Aviation Short Course

Aviation Infrastructure Economics

October 14-15, 2004

The Aerospace Center Building

901 D St. SW, Suite 850
Washington, DC 20024
Lecture BWI/Andrews Conference Rooms

Instructor:
Jasenka Rakas
University of California, Berkeley

EQUIPMENT FAILURE RATES AND PROBABILITITES
MARKOV TRANSITION MATRICES
DETERIORATION AND OBSOLESCENCE

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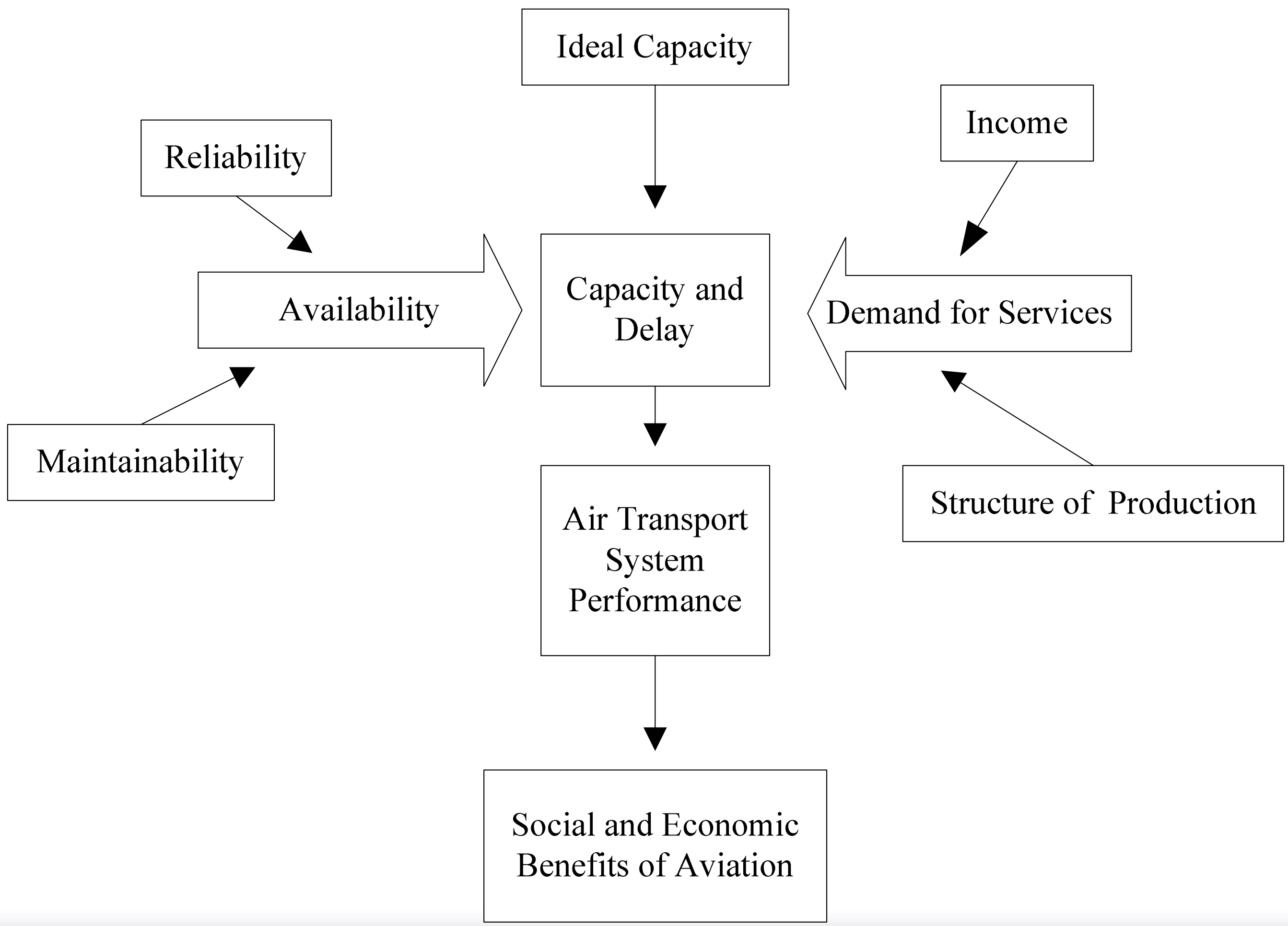
Background

Relevant NAS Measures of Performance and their Relations



EQUIPMENT FAILURE RATES AND PROBABILITITES

MARKOV TRANSITION MATRICES

DETERIORATION AND OBSOLESCENCE

How Do We Predict Equipment Failures?

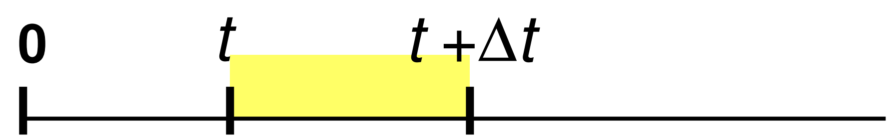
Failure Rate (λ)

For a stated period in the life of a piece of equipment, the ratio of the total number of failures N (or k for observed) to the total cumulative observed time T is the observed failure rate λ:

$$\lambda = k/T$$

The probability of a piece of equipment failing in the interval between t and $t + dt$ given that it has survived until time t:

$$\lambda(t)\,\mathrm{d}t$$



where $\lambda(t)$ is the failure rate.

The probability of failure in the interval t to $t + \mathrm{d}t$ unconditionally:

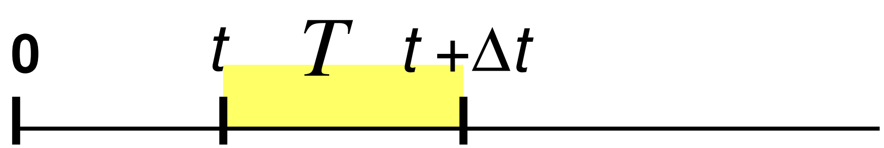
$$f(t)\,\mathrm{d}t$$

where $f(t)$ is the failure probability density function.

How Do We Predict Equipment Failures?

The failure rate $\lambda(t)$ is probability of failure in a period t to $t + \Delta t$ under the condition that no failure occurred before t, divided by Δt and Δt going to 0.

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{Prob\{\, t \leq T < t + \Delta t \,/\, T \geq t \,\}}{\Delta t}$$



How Do We Predict Equipment Failures?

Probability Density Function

Probability distributions are typically defined in terms of the probability density function. For a continuous function, the probability density function (pdf) is the probability that the variate has the value t.

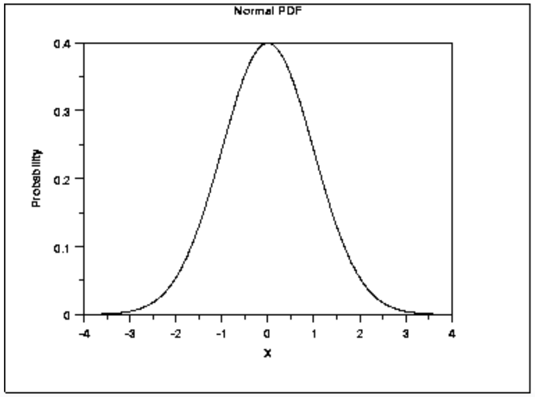
Since for continuous distributions the probability at a single point is zero, this is often expressed in terms of an integral between two points.

$$\int_{t}^{t+dt} f(t)dt = \Pr[t < T < t + dt]$$

For a discrete distribution, the pdf is the probability that the variate takes the value *t* (commonly denoted by *x*).

$$f(t) = \Pr[T = t]$$

The following is the plot of the normal probability density function.



How Do We Predict Equipment Failures?

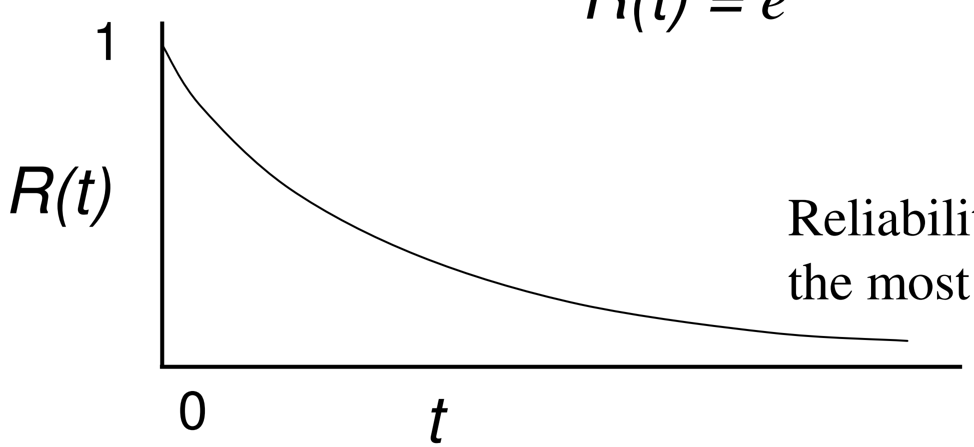
Reliability R(*t*)

The probability of survival to time *t* is defined as the reliability R(*t*).

$$F(t) = \int_{t}^{t+dt} f(t)dt \qquad R(t) = 1 - F(t) \qquad R(t) = e^{-\int_{0}^{t} \lambda(t)dt}$$

If λ(*t*) is constant then:

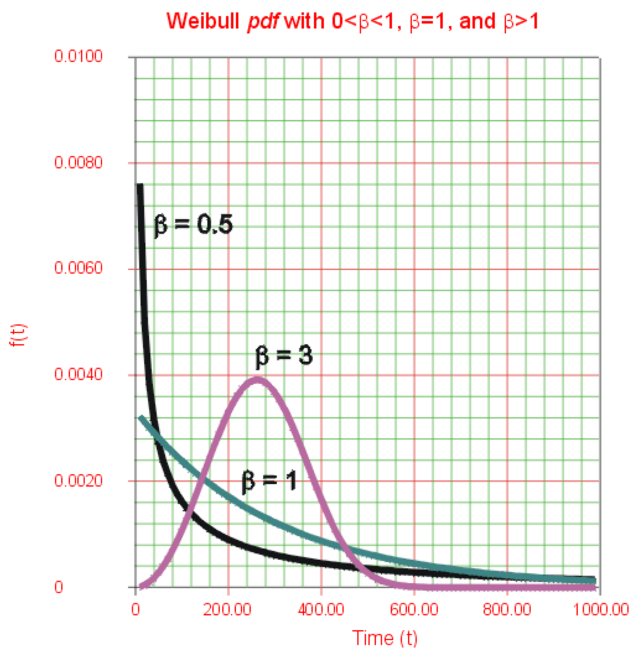
$$R(t) = e^{-\lambda t}$$



Reliability Function:
the most common shape

How Do We Predict Equipment Failures?

