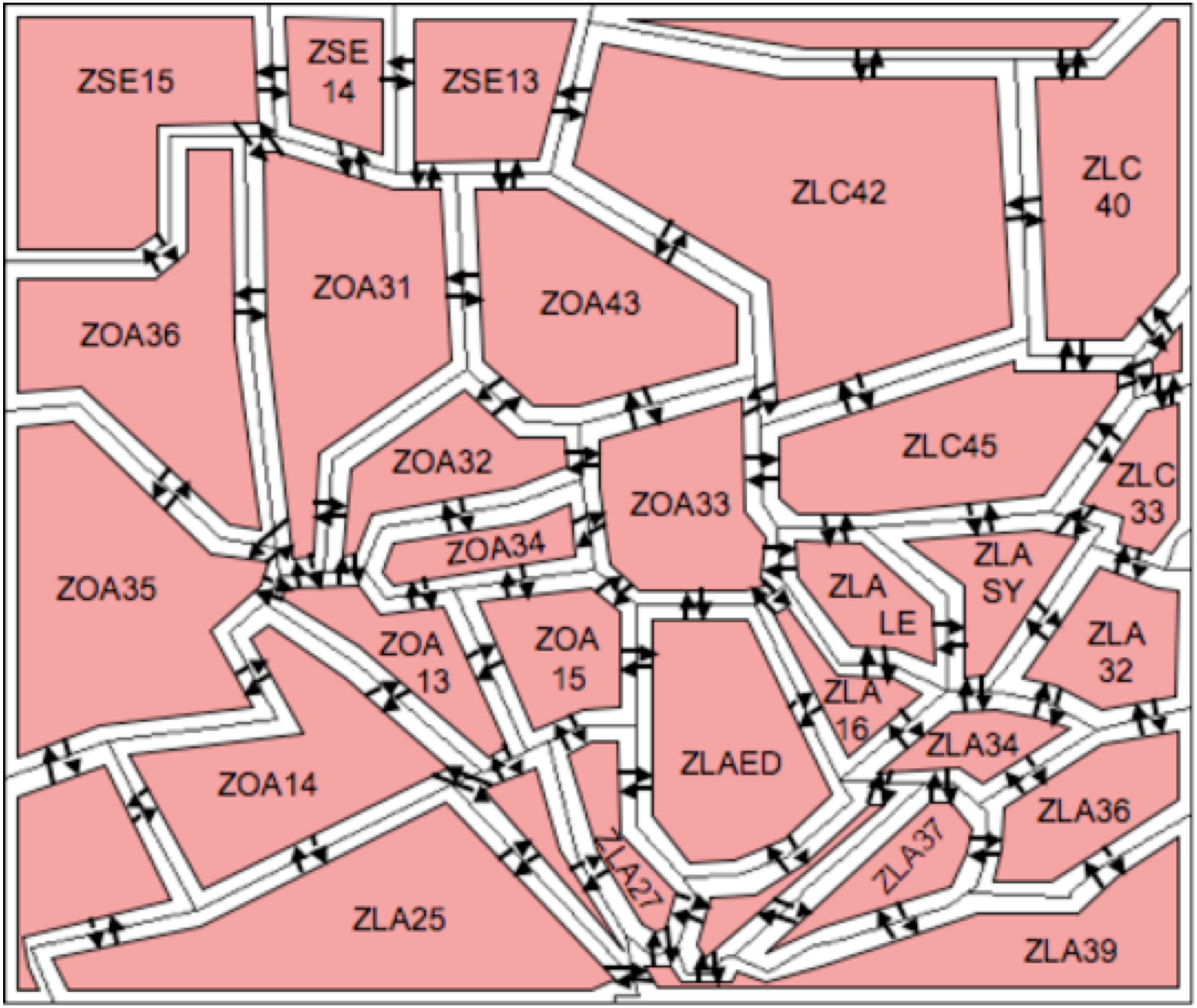
Data analysis procedure

Sequential (automated) algorithm

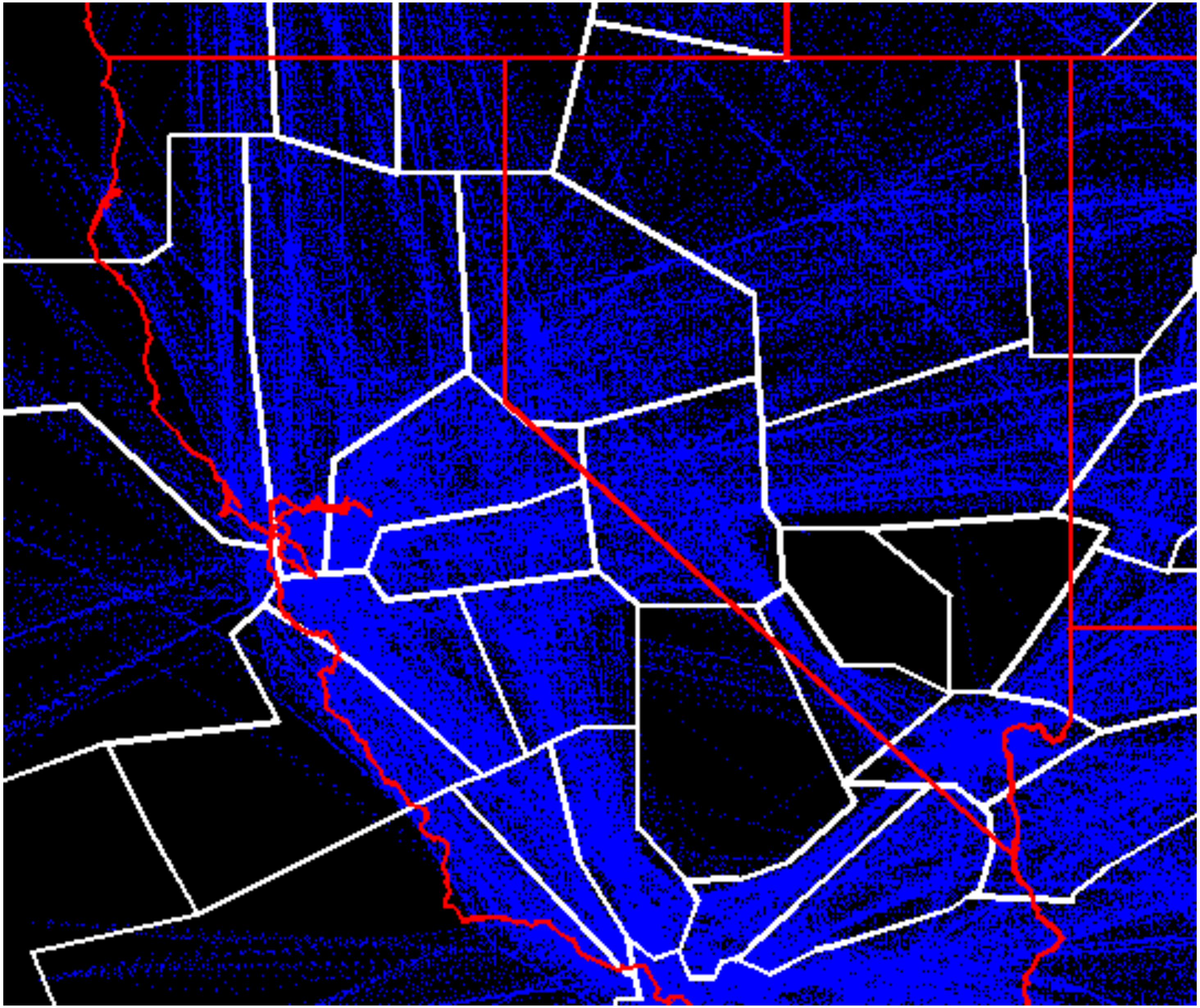
1. Airspace segmentation using sector boundaries
2. Link building using clustering techniques
3. Data aggregation using ASDI/ETMS information (flight plan information)
4. Filtering using LOAs, and observed flow patterns
5. Computation of the aggregate flow pattern features

