NEXTOR

Aviation Short Course

Aviation Infrastructure Economics

October 14-15, 2004

The Aerospace Center Building

901 D St. SW, Suite 850
Washington, DC 20024
Lecture BWI/Andrews Conference Rooms

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Introduction to Optimization Techniques for Infrastructure Management

Application – Markov Decision Processes for Infrastructure Management, Maintenance and Rehabilitation

October 15, 2004

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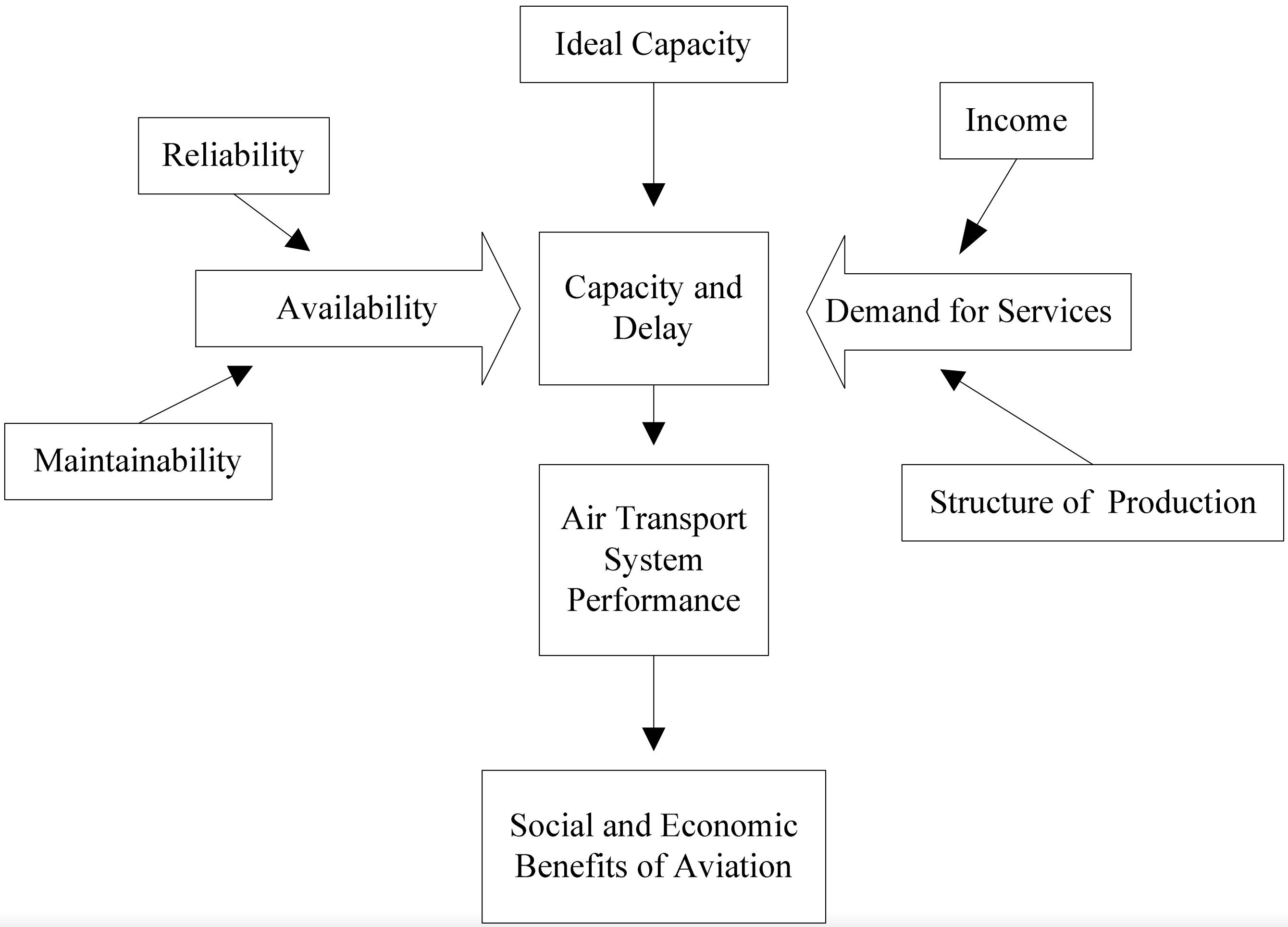
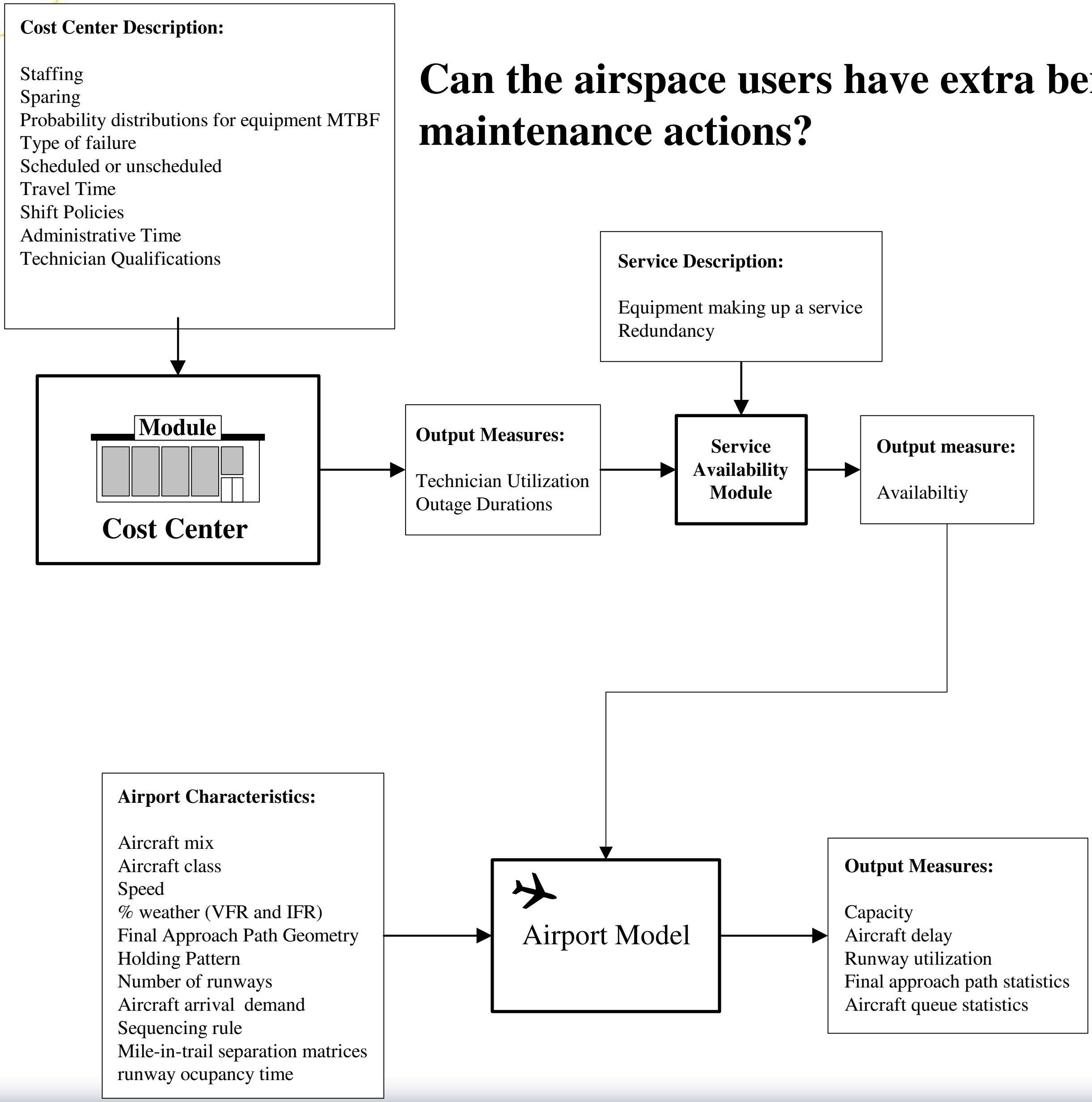
Background

Relevant NAS Measures of Performance and their Relations



Background

Can the airspace users have extra benefits from our maintenance actions?