The time between equipment failures can follow different probability distributions:



The effect of the Weibull shape parameter β on the *pdf*

The Weibull Distribution

The Weibull distribution is widely used in reliability and life data analysis due to its versatility. Depending on the values of the parameters, the Weibull distribution can be used to model a variety of equipment life behaviors.

$$f(T) = \frac{\beta}{\eta}\left(\frac{T-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{T-\gamma}{\eta}\right)^{\beta}}$$

$$f(T)\geq 0, T\geq 0 \text{ or } \gamma, \beta>0, \eta>0, -\infty<\gamma<\infty$$

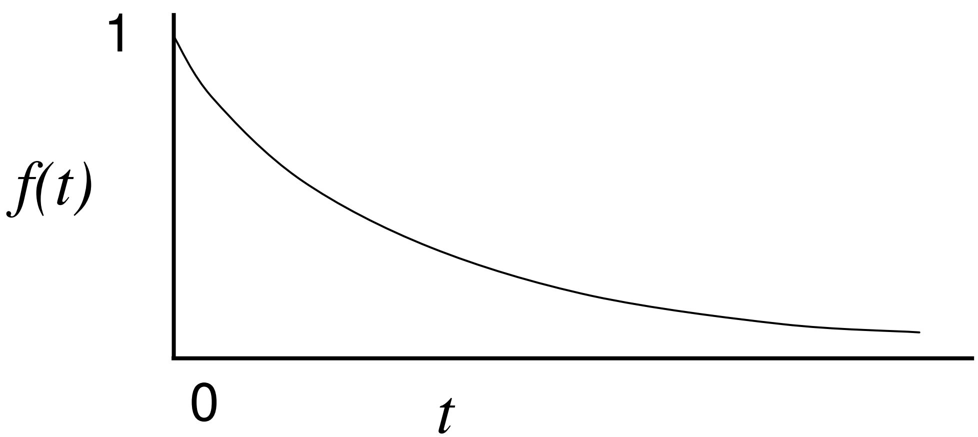
- η = scale parameter
- β = shape parameter (or slope)
- γ = location parameter

How Do We Predict Equipment Failures?

Exponential Distribution

The exponential distribution is a very commonly used distribution in reliability engineering. Due to its simplicity, it has been widely employed. The exponential distribution is used to describe units that have a constant failure rate λ.
The general formula for the probability density function (pdf) of the exponential distribution is:

$$f(t) = \lambda e^{-\lambda t}, \quad t > 0$$



Plot of the Exponential pdf

How Do We Predict Equipment Failures?

Other probability distributions used in modeling time of equipment failure occurrences:

Normal Distribution $$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$

Gama Distribution $$f(t) = \frac{\lambda(\lambda t)^{\alpha-1} e^{-\lambda t}}{\Gamma(\alpha)}, \; t > 0, \; \lambda > 0, \; \alpha > 0$$

where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$

Rayleigh Distribution $$f(t) = \frac{t}{\sigma^2} e^{-\frac{t^2}{2\sigma^2}}$$

How Do We Predict Equipment Failures?

Numerical Example:

If a piece of equipment fails according to Rayleigh Distribution with parameter $\sigma = 1860$ hours, what is the Reliability of this piece of equipment after 1000 hours of work, i.e. R(1000)?

$$R(t) = 1 - F(t) = 1 - \int_0^t f(t)dt = 1 - \int_0^t \frac{t}{\sigma^2} e^{-\frac{t^2}{2\sigma^2}} dt$$

$$R(1000) = 1 - \int_0^{1000} \frac{t}{1860^2} e^{-\frac{t^2}{2(1860)^2}} dt = 0.87$$

The probability of this piece of equipment still working at the 1000th hour is 0.87

How Do We Predict Equipment Failures?

Numerical Example:

Assume a piece of equipment fails with a constant rate $\lambda=0.82$ failures/hour. What is the probability that the equipment will still work after being utilized for 6 hours?

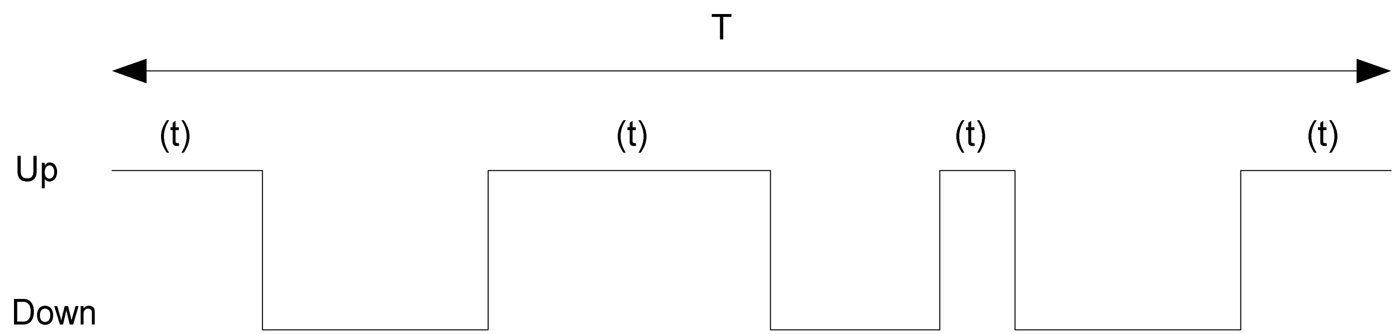
$$R(t) = 1 - \int_0^t f(t)dt = 1 - \int_0^t \lambda e^{-\lambda t} dt = e^{-\lambda t}$$

R(6)=0.995

The probability of this piece of equipment still working at the end of the 6^{th} hour is 0.995.

How Do We Predict Equipment Failures?

Mean Time Between Failures (MTBF):



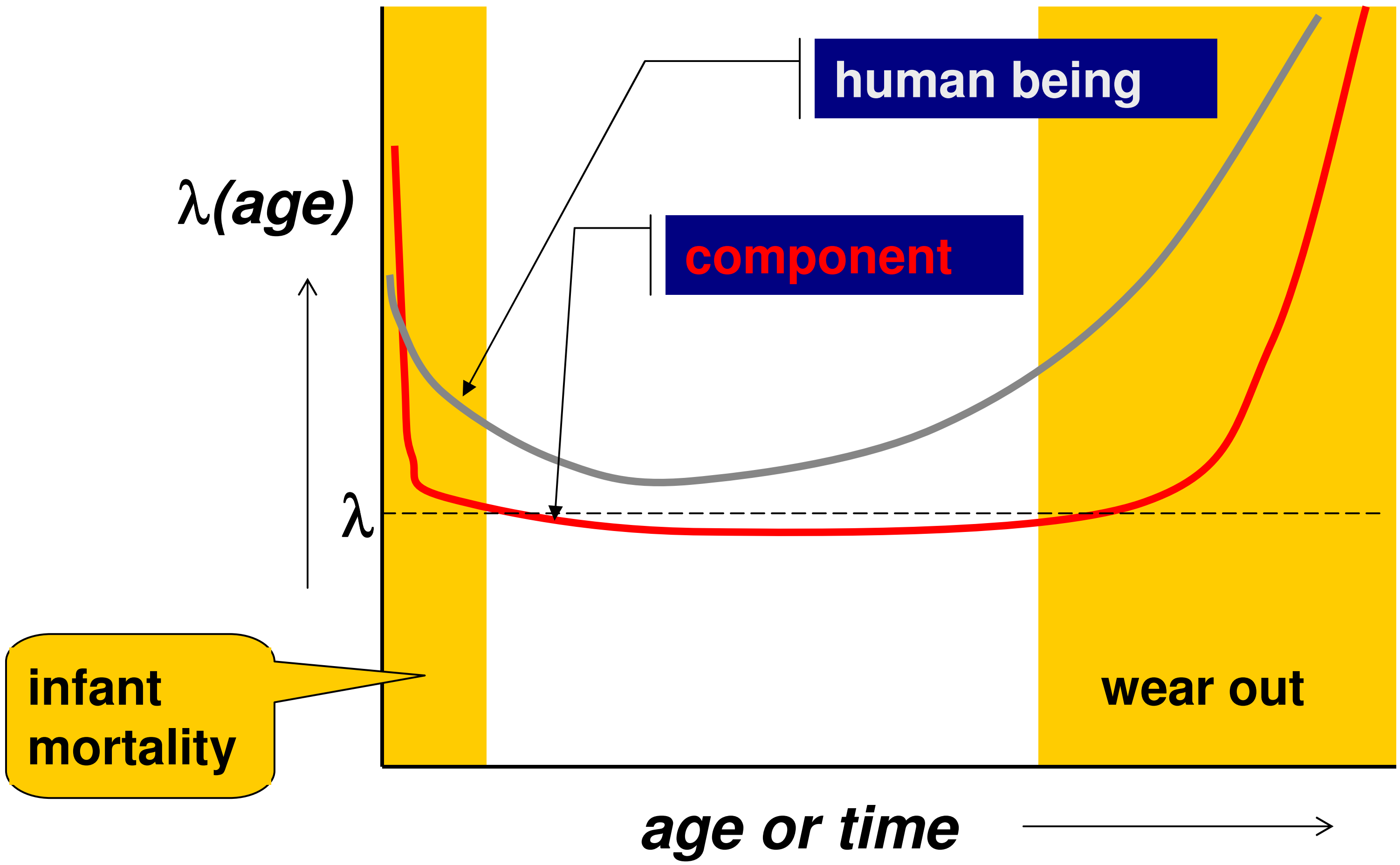
For a stated period in the life of a piece of equipment the mean value of the length of time between consecutive failures, computed as the ratio of the total cumulative observed time to the total number of failures *N* (or *k* for observed).

$$MTBF = T/k$$

MTBF is the mean Up time between failures. It is the average of values of (*t*).

When failure rate λ is constant, MTBF = 1/ λ.

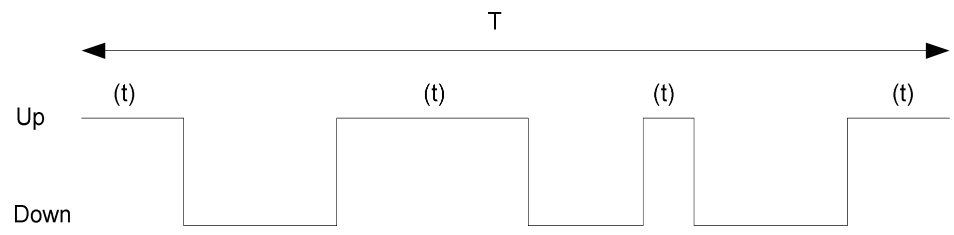
How Do We Predict Equipment Failures?



Bathtub Curve

How Do We Predict Equipment Failures?

Mean Time To Fail (MTTF):



For a stated period in the life of a piece of equipment computed as the ratio of the total cumulative observed time to the total number of failures N (or k for observed).

$$MTTF = T/k$$

The only difference between MTBF and MTTF is in their usage. MTTF is applied to equipment that are not repaired (transistors, bearings), and MTBF is applied to items which are repaired.

√ **EQUIPMENT FAILURE RATES AND PROBABILITITES**

MARKOV TRANSITION MATRICES

DETERIORATION AND OBSOLESCENCE

Markov Chains

A Markov chain is a sequence of random (stochastic) values whose probabilities at a time interval depend upon the value of the number at the **previous** time. A simple example is the non-returning random walk, where the walkers are restricted to not go back to the location just previously visited.

Markovian property: the conditional probability of any future "event" given any past "event" and the present state $X_t = i$, is independent of the past event and depends upon only the present state of the process.

$$P\{X_{t+1} = j \mid X_0 = k_0, X_1 = k_1, \ldots\ldots X_{t-1} = k_{t-1}, X_t = i\} = P\{X_{t+1} = j \mid X_t = i\}$$

for $t = 0,1,2,..$ and every sequence $i, j, k_0, k_1, k_2, \ldots k_{t-1}$.