Output: topology of the flows

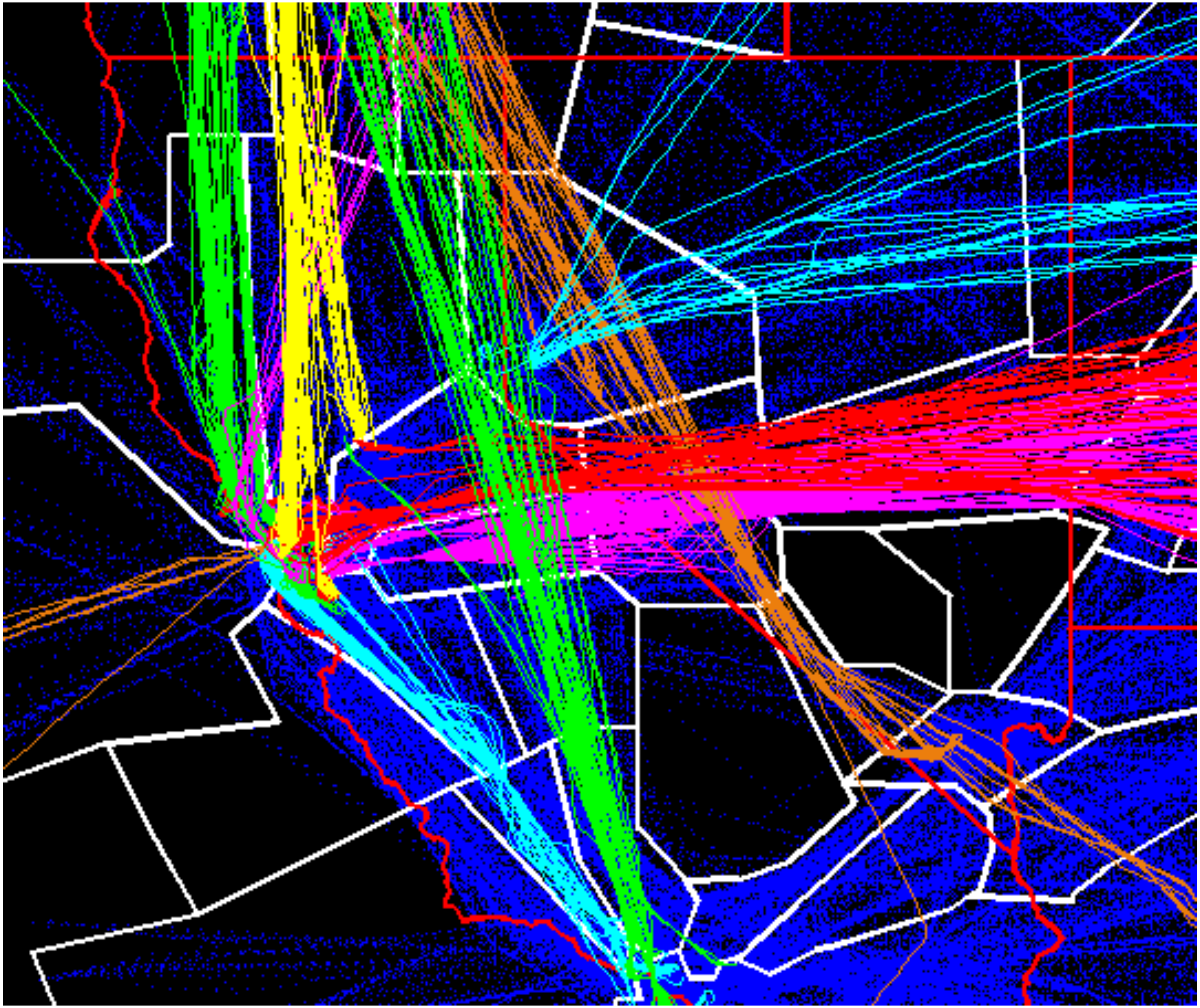
Data analysis procedure



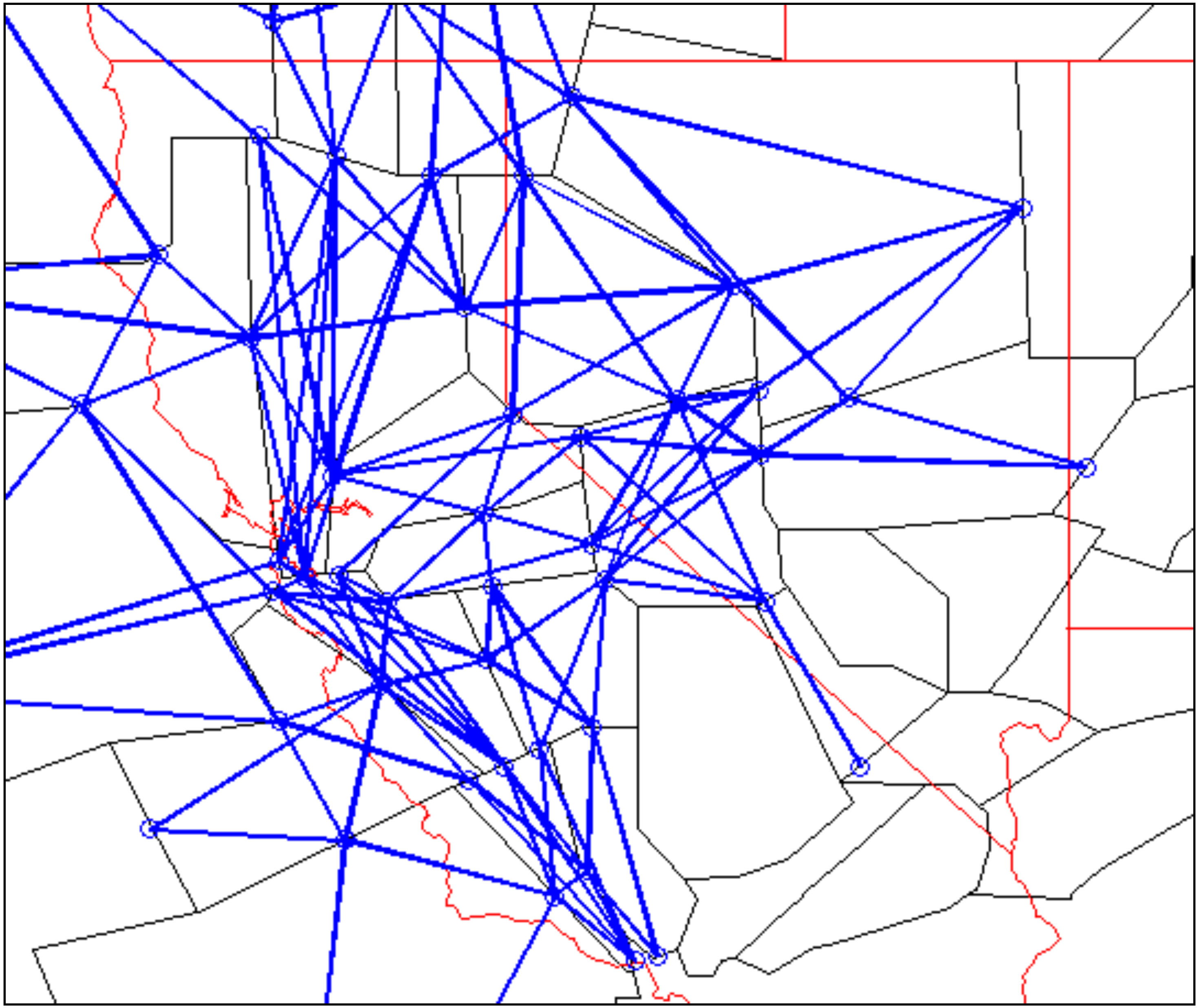
Data analysis procedure



Data analysis procedure



Data analysis procedure



Main question: what is the aggregate dynamics?