Models for The National Airspace System Infrastructure Performance and Investment Analysis

October 15, 2004

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Constrained Optimization for Steady State Maintenance, Repair & Rehabilitation (MR&R) Policy

The objective is to apply constrained optimization model to solve an optimal steady state NAS infrastructure management problem, focusing on Terminal Airspace/Runway navigational equipment.

Markov Decision Process is reduced to a linear programming formulation to determine the optimum policy.

Literature Review

Review of Special Types of Linear Programming problems:

- transportation problem
- transshipment problem
- assignment problem

Review of Dynamic Programming (a mathematical technique often useful for making a sequence of interrelated decisions):

- deterministic
- probabilistic

Literature Review

Review of Inventory Theory:

- components
- deterministic models
- stochastic models

Review of Markov Decision Processes:

- Markov decision models
- linear programming and optimal policies
- policy-improvement algorithms for finding optimal policies

Methodology

Markov Decision Processes

Decision	Cost / State (probability)	Expected cost due to caused traffic delays Cd	Maintenance Cost Cm	Total Cost Ct = Cd + Cm
1. Leave ASR as it is	0 = good as new 1 = operable – minor deterioration 2 = operable – major deterioration 3 = inoperable	$ 0 $ 1 000,000 (for example) $ 6 000,000 $ 20,000,000	$ 0 $ 0 $ 0 $ 0	$ 0 $ 1 000,000 $ 6 000,000 $ 20,000,000
2. Maintenance	0 = good as new 1 = operable – minor deterioration 2 = operable – major deterioration 3 = inoperable	If scheduled, $0; otherwise $X2 If scheduled, $0; otherwise $Y2 If scheduled, $0; otherwise $Z1 If scheduled, $M2; otherwise $N2	If scheduled $A2, otherwise $B2 If scheduled $C2, otherwise $D2 If scheduled $E2, otherwise $F2 If scheduled $G2, otherwise $ H2	Cd + Cm
3. Replace	0 = good as new 1 = operable – minor deterioration 2 = operable – major deterioration 3 = inoperable	If scheduled, $0; otherwise $X3 If scheduled, $0; otherwise $Y3 If scheduled, $0; otherwise $Z3 If scheduled, $M3; otherwise $N3	If scheduled $A3, otherwise $B3 If scheduled $C3, otherwise $D3 If scheduled $E3, otherwise $F3 If scheduled $G3, otherwise $ H3	Cd + Cm
4. Upgrade	0 = good as new 1 = operable – minor deterioration 2 = operable – major deterioration 3 = inoperable	If scheduled, $0; otherwise $X4 If scheduled, $0; otherwise $Y4 If scheduled, $0; otherwise $Z4 If scheduled, $M4; otherwise $N4	If scheduled $A4, otherwise $B4 If scheduled $C4, otherwise $D4 If scheduled $E4, otherwise $F4 If scheduled $G4, otherwise $ H4	Cd + Cm

Methodology

Markov Decision Processes

Markov Decision Processes studies sequential optimization of discrete time random systems.

The basic object is a discrete-time random system whose transition mechanism can be controlled over time.

Each control policy defines the random process and values of objective functions associated with this process. The goal is to select a “good’ control policy.

Methodology

Markov Decision Processes

Interrupt Condition	Entry Type	Code Cause
FL Full outage RS Reduced Service RE Like Reduced Service but no longer used	LIR Log Interrupt condition LCM Log Corrective Maintenance LPM Log Preventative Maintenance LEM Log Equipment Upgrade Logs	60 Scheduled Periodic Maintenance 61 Scheduled Commercial Lines 62 Scheduled Improvements 63 Scheduled Flight Inspection 64 Scheduled Administrative 65 Scheduled Corrective Maintenance 66 Scheduled Periodic Software Maintenance 67 Scheduled Corrective Software Maintenance 68 Scheduled Related Outage 69 Scheduled Other 80 Unscheduled Periodic Maintenance 81 Unscheduled Commercial Lines 82 Unscheduled Prime Power 83 Unscheduled Standby Power 84 Unscheduled Interface Condition 85 Unscheduled Weather Effects 86 Unscheduled Software 87 Unscheduled Unknown 88 Unscheduled Related Outage 89 Unscheduled Other