Markov Chains

Transition Probabilities

The controlling factor in a Markov chain is the **transition probability**. It is a conditional probability for the system to go to a particular new state, given the current state of the system.
For many problems, the Markov chain obtains the much desired importance sampling. This means that we get fairly efficient estimates if we can determine the proper transition probabilities.

The conditional probabilities $P\{X_{t+1} = j \mid X_t = i\}$ are called transition probabilities. If, for each *i* and *j*,

$$P\{X_{t+1} = j \mid X_t = i\} = P\{X_1 = j \mid X_0 = i\}, \text{ for all } t = 0,1,..$$

then the (one step) transition probabilities are said to be *stationary* and are denoted by p_{ij}.

Defining a Markov Chain

A stochastic process $\{X_t\}$ (t = 0,1, …) is a finite-state Markov chain if it has the following:

1. A finite number of states,
2. The Markovian property,
3. Stationary transition probabilities,
4. A set of initial probabilities $P\{X_0 = i\}$ for all i.

Markov Chains

Defining a Markov Chain

A convenient notation for representing the transition probabilities is the matrix form:

	State	0	1	…	M
	0	$p_{00}^{(n)}$	$p_{01}^{(n)}$	…	$p_{0M}^{(n)}$
$P^{(n)} =$	1	$p_{10}^{(n)}$	$p_{11}^{(n)}$		
		:	:		
	M	$p_{M0}^{(n)}$	$p_{M1}^{(n)}$	…	$p_{MM}^{(n)}$

for $n = 1,2,\ldots.$ where n denotes the number of steps or time units

$$p_{ij}^{(n)} \geq 0, \text{ for all } i \text{ and } j, \text{ and } n = 0,1,2....$$

$$\sum_{j=0}^{M} p_{ij}^{(n)} = 1 \quad \text{for all } i \text{ and } n = 0,1,2....$$

Defining a Markov Chain

$p_{ij}^{(n)}$ is just the conditional probability that the random variable *X*, starting in state *i*, will be in state *j* after *n* steps

Equivalently:

$$P^{(n)} = \begin{bmatrix} p_{00}^{(n)} & \cdots & p_{0M}^{(n)} \\ \vdots & & \vdots \\ p_{M0}^{(n)} & \cdots & p_{MM}^{(n)} \end{bmatrix}$$

Markov Chains

Chapman-Kolmogorov Equations

The n-step transition probability $p_{ij}^{(n)}$ is useful when the process is in state *i* and we want to calculate the probability that the process will be in in state *j* after *n* periods.

Chapman-Kolmogorov equations provide a method for computing these *n*-step transition probabilities:

$$p_{ij}^{(n)} = \sum_{k=0}^{M} p_{ik}^{(v)} p_{kj}^{(n-v)}, \text{ for all } i, j, n \text{ and } 0 \le v \le n$$

Explanation:
In going from state *i* to state *j* in n steps the process will be in some state *k* after exactly *v* steps.

$p_{ik}^{(v)} p_{kj}^{(n-v)}$ is just the conditional probability that, starting from state *i*, the process goes to state *k* after *v* steps and then to state *j* in *n* – *v* steps. Summing these conditional probabilities over all possible *k* must yield $p_{ij}^{(n)}$.

Steady-State Probability

Steady state probability means that the probability of finding the process in certain state, say j, after a large number of transitions tends to the value π_j, *independent* of the initial probability distribution defined over states. It is important to note that steady-state probability does *not* imply that the process settles down into one state. On the contrary, the process continues to make transitions from state to state, and at any step n the transition probability from state i to state j is still p_{ij}.

Markov Chains

Steady-State Probability

Why is it useful?

Why is it important?

$$P = \begin{bmatrix} 0.080 & 0.184 & 0.368 & 0.368 \\ 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \end{bmatrix}$$

the one-step transition matrix

$$P^{(2)} = P^2 = \begin{bmatrix} 0.080 & 0.184 & 0.368 & 0.368 \\ 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \end{bmatrix} \times \begin{bmatrix} 0.080 & 0.184 & 0.368 & 0.368 \\ 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \end{bmatrix}$$

the two-step transition matrix

$$P^2 = \begin{bmatrix} 0.249 & 0.286 & 0.300 & 0.168 \\ 0.283 & 0.252 & 0.233 & 0.233 \\ 0.351 & 0.319 & 0.233 & 0.097 \\ 0.249 & 0.286 & 0.300 & 0.165 \end{bmatrix}$$

Markov Chains

Steady-State Probability

Why is it useful?

Why is it important?

$$P^{(4)} = P^{(2)} \times P^{(2)} = \begin{bmatrix} 0.249 & 0.286 & 0.300 & 0.168 \\ 0.283 & 0.252 & 0.233 & 0.233 \\ 0.351 & 0.319 & 0.233 & 0.097 \\ 0.249 & 0.286 & 0.300 & 0.165 \end{bmatrix} \times \begin{bmatrix} 0.249 & 0.286 & 0.300 & 0.168 \\ 0.283 & 0.252 & 0.233 & 0.233 \\ 0.351 & 0.319 & 0.233 & 0.097 \\ 0.249 & 0.286 & 0.300 & 0.165 \end{bmatrix}$$

$$P^{(4)} = \begin{bmatrix} 0.289 & 0.286 & 0.261 & 0.164 \\ 0.282 & 0.285 & 0.268 & 0.166 \\ 0.284 & 0.283 & 0.263 & 0.171 \\ 0.289 & 0.286 & 0.261 & 0.164 \end{bmatrix}$$

the four-step transition matrix

$$P^{(8)} = P^{8} = P^{(4)} \times P^{(4)} = \begin{bmatrix} 0.289 & 0.285 & 0.264 & 0.166 \\ 0.289 & 0.285 & 0.264 & 0.166 \\ 0.289 & 0.285 & 0.264 & 0.166 \\ 0.289 & 0.285 & 0.264 & 0.166 \end{bmatrix}$$

the eight-step transition matrix

Markov Chains

Steady-State Probability

Why is it useful?

Why is it important?

$$P^{(8)} = P^{8} = P^{(4)} \times P^{(4)} = \begin{bmatrix} 0.289 & 0.285 & 0.264 & 0.166 \\ 0.289 & 0.285 & 0.264 & 0.166 \\ 0.289 & 0.285 & 0.264 & 0.166 \\ 0.289 & 0.285 & 0.264 & 0.166 \end{bmatrix}$$

the eight-step transition matrix

The probability of being in state *j* after 8 steps (weeks, days…--any time units) appears to be independent of the initial state.
In other words, there is a limiting probability that the system will be in state *j* after a large number of transitions, and this probability is independent of the initial state *i*.

$$\lim_{n \to \infty} p_{ij}^{(n)} = \pi_j$$ and π_j's satisfy the following conditions:

$$\pi_j > 0$$

$$\pi_j = \sum_{i=0}^{M} \pi_i p_{ij}, \text{ for } j = 0,1,..M$$

$$\sum_{j=0}^{M} \pi_j = 1$$

Markov Chains

Expected Average Cost per Unit Time

Why is it useful?

Why is it important?

The long-run average cost associated with a Markov chain:

If a cost $C(X_t)$ is incurred when the process is in state X_t at time t, then the expected average cost incurred over the first n periods is:

$$E\left[\frac{1}{n}\sum_{t=1}^{n} C(X_t)\right]$$

If $\lim_{n\to\infty}\left\{\frac{1}{n}\sum_{k=1}^{n} p_{ij}^{(k)}\right\} = \pi_j$ then *the long run expected average cost per unit time* is:

$$\lim_{n\to n}\left\{E\left[\frac{1}{n}\sum_{t=1}^{n} C(X_t)\right]\right\} = \sum_{j=0}^{M} C(j)\pi_j$$

Expected Average Cost per Unit Time

Numerical Example:

Before the end of one inspection period (t) we are concerned about our maintenance budget and want to know if we can perform maintenance of (for example) a radar system. Assume that the following costs for each type of radar maintenance are incurred: For j=0, i.e., regular maintenance, C(j=0)= \$2 units

j=1, i.e., minor repair, C(j=1) = \$3 units

j=2, i.e., major repair, C(j=2) = \$5 units

j=3, i.e., replacement, C(j=3) = \$20 units

If the long-run transition probabilities are $P^{(8)} = P^8 = P^{(4)} \times P^{(4)} = \begin{bmatrix} 0.289 & 0.285 & 0.264 & 0.166 \\ 0.289 & 0.285 & 0.264 & 0.166 \\ 0.289 & 0.285 & 0.264 & 0.166 \\ 0.289 & 0.285 & 0.264 & 0.166 \end{bmatrix}$

Then the long-run expected cost of maintaining radar at the end of the inspection period t is:

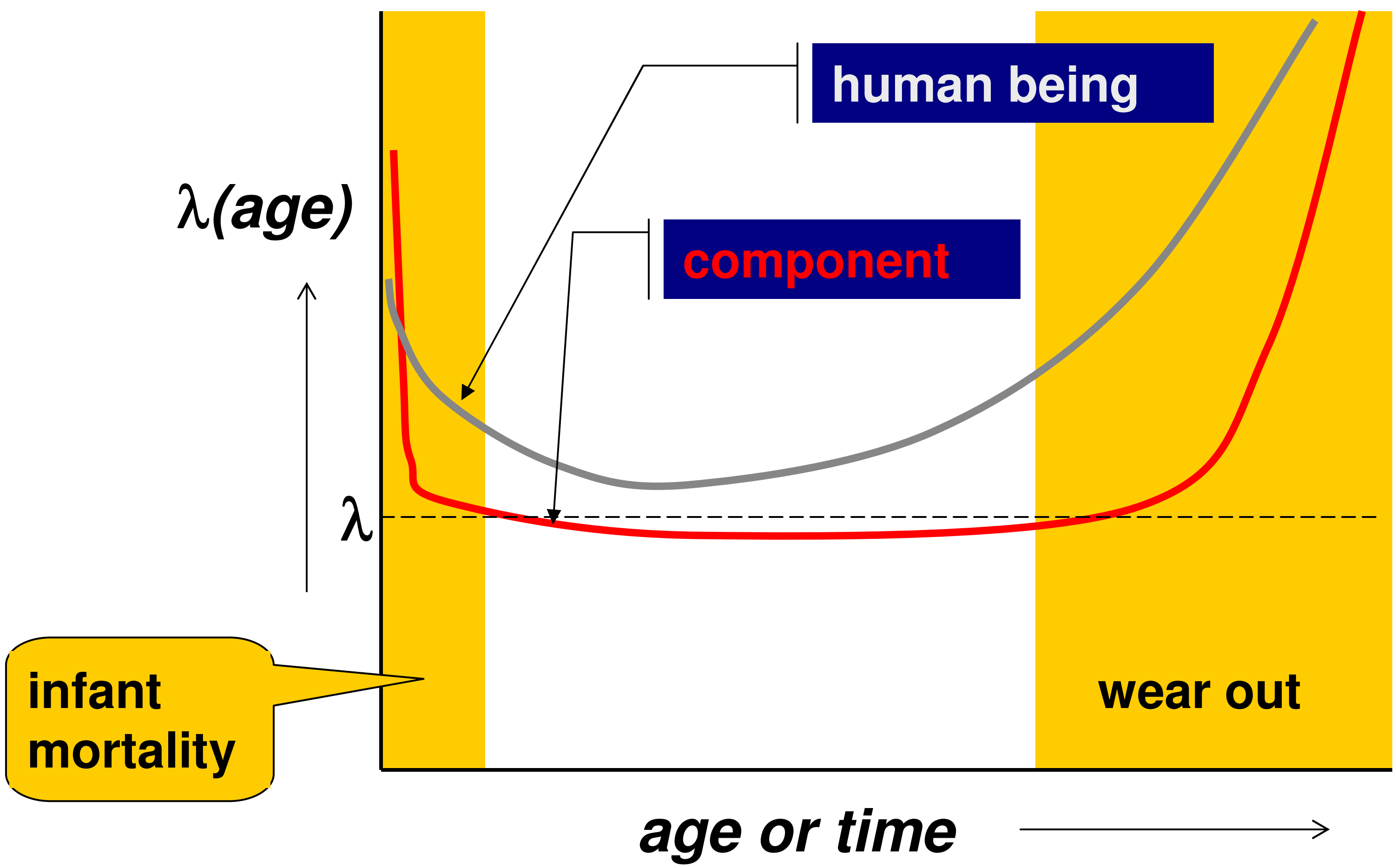
$$E(C) = \sum_{j=0}^{M} C(j)\pi_j = (2)(0.289)+(3)(0.285)+(5)(0.264)+(20)(0.166)=\$6.073 \text{ units}$$