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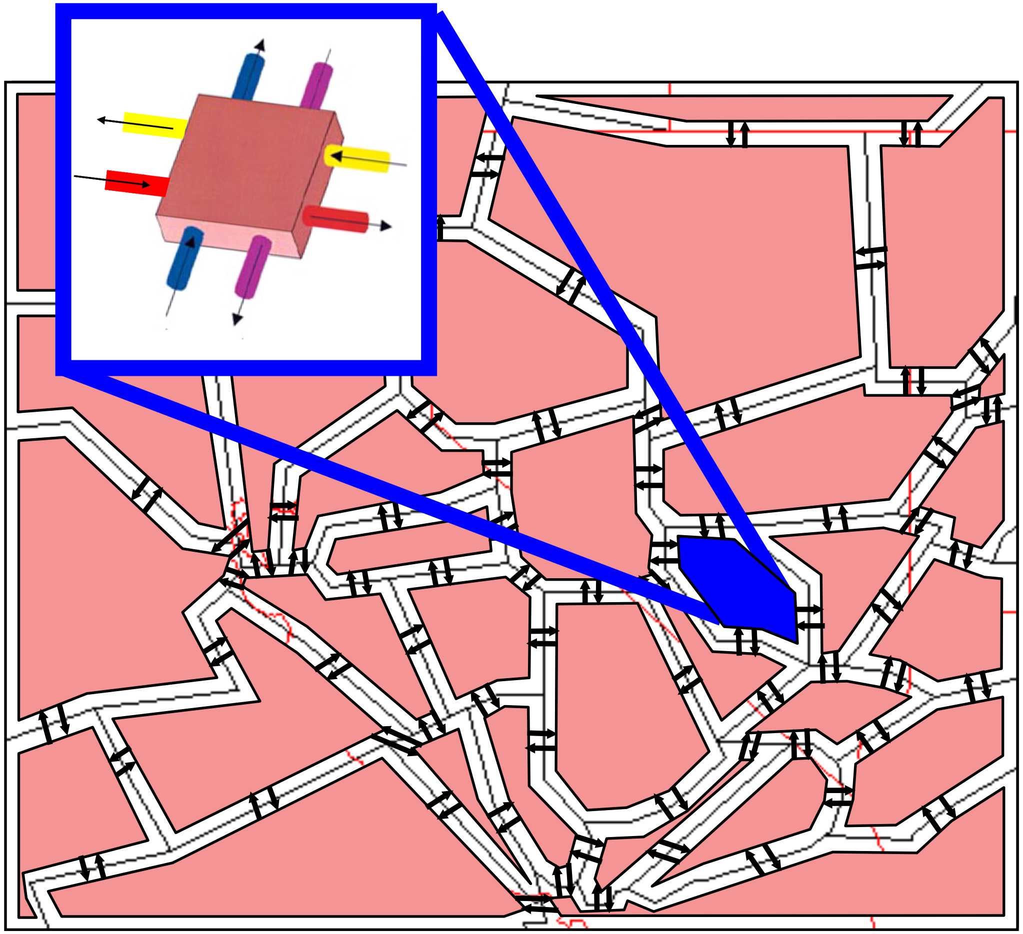
How to relate inflow to outflow (MIMO)?

What is the internal dynamics?

How to use these models to assess capacity?



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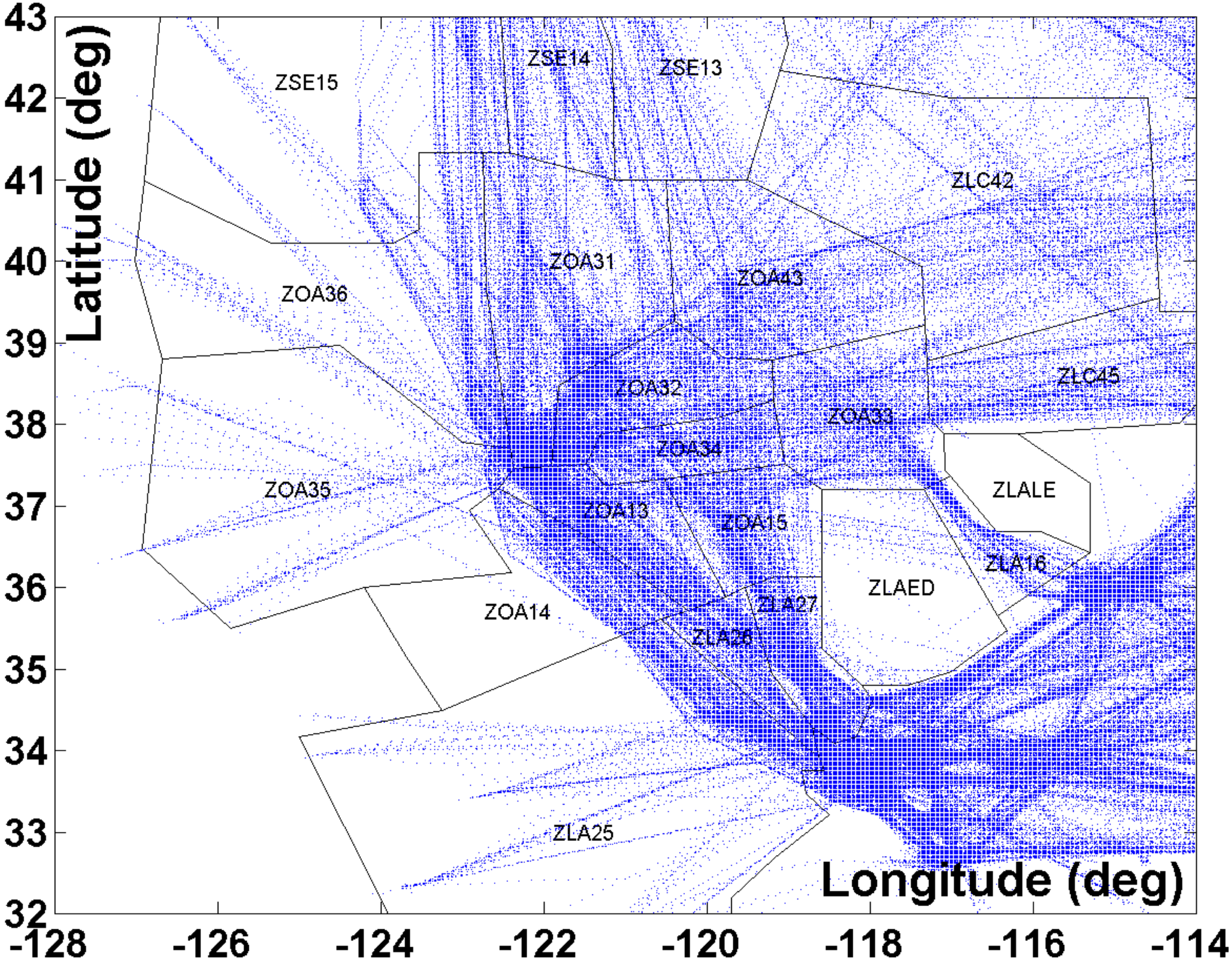
System identification (example: one link)