Linear Programming and Optimal Policies

General Formulation

C_{ik} Expected cost incurred during next transition if system is in state *i* and decision *k* is made

y_{ik} Steady state unconditional probability that the system is in state *i* *AND* decision *k* is made

$$y_{ik} = P\{\text{state} = i \text{ and decision} = k\}$$

Linear Programming and Optimal Policies

General Formulation

OF $$Min\sum_{i=0}^{M}\sum_{k=1}^{K}C_{ik}\,y_{ik}$$

subject to the constraints

(1) $$\sum_{i=0}^{M}\sum_{k=1}^{K}y_{ik}=1$$

(2) $$\sum_{k=1}^{K}y_{jk}-\sum_{i=0}^{M}\sum_{k=1}^{K}y_{ik}\,p_{ij}(k)=0$$, for j = 0,1,…M

(3) $$y_{ik}\geq 0$$, i = 0,1,….M; k = 1,2,……,K

Conditional probability that the decision *k* is made, given the system is in state *i*:

$$D_{ik} = P\{decision = k \mid state = i\}$$

$$\begin{array}{cc} & desision, k \\ state, i & \begin{bmatrix} D_{01} & D_{02} & \ldots & D_{0k} \\ D_{11} & D_{12} & \ldots & D_{1k} \\ \vdots & \vdots & \vdots & \vdots \\ D_{M1} & D_{M2} & \ldots & D_{MK} \end{bmatrix} \end{array}$$

Linear Programming and Optimal Policies

Assumptions

- network-level problem

non-homogeneous network (contribution)

Dynamic Programming (DP) used for single facility problems

Linear Programming (LP) used for network-level problems

Linear Programming and Optimal Policies

Assumptions

- deterioration process
  - constant over the planning horizon
- inspections
  - reveal true condition
  - performed at the beginning of every year for all facilities

Markov Decision Process
Linear Programming and Optimal Policies

Transition Probability Matrix

P(k|i,a) is an element in the matrix which gives the probability of equipment j being in state k in the next year, given that it is in the state i in the current year when action a is taken.

Specific Problem

Data:

Note: i is a condition
j is an equipment
a is an action

The cost $\boldsymbol{C}_{iaj}$ of equipment j in condition i when action a is employed.

The user cost $\boldsymbol{U}$ is calculated from the overall condition of the airport.

$\boldsymbol{Budget}_j$ The budget for equipment j

Specific Problem

Decision Variable:

W_{iaj} Fraction of equipment j in condition i when action a is taken.

Note that some types of equipment have only one or two items per type of equipment. Therefore, some W_{iaj} are equal to 1.

Specific Problem

Objective Function:

Minimize the total cost per year (long term):

Minimize $\sum_{i}\sum_{a}\sum_{j}[C(i,a,j)]\times W_{iaj}+U(f(A,\eta,\text{pax-cost}))$

Specific Problem

W_{iaj} fraction of equipment j in condition i when action a is taken.

Constraint (1): mass conservation constraint

In order to make sure that the mass conservation holds, the sum of all fractions has to be 1.

$$\sum_{i}\sum_{a} W_{iaj} = 1 \qquad \forall j$$

Specific Problem

C_{iaj}:

Cost of equipment j in condition i when action a is employed.

U cost:

A airport service availability

η passenger load (per aircraft)

pax-cost

$$\sum_i \sum_a \sum_j [C(i,a,j)] \times W_{iaj} + U(f(A, \eta, \text{pax-cost}))$$

Specific Problem

Constraint (2): All fractions are greater than 0

$$W_{ia} \geq 0 \qquad \forall a, \forall i$$

Constraint (3): Steady-state constraint is added to verify that the Chapman-Kolmogorov equation holds.

$$\sum_i \sum_a W_{iaj} * P_j(k \mid i, a) = \sum_a W_{kaj} \qquad \forall j$$

Specific Problem

Constraint (4): This constraint is added to make sure that there will be less than 0.1 in the worst state.

$$\sum_{a} W_{3aj} < 0.1$$

Constraint (5): This constraint is added to make sure that there will be more than 0.3 in the best state.

$$\sum_{a} W_{1aj} > 0.3$$

Specific Problem

Constraint (6): Non-negativity constraint

$$C(i, a, j) \geq 0 \qquad \forall i, a$$

Constraint (7): Budget constraint

$$\sum_{i} \sum_{a} C(i, a, j) \text{ x } W_{iaj} \leq Budget_{j} \qquad \forall j$$

Additional assumptions:

1) All pieces of equipment are independent. This assumption allows the steady-state constraint to be considered independently; that is, the probability of the next year condition depends only on the action taken on that equipment only.