√ **EQUIPMENT FAILURE RATES AND PROBABILITITES**

√ **MARKOV TRANSITION MATRICES**

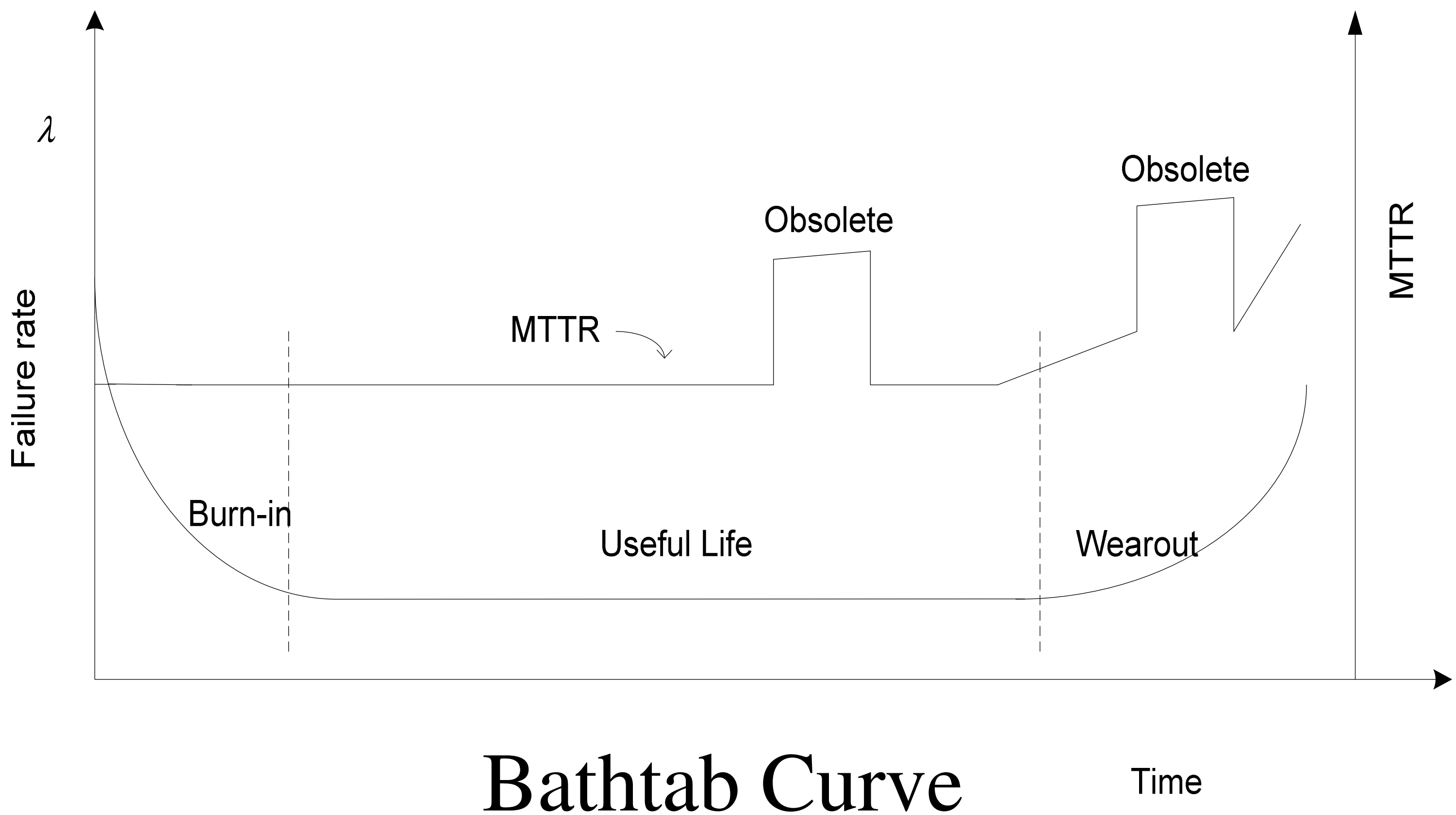
DETERIORATION AND OBSOLESCENCE

Obsolescence Analysis



Bathtub Curve

Obsolescence Analysis



Bathtab Curve

Traditional Elements of Obsolescence

An "obsolescence" event occurs if:

- There is a lack of technician training ("basic obsolescence")

  The equipment could be in either the useful life phase or the wearout phase. The absence of appropriately trained technicians increases MTTRs making it economically unjustifiable to keep such assets in the system.

- There is a lack of spare parts ("basic obsolescence").

  Inability to obtain spare parts increases MTTRs and reduces assets' AVAILABILITY ($A = MTBO / (MTBO + MTTR)$). If spare parts are not attainable, an asset will become obsolete even if its failure rate is in the useful life phase.

- functionality of a piece of equipment cannot be changed ("functional obsolescence").

  Automation tools (Host computer or ARTS) have aged and are no longer able to "absorb" additional functions required to modernize these tool.

Traditional Elements of Obsolescence

Cost Issues:

- operation and maintenance costs exceed the FAA's designated budget
- maintenance cost exceeds replacement cost

How do we predict the time at which a piece of equipment becomes obsolete?

Background

How does the Obsolescence Model fit into our overall NAS Model for Infrastructure Performance and Analysis?

What distinguishes the Obsolescence Model from the overall NAS Model for Infrastructure Performance and Analysis?

What is so specific about the Obsolescence Model?

Equipment States and Maintenance Decisions

Where does the Obsolescence model fit within the overall NAS Model for Infrastructure Management?

Decision	Cost / State (probability)	Expected cost due to caused traffic delays Cd	Maintenance Cost Cm	Total Cost Ct = Cd + Cm
1. Leave ASR as it is	0 = good as new 1 = operable – minor deterioration 2 = operable – major deterioration 3 = inoperable	\$ 0 \$ 1 000,000 (for example) \$ 6 000,000 \$ 20,000,000	\$ 0 \$ 0 \$ 0 \$ 0	\$ 0 \$ 1 000,000 \$ 6 000,000 \$ 20,000,000
2. Maintenance	0 = good as new 1 = operable – minor deterioration 2 = operable – major deterioration 3 = inoperable	If scheduled, \$0; otherwise \$X2 If scheduled, \$0; otherwise \$Y2 If scheduled, \$0; otherwise \$Z1 If scheduled, \$M2; otherwise \$N2	If scheduled \$A2, otherwise \$B2 If scheduled \$C2, otherwise \$D2 If scheduled \$E2, otherwise \$F2 If scheduled \$G2, otherwise \$ H2	Cd + Cm
3. Replace	0 = good as new 1 = operable – minor deterioration 2 = operable – major deterioration 3 = inoperable	If scheduled, \$0; otherwise \$X3 If scheduled, \$0; otherwise \$Y3 If scheduled, \$0; otherwise \$Z3 If scheduled, \$M3; otherwise \$N3	If scheduled \$A3, otherwise \$B3 If scheduled \$C3, otherwise \$D3 If scheduled \$E3, otherwise \$F3 If scheduled \$G3, otherwise \$ H3	Cd + Cm
4. Upgrade	0 = good as new 1 = operable – minor deterioration 2 = operable – major deterioration 3 = inoperable	If scheduled, \$0; otherwise \$X4 If scheduled, \$0; otherwise \$Y4 If scheduled, \$0; otherwise \$Z4 If scheduled, \$M4; otherwise \$N4	If scheduled \$A4, otherwise \$B4 If scheduled \$C4, otherwise \$D4 If scheduled \$E4, otherwise \$F4 If scheduled \$G4, otherwise \$ H4	Cd + Cm

Obsolescence

Binary Decision Variable S (keep=0, upgrade=1)

$$S = \begin{cases} 0, & \text{if equipment age, performance and/or market competition are } \textit{not} \text{ issues} \\ 1, & \text{otherwise} \end{cases}$$

Computers, software or electronics are more market driven than (for example) radars.

New Thinking

Classification of different methodologies as a function of:

- obsolescence definitions
- types of equipment analyzed

Obsolescence Model should be applicable to equipment:

- whose upgrades are age-dependent but also include market consideration
- whose upgrades are primarily market driven

New Thinking

Technology is improving: old systems are phased out and eventually replaced by newer models.

When making decisions on whether to keep a piece of equipment or replace it with a new-technology (currently available on the market), we take into consideration that it might be better to keep the old equipment and wait until it is replaced with an even newer and more advanced technology.

Technology changes stochastically: costs associated with technology can vary with time; introduction of technology has a probabilistic nature.

New Thinking

Consider the following variables as uncertain:

- the time at which the new technology becomes available
- the cost of the new technology

These are important issues when making maintenance decisions.

Proposed Methodology

Optimization Technique:

Methodology to obtain optimal solutions by working backward from the end of a problem to the beginning, by breaking up a larger problem into a series of smaller, more tractable problems.

Dynamic Programming (DP) is often used to solve network, inventory, and resource allocation problems.

DP is used as a central methodology to find optimal replacing.

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SERVICE AVAILABILITY

FAULT TREE ANALYSIS

AIRPORT PERFORMANCE ASSESSMENTS

October 14, 2004

Second Part of the Afternoon Session

What is availability?

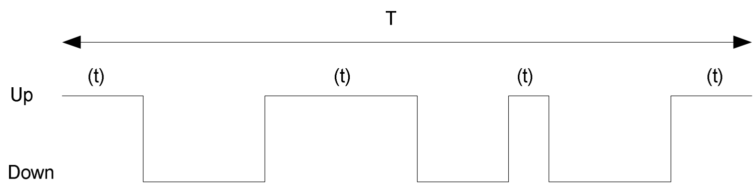
What is availability?

What is service availability?

What factors affect airport and terminal area availability?

How do we determine airport/airspace availability?

Availability: probability (or fraction of time) the system is operating.



$$A = \frac{\text{Up time}}{\text{Total time}} = \frac{\text{Up time}}{\text{Up time + Down time}}$$

Traditional availability estimates consider weather and equipment availability separately.