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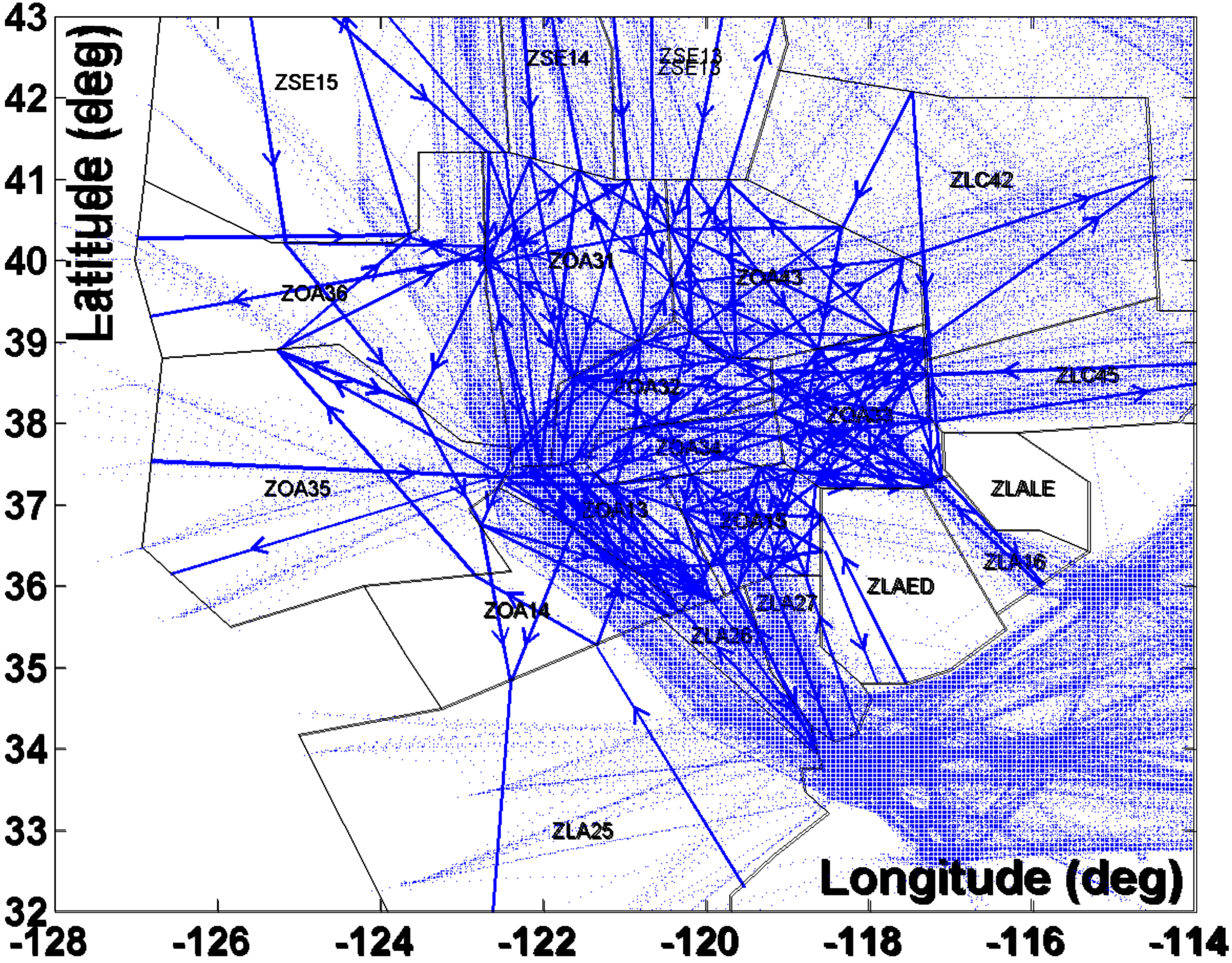
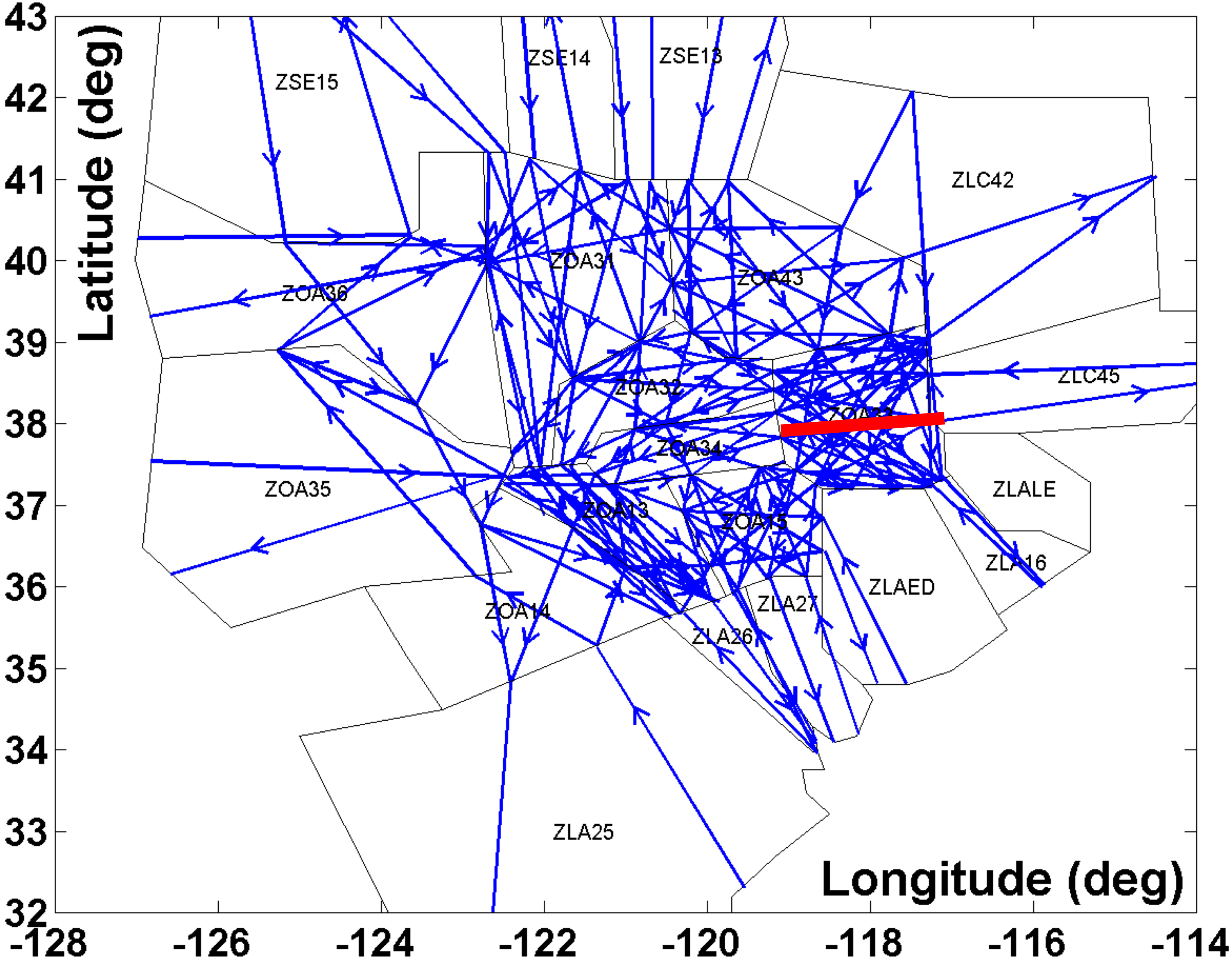


System identification (example: one link)



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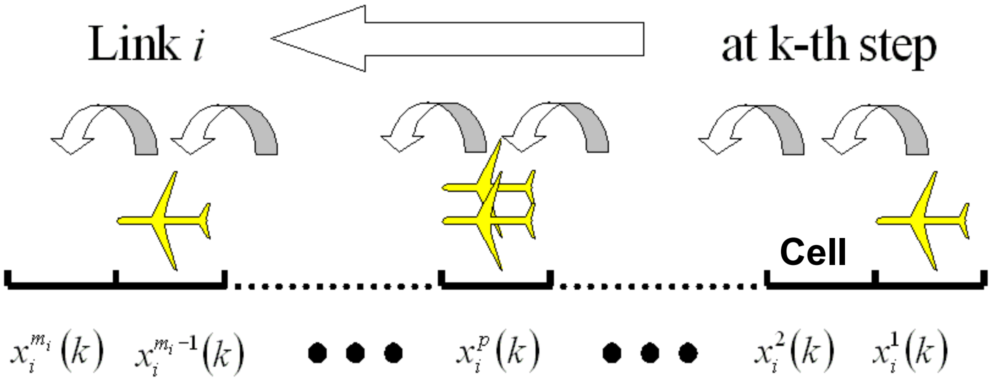
Delay system: Link level