2) During the scheduled maintenance, it is assumed that the equipment is still working properly although it is actually turned off. This assumption is based on the fact that before any scheduled maintenance, there is a preparation or a back-up provided in order to maintain the same level of service.

3) We assume the VFR condition is 70% of the total operating time; and IFR CATI, II, III are 10% of the total operating time, respectively.

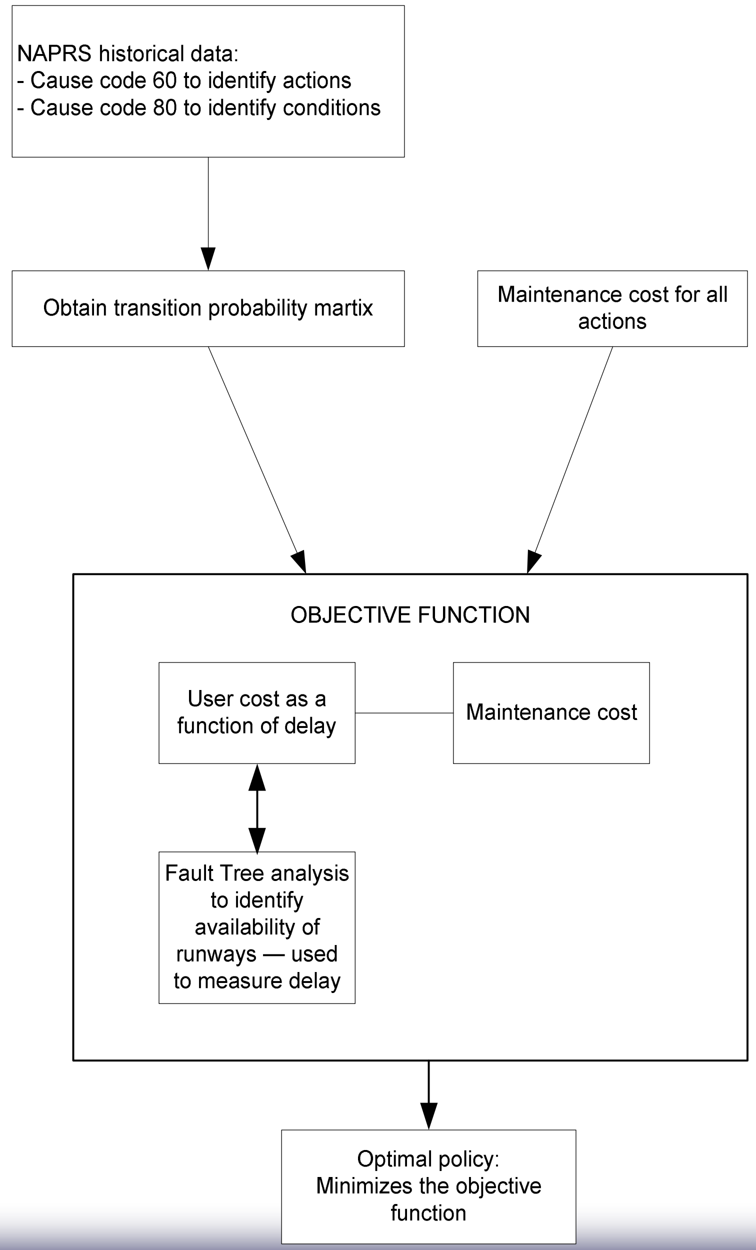
Methodology

The time period in the probability matrix is 1 year. Unscheduled maintenance actions (outages, cause code 80-89) represent the condition *i* of an equipment piece.

The scheduled maintenance actions (code 60-69) represent an action *a* taken in each year.

Given the total time of outages and scheduled maintenances from the historical data, obtained are transitional probability matrices.

Methodology



Numerical Example

- Single airport with 1 runway.
- During IFR conditions, an arriving runway requires 7 types of equipment. If assumed that all types of equipment have the same transition probability matrix, all pieces of equipment are homogeneous. Otherwise, they are non-homogeneous.
- Airport is under IFR conditions 30% of the time. Half of the time is used for departures and the other half is utilized by arrivals.