Equipment Availability: $A = MTBF \,/\, (MTBF + MTTR)$

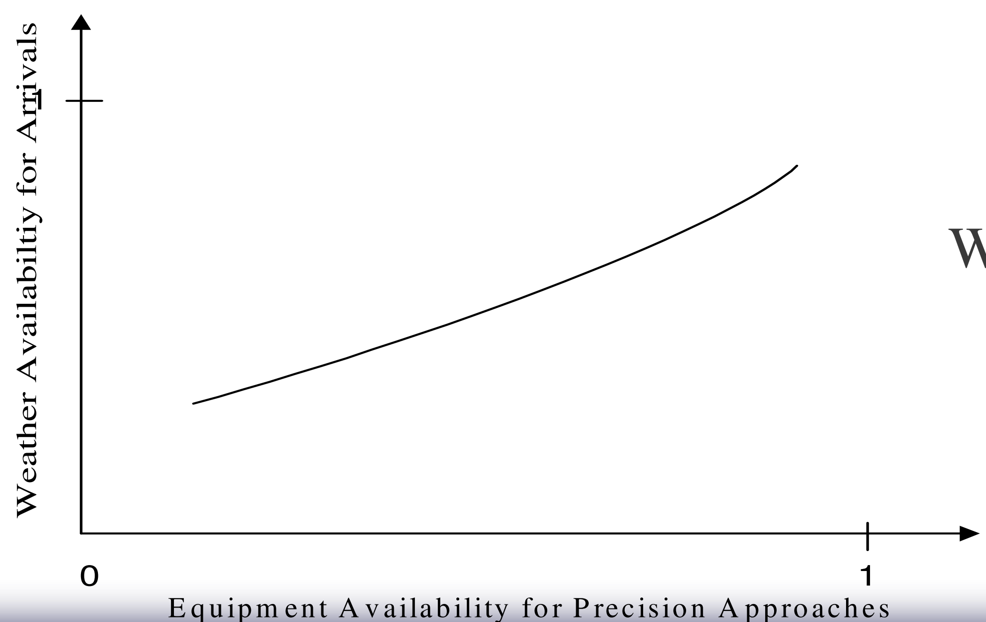
$$A_{op} = (t_s - t_{down}) \,/\, t_s$$

Weather Availability: $$A_w = \frac{MTBC}{MTBC + MTTC_w}$$

Availability Modeling for Airports

However, during bad weather conditions airport availability for arrivals is different from the availability for departures due to different ceiling and visibility requirements.

Airport equipage influences weather availability: if an airport is not CAT III equipped, weather related availability is lower.

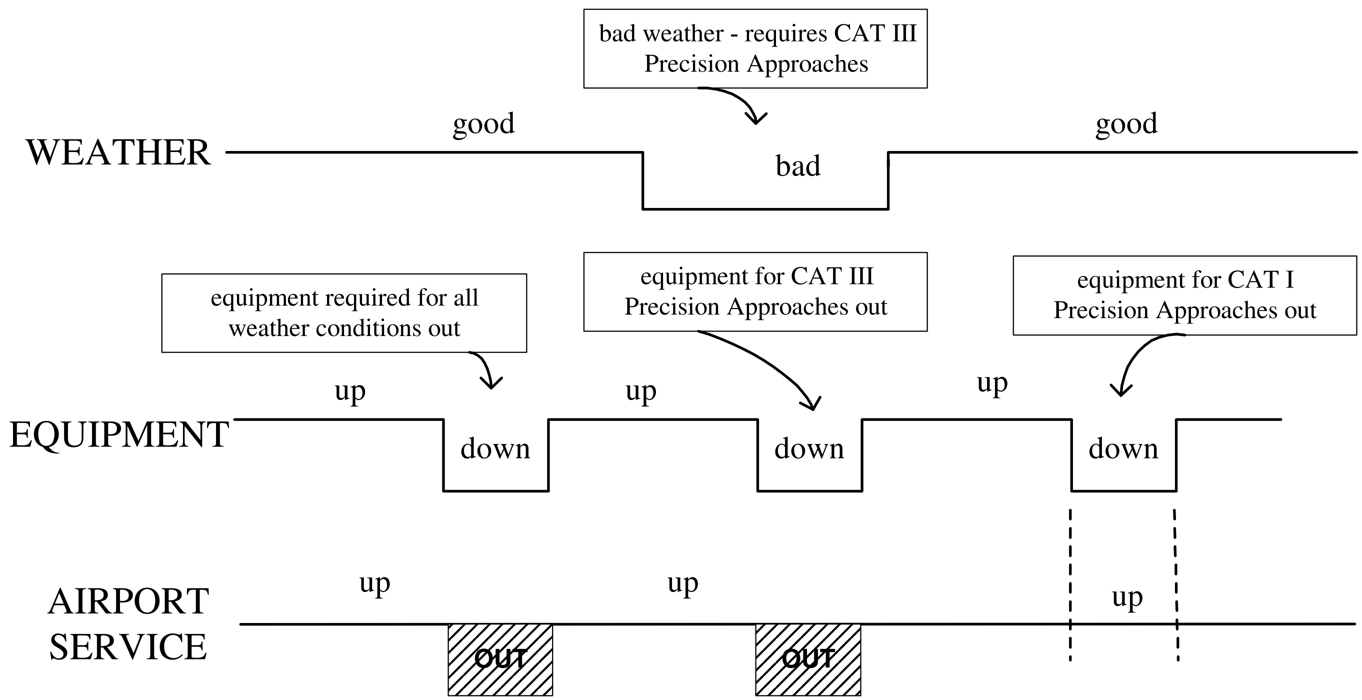


Relation between
Weather Availability for Arrivals
and Equipment Availability
for CAT III Approaches

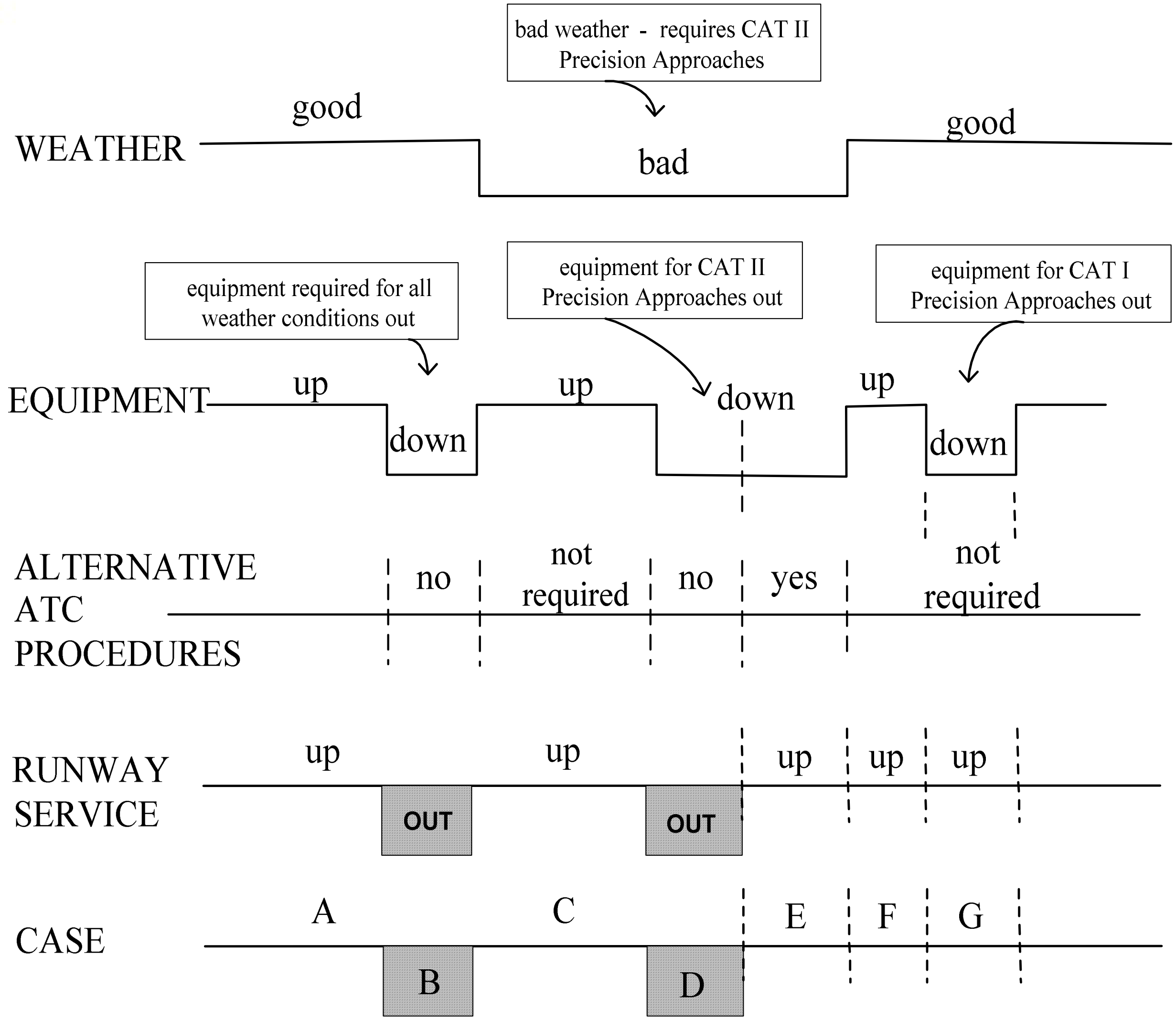
Airport Service Availability

Airport arrival service availability and departure service availability: includes weather and equipment availability for each primary wind direction and noise constraint.

It is a percentage of time (or probability) that a service for arrivals and departures is being provided.



Airport Service Availability



Arrival/Runway Service Availability

Airport Service Availability

Conceptual approach for airport service availability:

1) arrival and departure equipment availability estimated separately for each weather condition

(VFR, IFR CAT I, CAT II and CAT III) using

Fault Tree Analysis (FTA)

2) single runway availability is combined with that of other runways used within a particular runway configuration.