Link level model



k: time step

p: cell number

i: link number

$x_i^p(k)$: number of aircraft

$f_i(k)$: entry input

$u_i(k)$: delay control

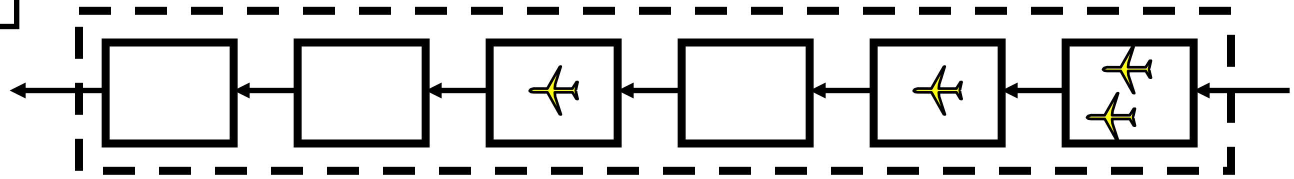
$$x_i(k) = [x_i^{m_i}(k), \ldots, x_i^1(k)]^T$$

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + B_i^f f_i(k) + B_i^u u_i(k) \\ y(k) &= C_i x_i(k) \end{aligned}$$

Illustration of the model: state vector

time step
1

delay system at the link level



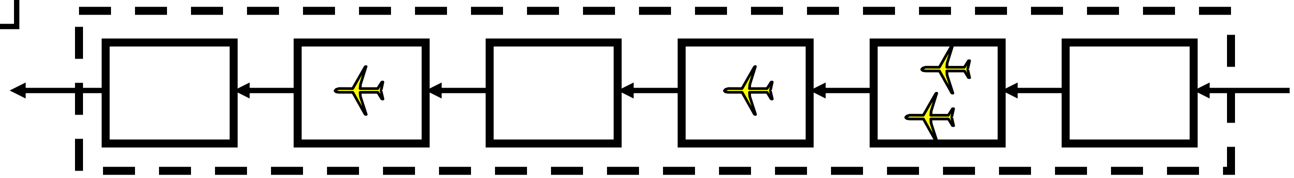
state: cell counts

$$x(1) = \begin{bmatrix} x_6(1) \\ x_5(1) \\ x_4(1) \\ x_3(1) \\ x_2(1) \\ x_1(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

Illustration of the model: states transition

time step
2

delay system at the link level



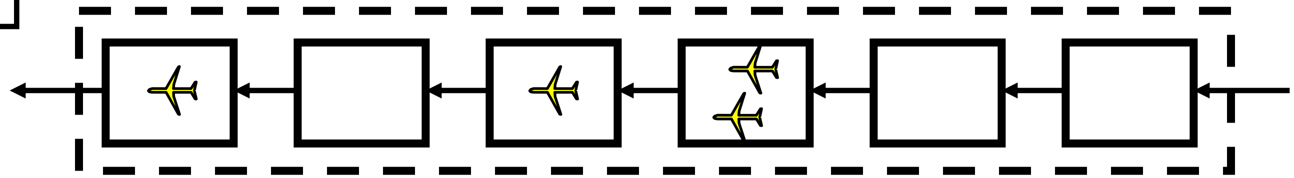
$$
\underbrace{\begin{bmatrix}0\\1\\0\\1\\2\\0\end{bmatrix}}_{x(2)} = \begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\1\\0\\1\\2\end{bmatrix}}_{x(1)} + \begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}}_{u(1)} + \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\underbrace{0}_{f(1)}
$$

state: cell counts

Illustration of the model: states transition

time step
3

delay system at the link level



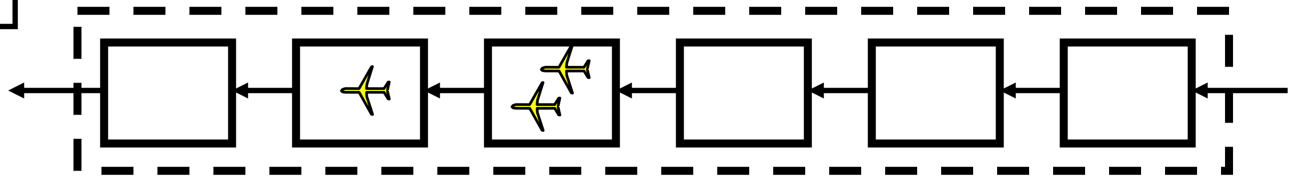
$$\underbrace{\begin{bmatrix}1\\0\\1\\2\\0\\0\end{bmatrix}}_{x(3)} = \begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}0\\1\\0\\1\\2\\0\end{bmatrix}}_{x(2)} + \begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}}_{u(2)} + \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\underbrace{0}_{f(2)}$$

Illustration of the model: states transition

time step

4

delay system at the link level

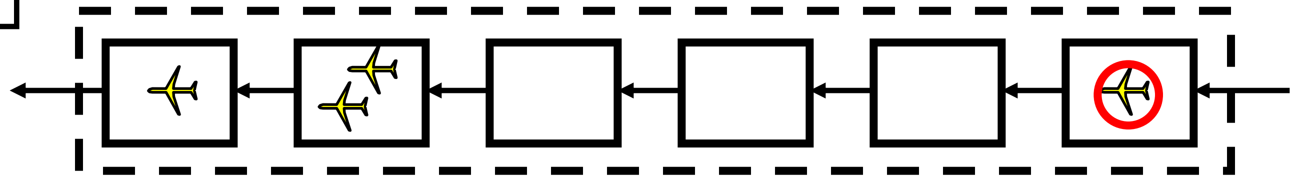


$$\underbrace{\begin{bmatrix}0\\1\\2\\0\\0\\0\end{bmatrix}}_{x(4)} = \begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}1\\0\\1\\2\\0\\0\end{bmatrix}}_{x(3)} + \begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}}_{u(3)} + \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\underbrace{0}_{f(3)}$$

Illustration of the model: entry input

time step
5

delay system at the link level



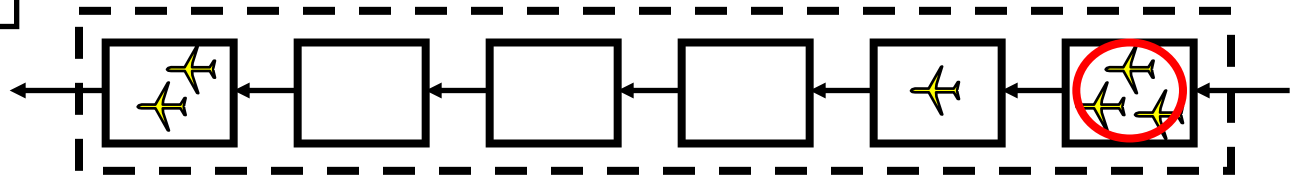
$$\underbrace{\begin{bmatrix}1\\2\\0\\0\\0\\1\end{bmatrix}}_{x(5)} = \begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}0\\1\\2\\0\\0\\0\end{bmatrix}}_{x(4)} + \begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}}_{u(4)} + \underbrace{\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}}_{f(4)} \quad (1)$$

input to the link (forcing) → $f(4)$

Illustration of the model: entry input

time step
6

delay system at the link level

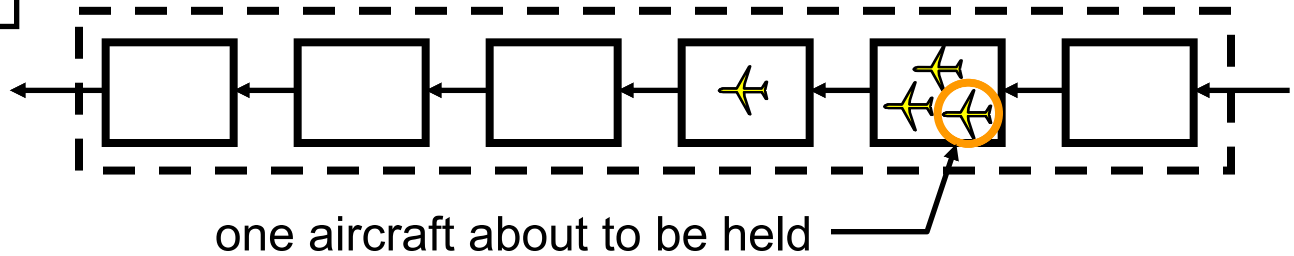


$$\underbrace{\begin{bmatrix}2\\0\\0\\0\\1\\3\end{bmatrix}}_{x(6)} = \begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}1\\2\\0\\0\\0\\1\end{bmatrix}}_{x(5)} + \begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}}_{u(5)} + \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\underbrace{3}_{f(5)}$$