Numerical Example

- We define conditions and actions as follows:

  action 1: maintenance actions have low frequency

  action 2: maintenance actions have medium frequency

  action 3: maintenance actions have high frequency

  condition 1: availability is less than 99%

  condition 2: availability is 99%-99.5%

  condition 3: availability is 99.5%-100%

- The maintenance cost varies by actions and conditions taken.

Assumptions

Maintenance cost ($/hr)			
	action 1	action 2	action 3
condition 1	1000	1500	2000
condition 2	800	1200	1500
condition 3	600	900	1000

Numerical Example

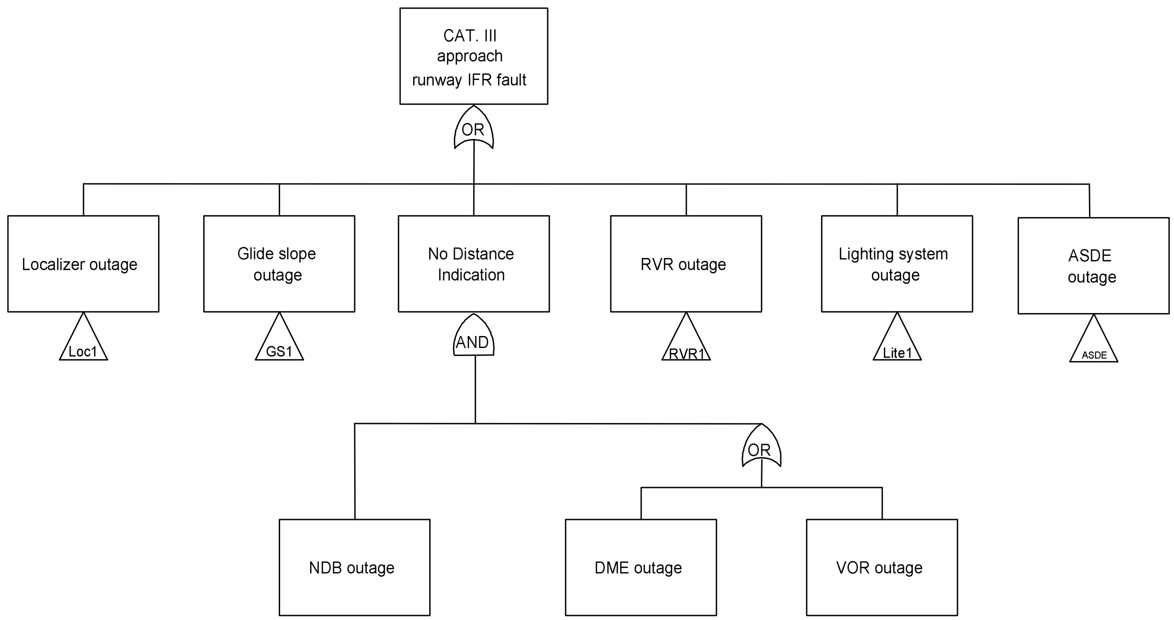
- The availability of the runway is calculated from the fault tree. Fault trees for arrivals and departures are different.
- To calculate the user cost, we use the availability for each condition state to calculate the expected down-time/year (the period that the airport can't operate due to outages). Then, we use the average load factor multiplied by the average passenger/plane and by the average plane/hour to find the total lost time for all passengers. Then, we use the value $28.6/hour as a value of time for each passenger.

Numerical Example

- Each piece of equipment affect airport performance differently, depending on the visibility, wind conditions, noise constrains, primary runway configuration in use and ATC procedures.

- Consequences of equipment outages are also airport specific.

Numerical Example



Top Level Category III IFR Arrival Failure Fault Tree

Numerical Example

We vary our budget in the budget constraint for maintenance costs. Then, we perform the sensitivity analysis.

Assume: budget = $250000/year

W_{iaj}	action			
condition		1	2	3
	1	0	0	0
	2	0	0	0
	3	0	0	1

Total cost is $W_{iaj} \times C_{iaj}$ + U = 210000 + 0 = $210000/year

Assume: budget = $200000/year

W_{iaj}	Action			
condition		1	2	3
	1	0	0	0.05069
	2	0.101378	0	0
	3	0	0.10138	0.746553

Total cost is = **196516.8** + **126875.4** = $**323392.2/year**