3) arrival and departure availability for each runway configuration used for service availability

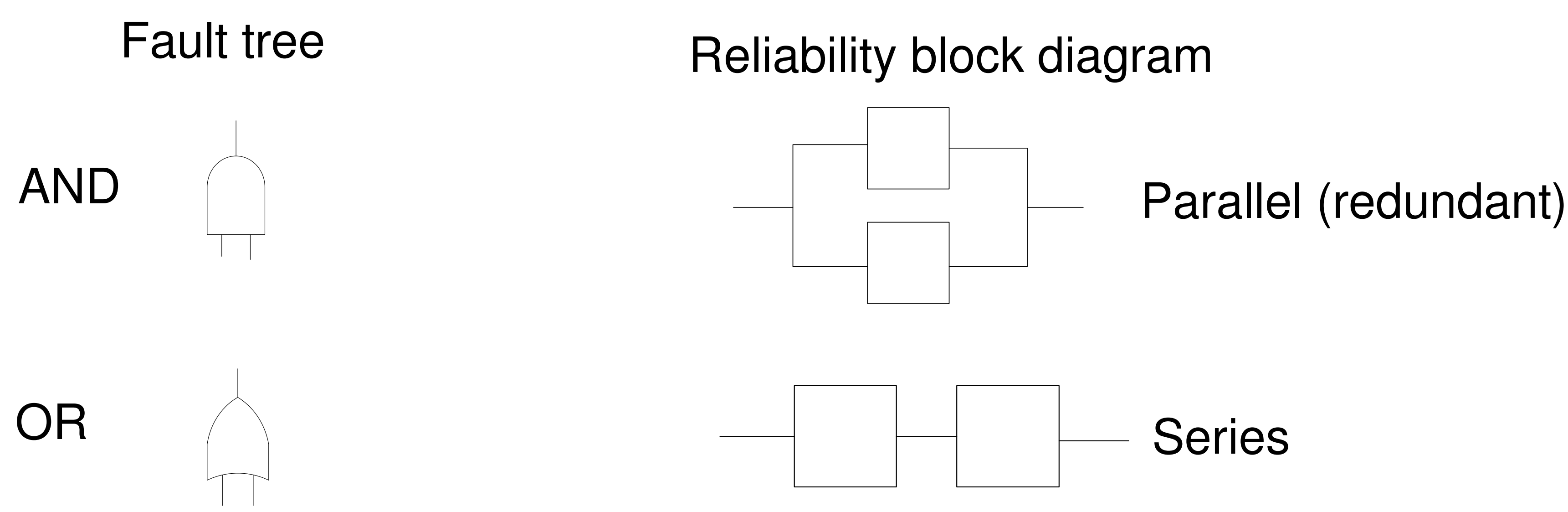
√ **SERVICE AVAILABILITY**

FAULT TREE ANALYSIS

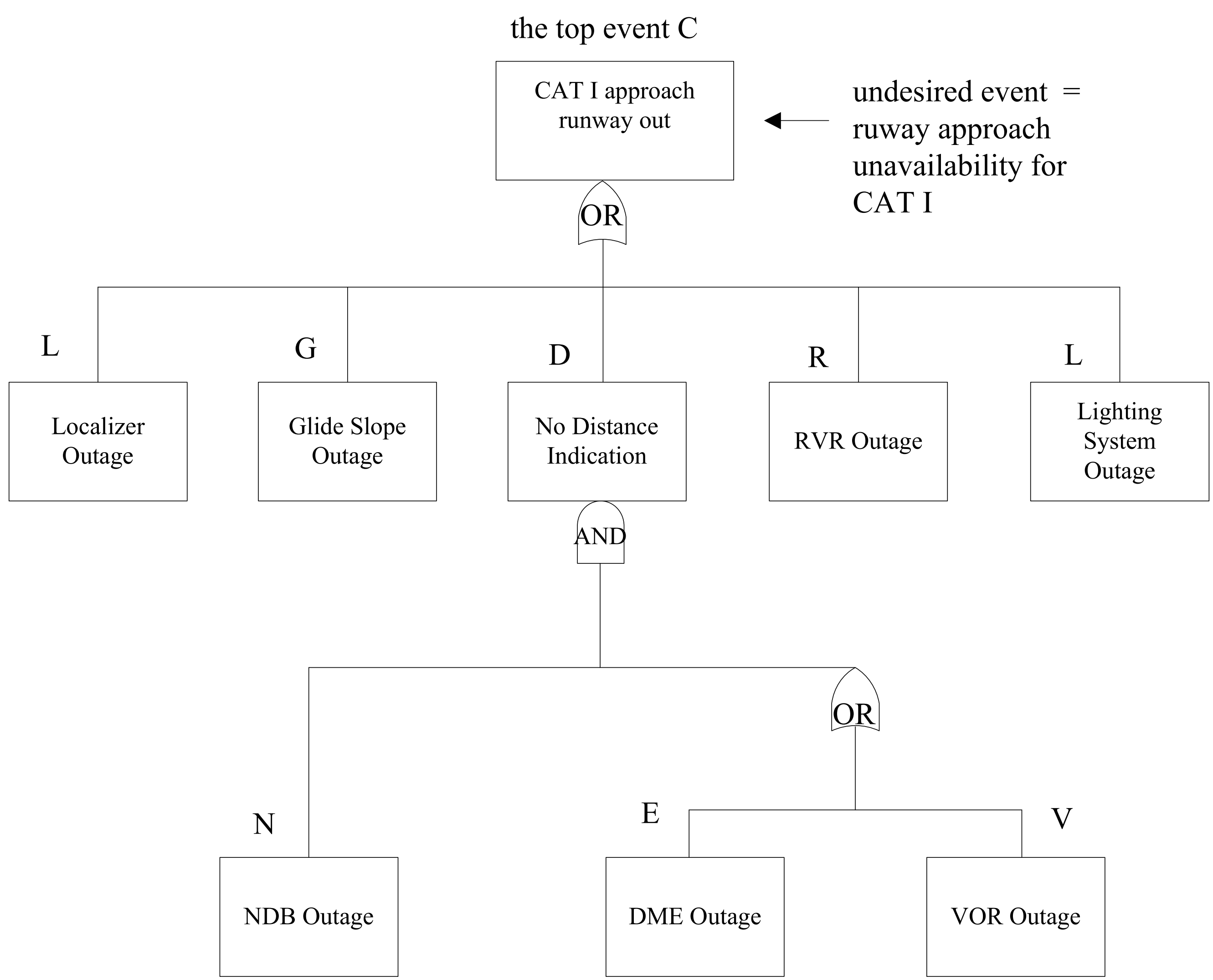
AIRPORT PERFORMANCE ASSESSMENTS

Fault Tree Analysis (FTA)

A Fault Tree is a graphical method of describing the combination of events leading to a defined system failure.
In fault tree terminology the system failure mode is known as the top event. The fault tree involves three logical possibilities and two main symbols.

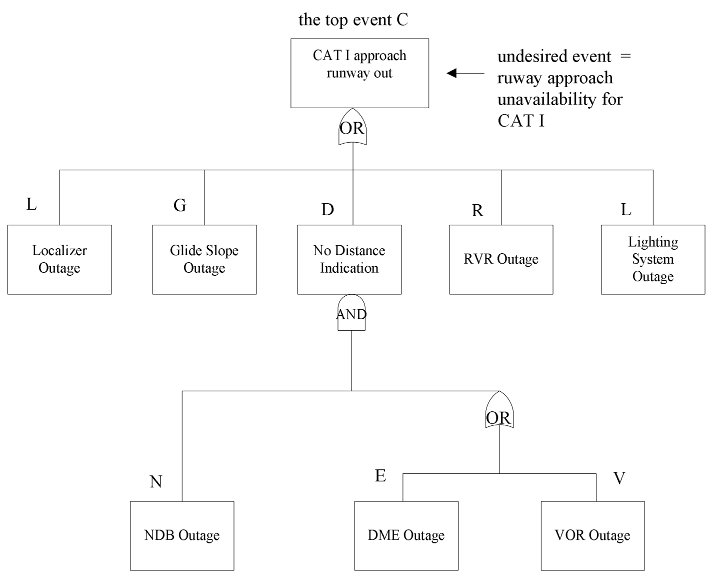


Fault Tree Analysis (FTA)



Fault Tree Analysis (FTA)

The OR gate: any input causes the output to occur.
The AND gate: all inputs need to occur for the output to occur.
The voted gate: two or more inputs are needed for the output to occur.



Boolean algebraic equations:

C = L + G + D + R + L

D = N x (E + V)

Unavailability C:

C = L + G + (N x E) + (N x V) + D + R + L

Runway Availability for Arrivals

The runway availability for arrivals a
on runway r in configuration f
(for a primary wind direction w and noise constraint n) A^a_{wnfr} is:

$$A^a_{wnfr} = \sum_{c=1}^{n} x_c A^a_{cr}$$

A^a_{cr} : arrival availability for weather category c, for runway r

x_c : percentage of time weather category c is use

C : weather category

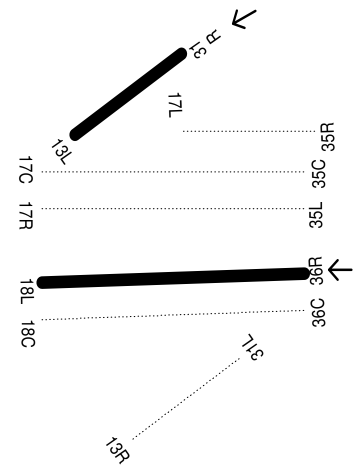
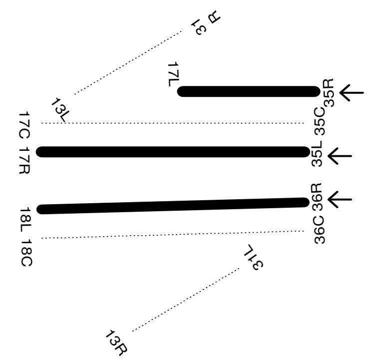
2) single runway availability is combined with that of other runways used within a particular runway configuration.

$$A^{\alpha}_{wnf} = 1 - (1 - A^{a}_{wnfr})$$ single runway availability

$$A^{\alpha}_{wnf} = 1 - (1 - A^{a}_{wnfr_1})(1 - A^{a}_{wnfr_2}).....(1 - A^{a}_{wnfr_n}), \quad \text{for } r_i = r_1 .... r_n$$

where n is the number of runways

NEXTOR

Primary wind direction w	Noise Constraint N	Runway configuration f	Primary Runways in Use R
w_1 = North	None	f_1	 runways: 31R and 36R
w_1 = North	None	f_2	 runways: 35R, 35L, and 36R
w_1 = North	None	f_3	Runways: 35C and 36C
w_2 = South	None	f_1	Runways: 13R, 17L
w_2 = South	None	f_2	Runways: 13R, 17C and 18R

3) arrival availability for each runway configuration used for service availability

The total airport arrival service availability A^{α} is weighted by the percentage of use of each previously calculated availability.

$$A^{\alpha} = \sum_{w=1}^{W} \sum_{n=1}^{N} \sum_{f=1}^{F} y_{wnf} A_{wnf}^{\alpha}$$

W : number of primary wind directions

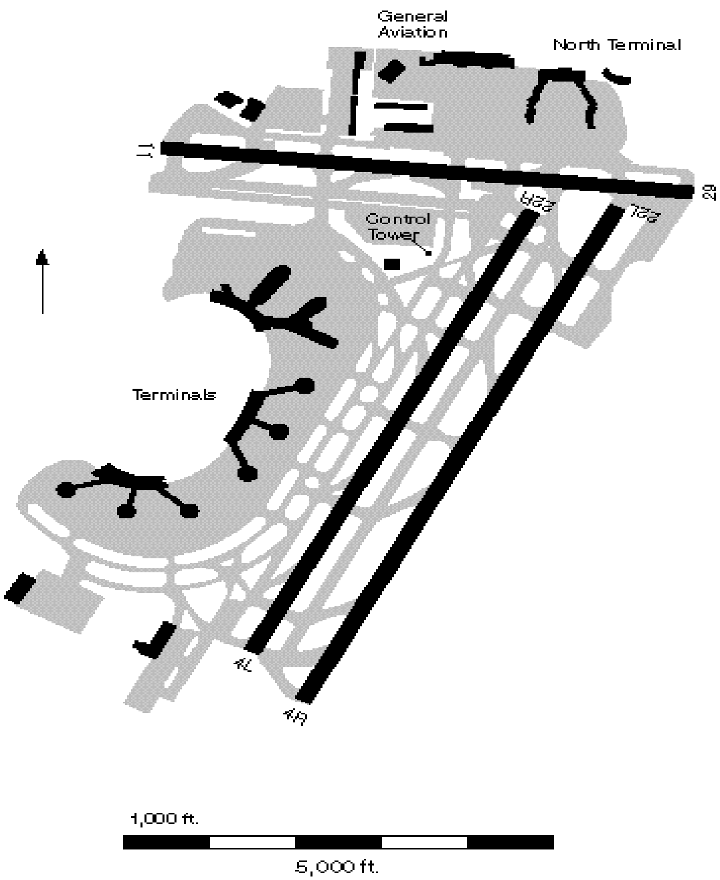
N : number of noise constraints

F : number of runway configurations

y_{wnf} : percentage of time each runway configuration *f* was in use in primary wind direction *w* and noise constraint *n*