Illustration of the model: delay control

time step
7

delay system at the link level



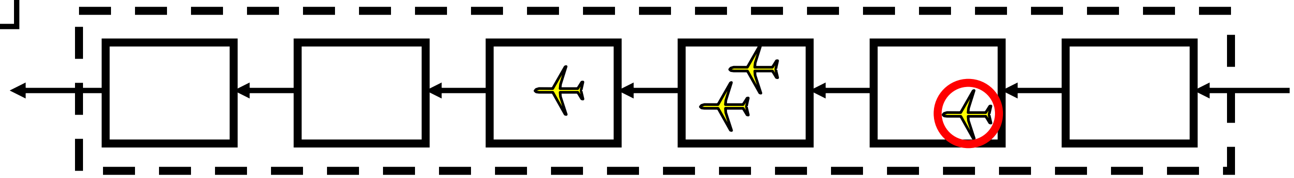
$$\underbrace{\begin{bmatrix}0\\0\\0\\1\\3\\0\end{bmatrix}}_{x(7)} = \begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}2\\0\\0\\0\\1\\3\end{bmatrix}}_{x(6)} + \begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}}_{u(6)} + \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\underbrace{0}_{f(6)}$$

Illustration of the model: delay control

time step

8

delay system at the link level



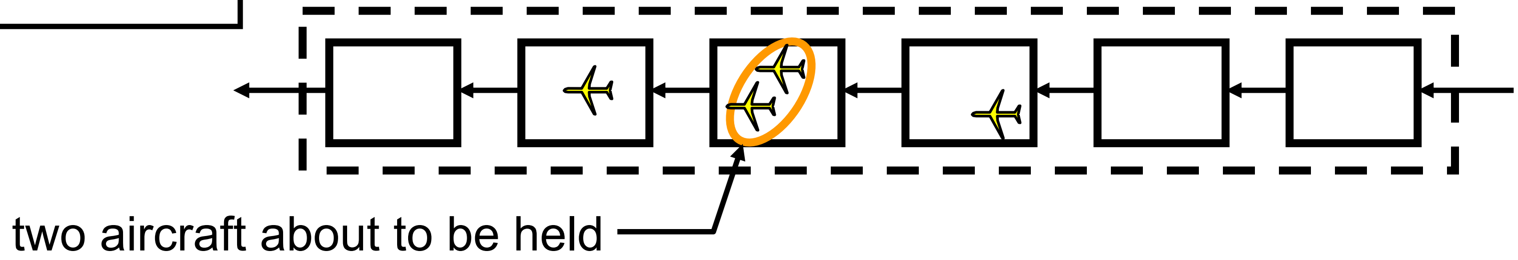
$$\underbrace{\begin{bmatrix}0\\0\\1\\2\\1\\0\end{bmatrix}}_{x(8)} = \begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\0\\1\\3\\0\end{bmatrix}}_{x(7)} + \begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix}}_{u(7)} + \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\underbrace{0}_{f(7)}$$

control input

Illustration of the model: delay control

time step
9

delay system at the link level

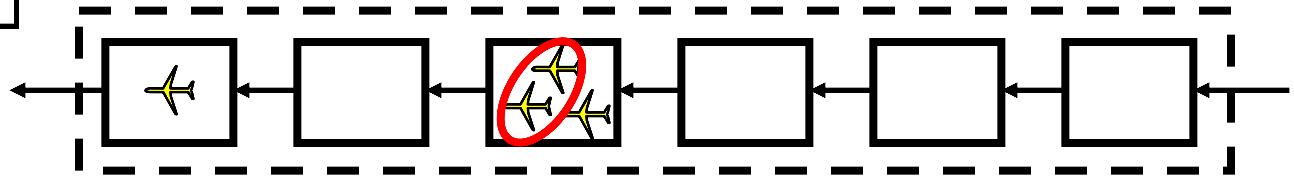


$$\underbrace{\begin{bmatrix}0\\1\\2\\1\\0\\0\end{bmatrix}}_{x(9)} = \begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\1\\2\\1\\0\end{bmatrix}}_{x(8)} + \begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}}_{u(8)} + \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\underbrace{0}_{f(8)}$$

Illustration of the model: delay control

time step
10

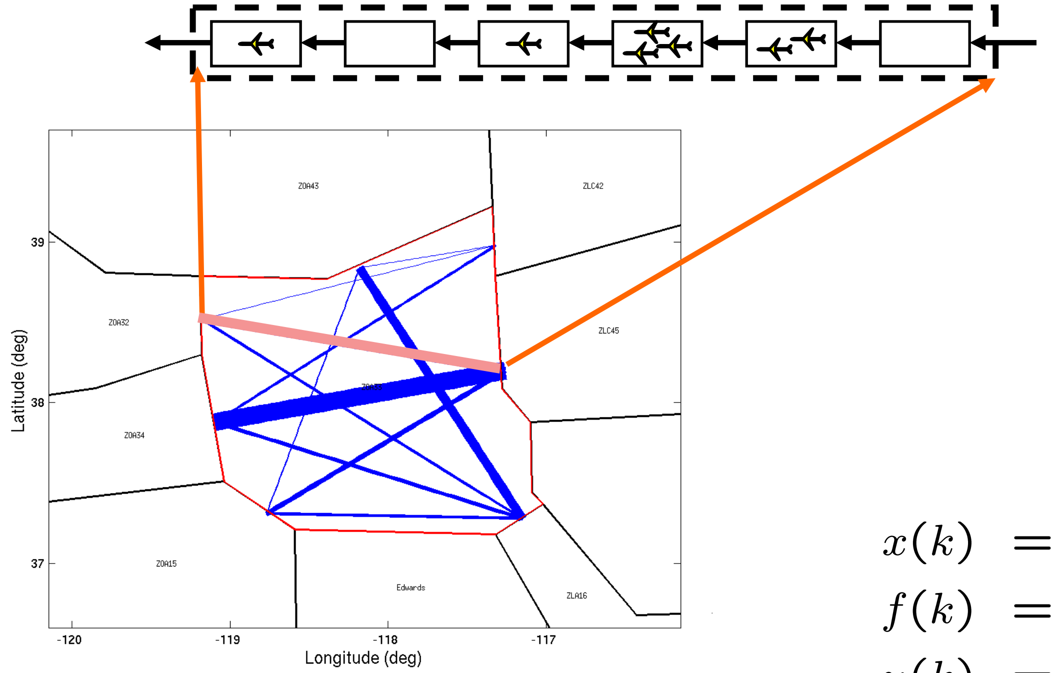
delay system at the link level



$$\underbrace{\begin{bmatrix}1\\0\\3\\0\\0\\0\end{bmatrix}}_{x(10)} = \begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\underbrace{\begin{bmatrix}0\\1\\2\\1\\9\\0\end{bmatrix}}_{x(9)} + \begin{bmatrix}1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix}\underbrace{\begin{bmatrix}0\\0\\2\\0\\0\\0\end{bmatrix}}_{u(9)} + \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\underbrace{0}_{f(9)}$$