Airport Availability Estimates

Case Study: Newark International Airport (EWR)



EWR
Runway Geometry

Required Data

EWR Runway IFR Capability

Runway Configuration Information

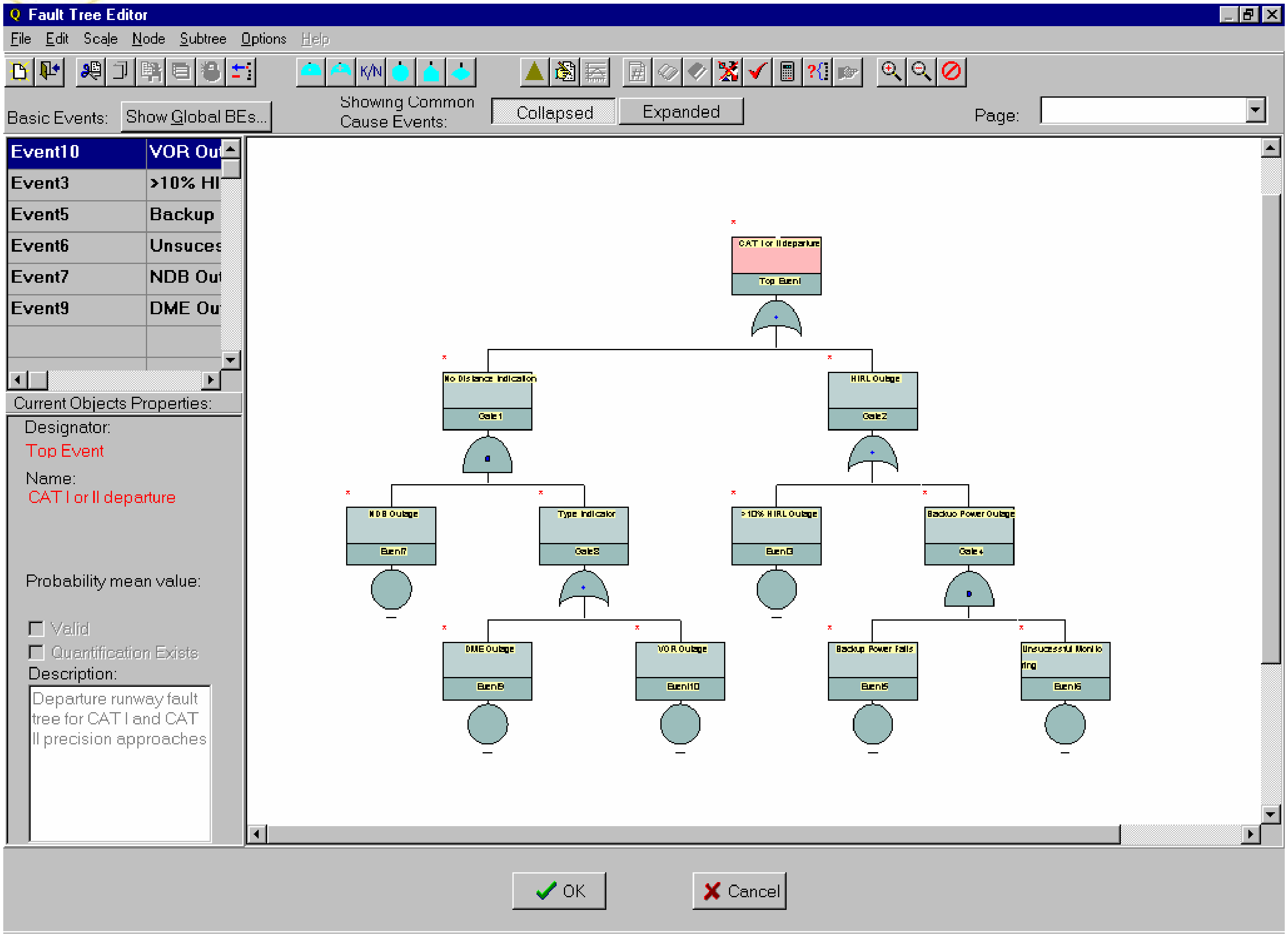
Outages by NAPRS Cause Code

Total Downtime by NAPRS Cause Code

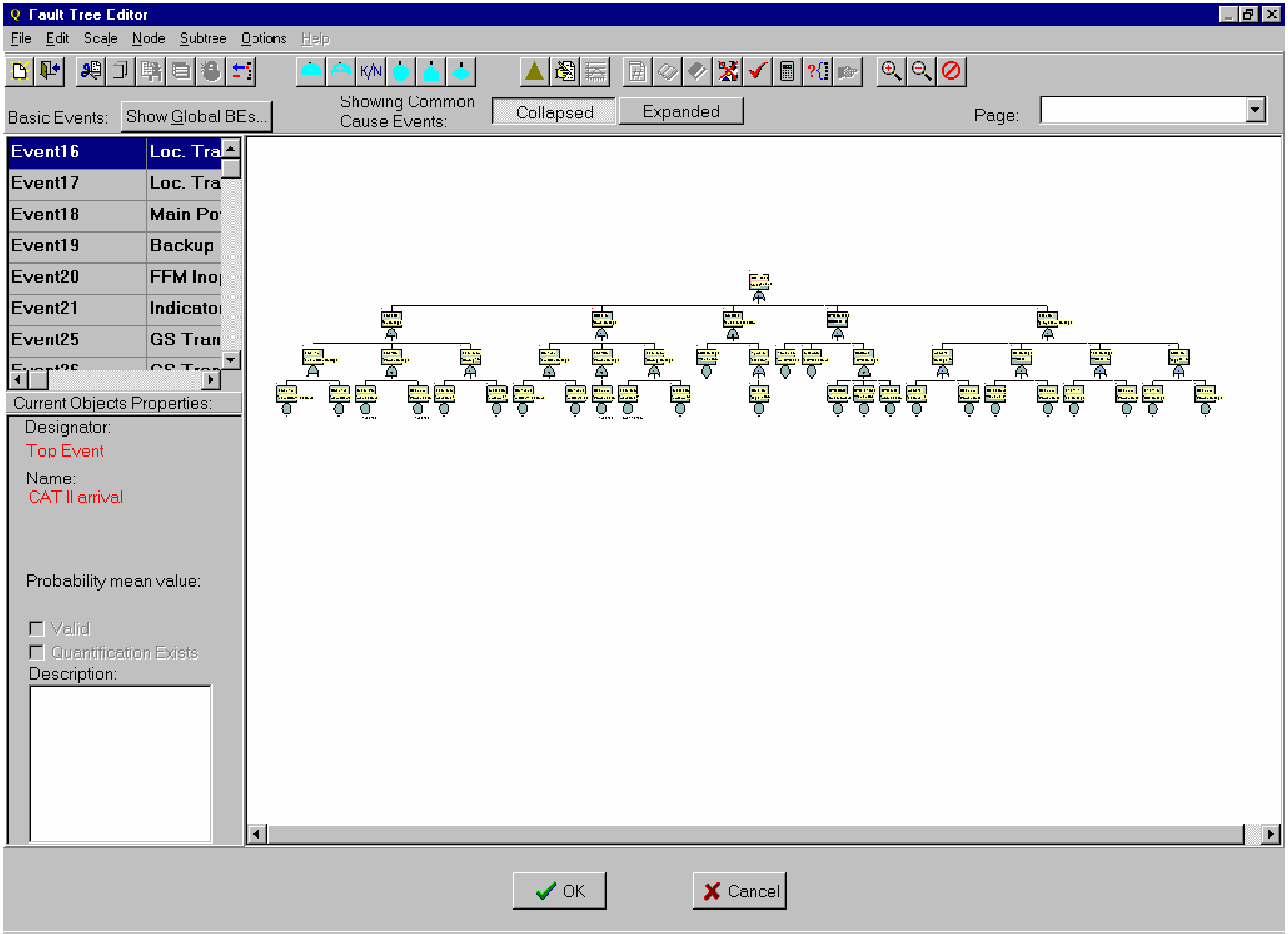
Runway Configuration Information

Percent Occurrence of Weather Categories by Month, Daytime Hours

QRAS Software



QRAS Software



Parameter	Description	Availability
A_A	Airport Arrival Availability	0.9950
A_D	Airport Departure Availability	0.9946
A_{AC1}	Arrival Availability for Configuration 1	0.9982
A_{DC1}	Departure Availability for Configuration 1	0.9931
A_{AC2}	Arrival Availability for Configuration 2	0.9573
A_{DC2}	Departure Availability for Configuration 2	0.9931
A_{AC3}	Arrival Availability for Configuration 3	1.0000
A_{DC3}	Departure Availability for Configuration 3	0.9965
A_{AC4}	Arrival Availability for Configuration 4	0.9989
A_{DC4}	Departure Availability for Configuration 4	0.9965

Arrival and Departure Configuration Availabilities

Parameter	Description	Availability
A_{AR4L}	Arrival Availability, Runway 4L	0.9573
A_{DR4L}	Departure Availability, Runway 4L	0.9580
A_{AR4R}	Arrival Availability, Runway 4R	0.9989
A_{DR4R}	Departure Availability, Runway 4R	1.0000
A_{AR11}	Arrival Availability, Runway 11	0.9573
A_{DR11}	Departure Availability, Runway 11	0.9580
A_{AR22L}	Arrival Availability, Runway 22L	0.9573
A_{DR22L}	Departure Availability, Runway 22L	0.9580
A_{AR22R}	Arrival Availability, Runway 22R	0.9170
A_{DR22R}	Departure Availability, Runway 22R	0.9170
A_{AR29}	Arrival Availability, Runway 29R	0.9170
A_{DR29}	Departure Availability, Runway 29R	0.9170

Arrival and Departure Configuration Availabilities

√ ***SERVICE AVAILABILITY***

√ ***FAULT TREE ANALYSIS***

AIRPORT PERFORMANCE ASSESSMENTS:

Censored Regression – Tobit Model

Deterministic Queuing Model

Factors Affecting Airport Performance

- Equipment outages (scheduled/unscheduled)
- Weather (wind/visibility…)
- Air traffic control procedures