Delay system: Sector level

Sector level



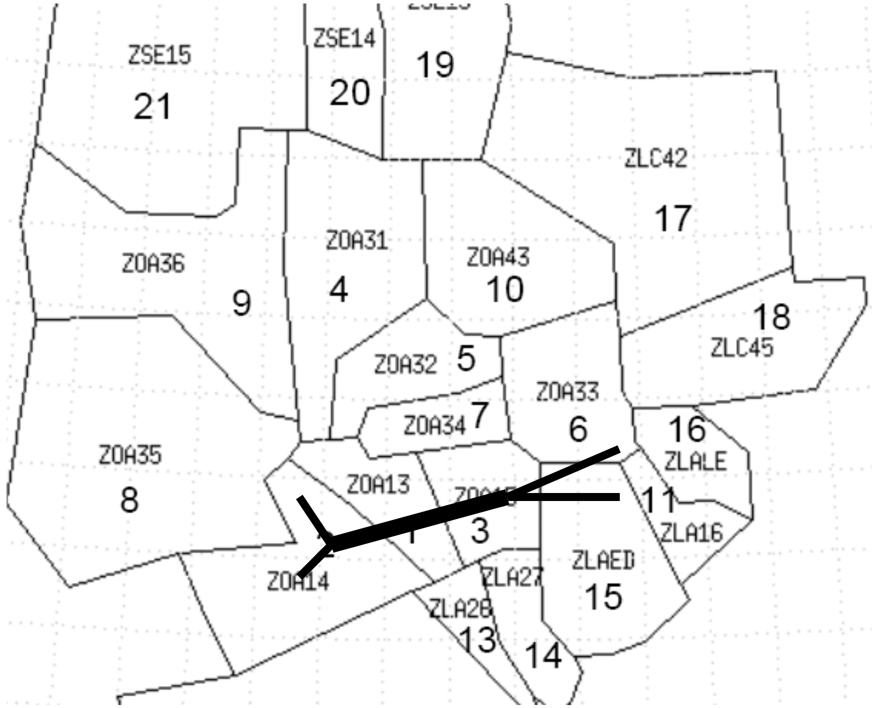
$$x(k+1) = Ax(k) + B^f f(k) + B^u u(k)$$
$$y(k) = Cx(k)$$

$$x(k) = [x_n(k), ..., x_1(k)]^T$$
$$f(k) = [f_n(k), ..., f_1(k)]^T$$
$$u(k) = [u_n(k), ..., u_1(k)]^T$$
$$A = \text{diag}(A_n, \ldots, A_1)$$
$$B^f = \text{diag}(B^f_n, \ldots, B^f_1)$$
$$B^u = \text{diag}(B^u_n, \ldots, B^u_1)$$

[Robelin, Sun, Wu, Bayen 2006]

Delay system: Center (ARTCC) level

Center level:



$$x(t+1) = A\, x(t) + B^u\, u(t) + B^f\, f(t)$$

[Robelin, Sun, Wu, Bayen 2006]

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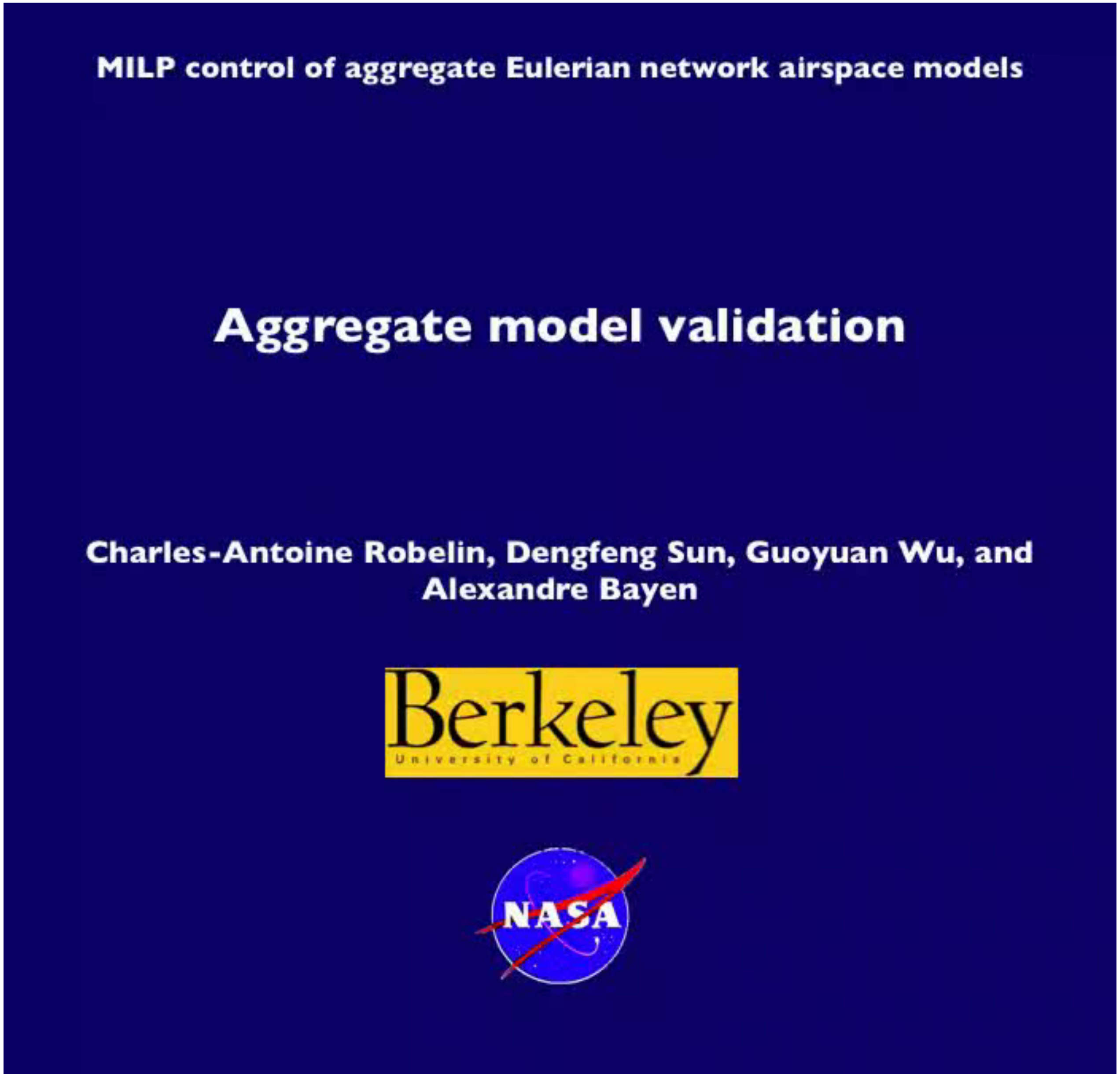
Aggregate model validation



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IP: Formulation

min: $\sum_{k=0}^{N} c^T x_k$

subject to:

$$Ex_k + Lu_k \leq M, \; k \in \{0, \cdots, N-1\}$$

$$x_{k+1} = Ax_k + B^f f_k + B^u u_k, \; k \in \{0, \cdots, N-1\}$$

$$x_0 = B^f f_0$$

N: number of time steps

c: vector of 1's

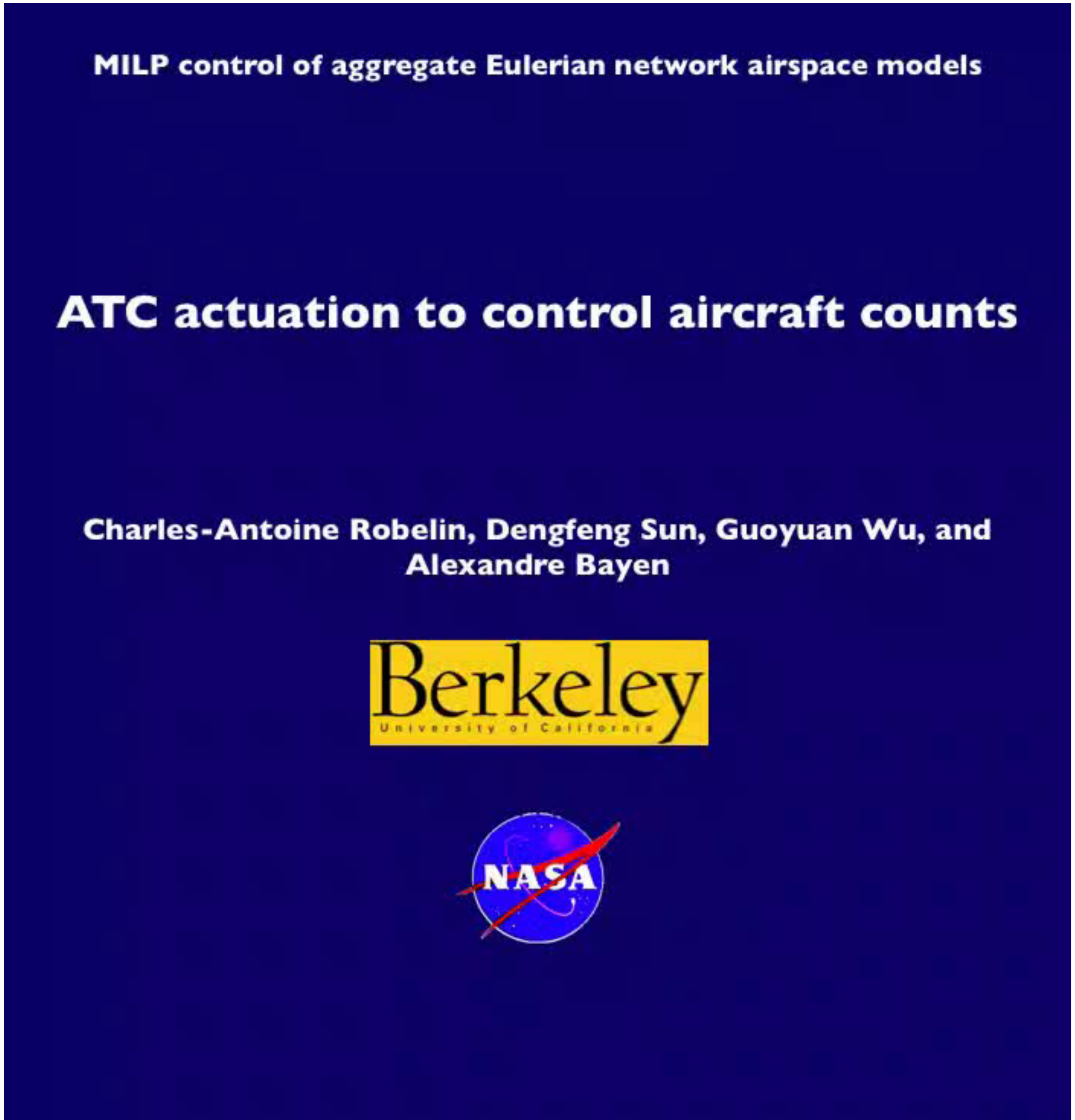
E, L, M: implement user-specified constraints (capacity, non-negativity, etc.)

Challenges >1M variables, >1M constrains.

CPLEX: <6 minute running time (LP)

[Robelin, Sun, Wu, Bayen 2006] **[Borelli 2003, Feron 2000, Bertsimas 1997]**

Overload control



Eulerian models

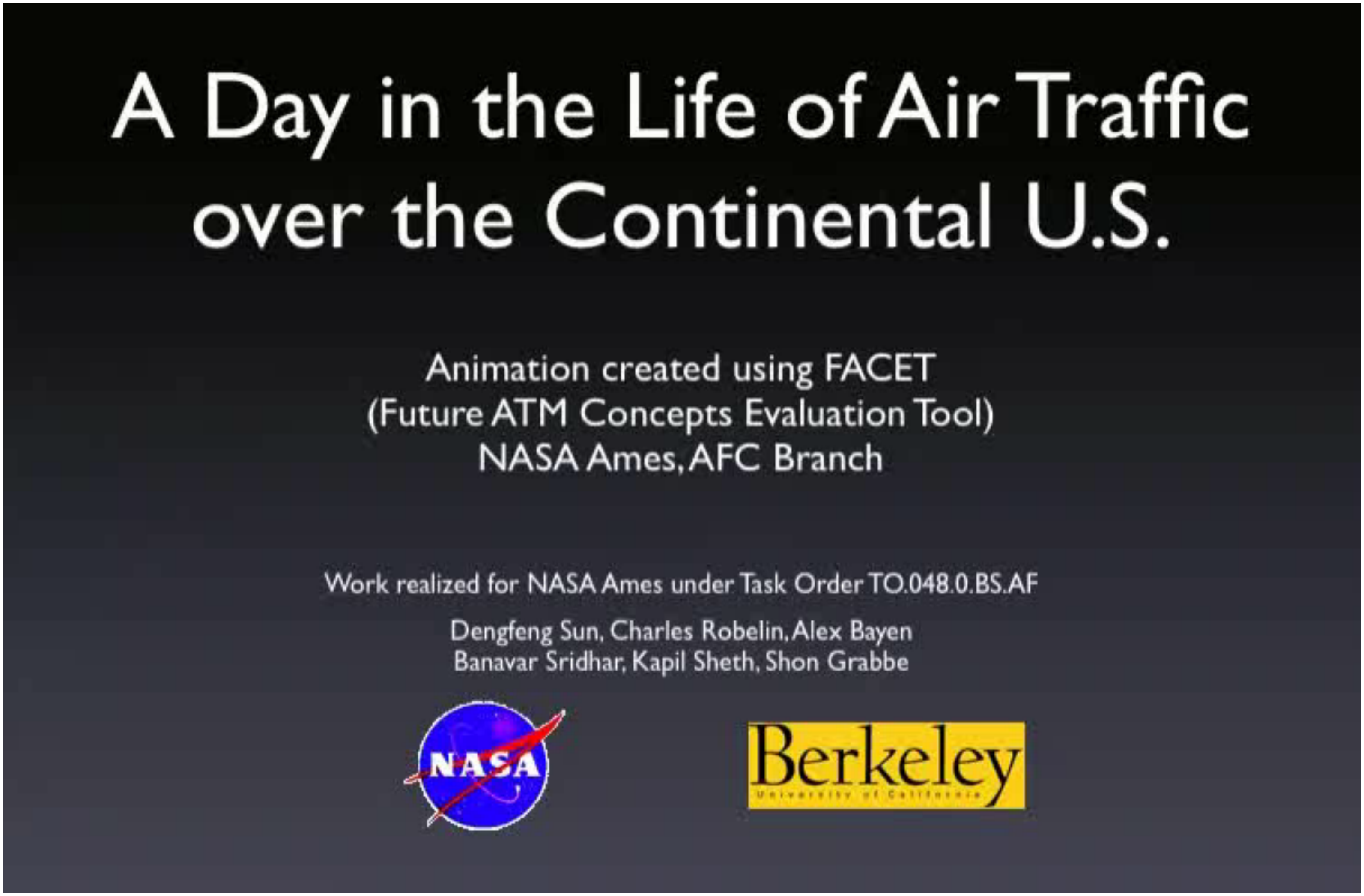
Generic features of Eulerian models

1. Eulerian models scale well: complexity is independent of number of aircraft
2. Control volume based: appropriate for capacity analysis
3. Linear features make them suitable for analysis
4. Can rely on control theory for controllability, observability
5. Combinatorial optimization algorithms can be applied

Features of the current model

1. Can take any set of ETMS/ASDI data as input
2. Eulerian model, validated against ETMS/ASDI data
3. Compared to 2 other existing models (AIAA GNC 2006)
4. Interface with FACET

Acknowledgments



UC Berkeley: Mark Hansen
NASA Ames: Banavar Sridhar, Kapil Sheth, Shon Grabbe, George Meyer
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