Censored Regression
Tobit Model

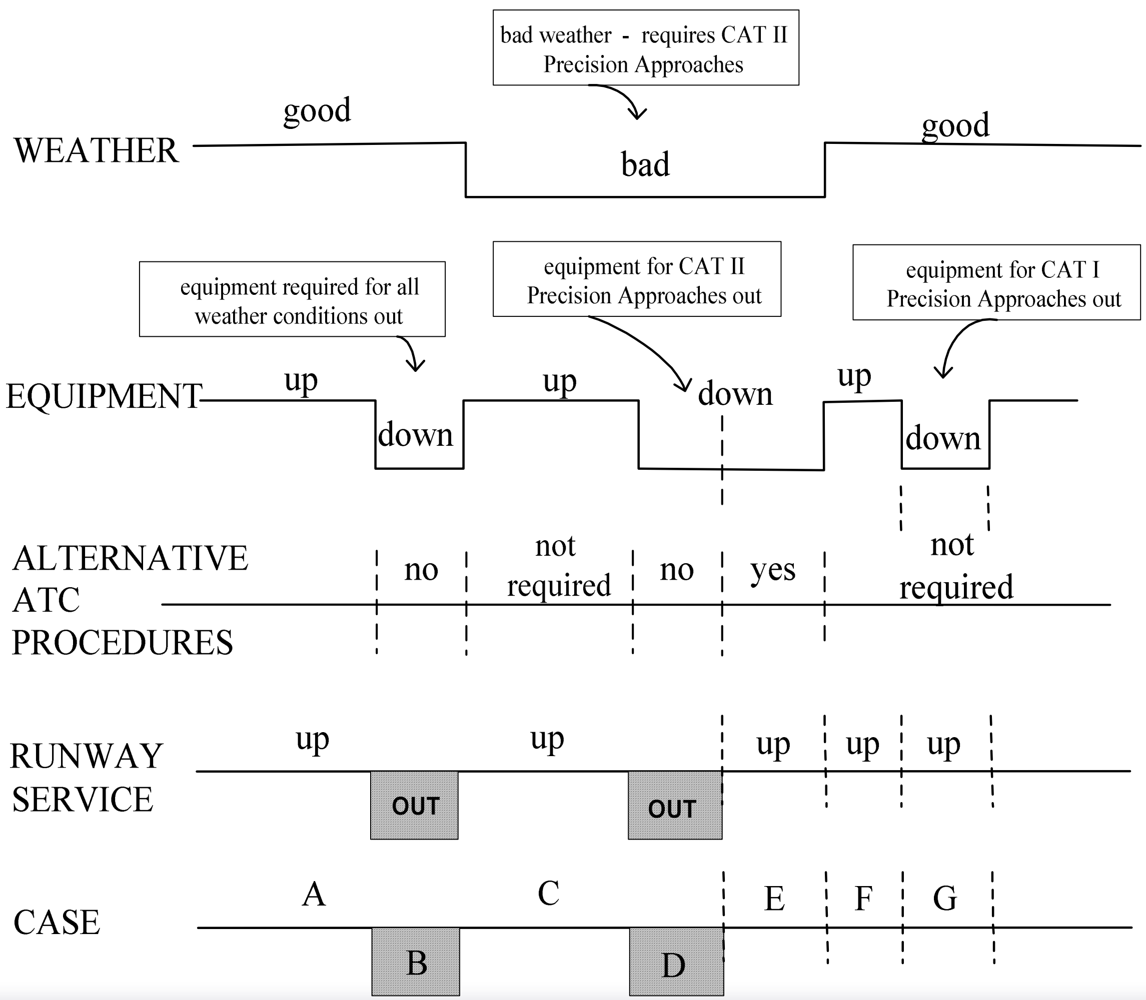
Objective:

To make a clear distinction between demand and capacity impacts on airport throughput.

To remove the impact of increased demand on airport throughput and to determine if unscheduled outages had any effect on airport performance

A special regression is used that included censored data. The censored data is defined as the smaller value between capacity and demand.

Runway Service Alternatives



Data

- FAA MMS: Maintenance Management System data base (equipment outages)
- ASPM: Aviation System Performance Metrics data base (airport quarter-hour throughput, weather conditions, flight rules)
- San Francisco International Airport
- Phoenix Sky Harbor International Airport

Censored Regression
Tobit Model

Dependent variable: Max throughput (capacity) in 15 minutes

Explanatory variables:

- Equipment outage: dummy variable
- Flight rule (IFR/VFR): dummy variable
- Wind direction/speed: by runway direction
- Visibility

  etc

Censored Regression
Tobit Model

The throughput of a particular runway configuration in a time period is determined by either the demand or the capacity during that period.

If demand is less than capacity, a runway could accommodate all demand.

On the other hand, if demand exceeds capacity, throughput would reach the capacity limit, resulting in unserved demand (i.e., delays), and a portion of demand would not be served.

Censored Regression - Tobit Model

$$Arr_t = \begin{cases} Capacity_t = \beta_{0t} + \sum_{n=1}^{N} \beta_{nt} x_{nt} + \varepsilon_t, & if\ \beta_{0t} + \sum_{n=1}^{N} \beta_{nt} x_{nt} + \varepsilon_t < Demand_t \\ Demand_t, & otherwise \end{cases}$$

Arr_t arrival throughput in time interval , which is usually 15 minutes;

β_{0t} constant to be estimated in the model in time interval ;

β_{nt} nth coefficients to be estimated in time interval ;

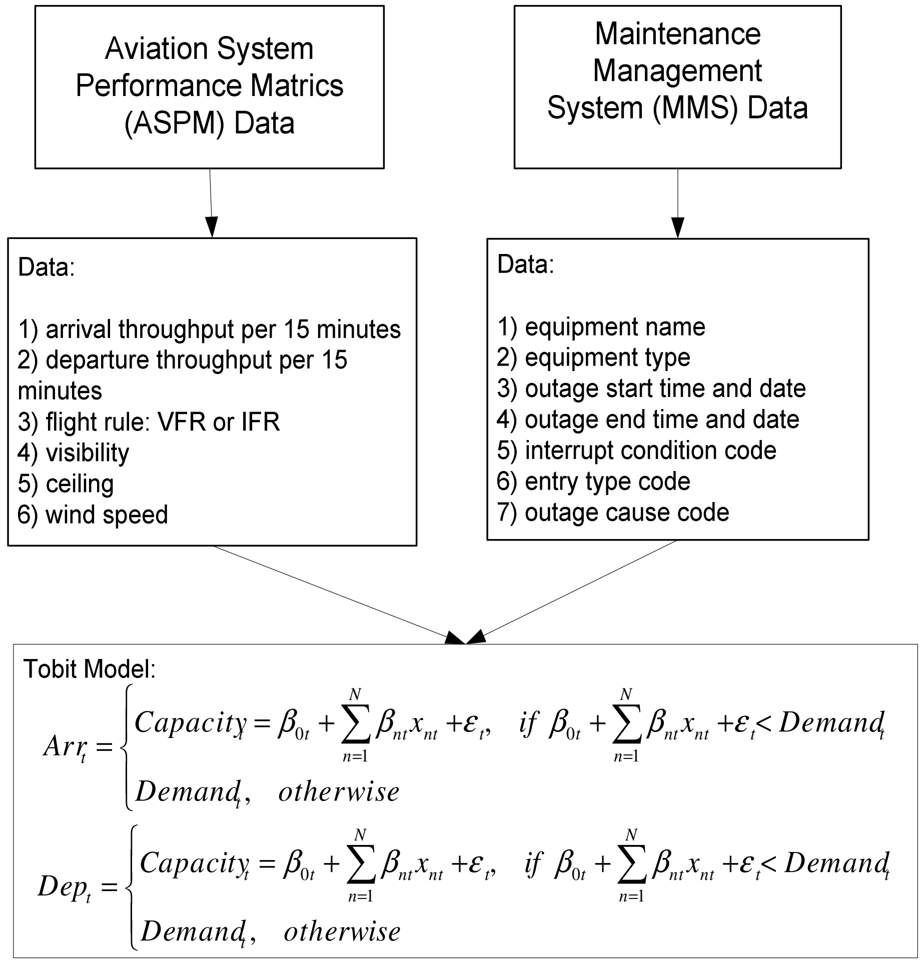
x_{nt} nth independent variable in time interval ;

ε_t error term of the model in time interval ;

$Capacity_t$ capacity in time interval ;

$Demand_t$ demand in time interval.

Tobit Model



Methodology for Aircraft Throughput during Outages

Analysis - VOR

Very High Frequency Omni-directional Range: determines aircraft position/distance

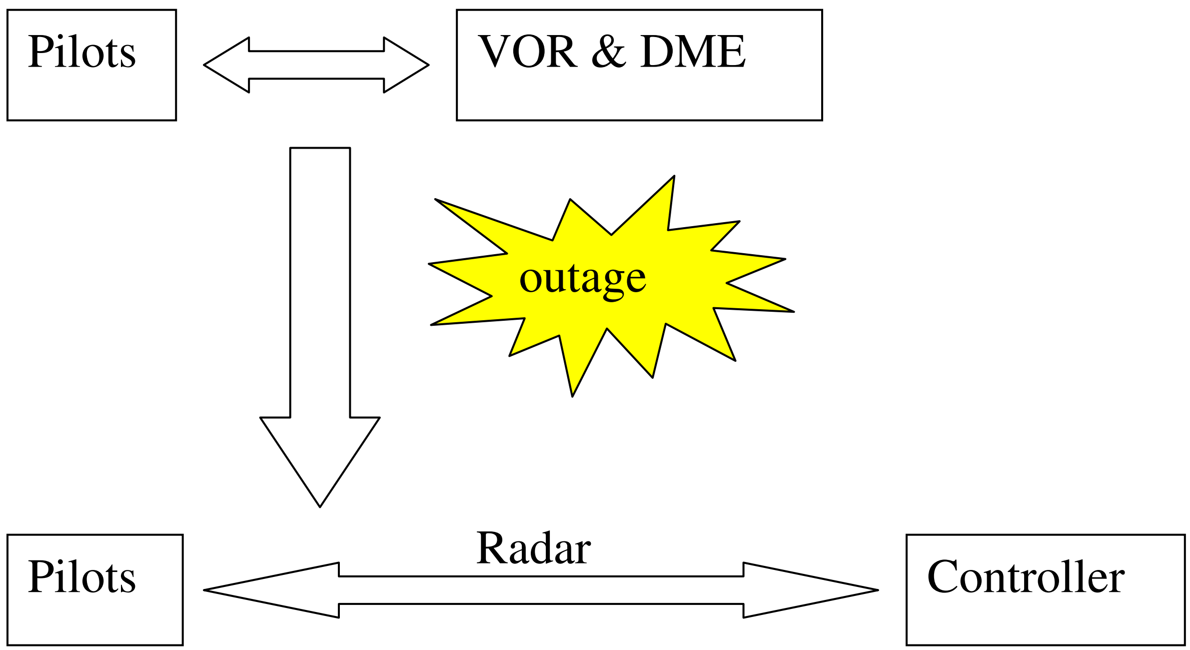
List of VOR Short Unscheduled Outages at SFO

List of VOR Outages at SFO				
Facility Type	**Code Category**	**Interrupt Condition**	**Outage Local Start Date and Time**	**Outage Local End Date and Time**
VOR	80	FL	5/8/01 16:25	5/8/01 18:50
VOR	80	FL	7/24/01 16:50	7/24/01 19:55
VOR	80	FL	8/23/01 14:25	8/23/01 15:25
VOR	80	FL	9/30/01 18:40	9/30/01 19:25
VOR	80	FL	10/14/01 16:12	10/14/01 17:40
VOR	80	FL	10/14/01 16:30	10/14/01 17:40
VOR	80	FL	4/12/02 16:30	4/12/02 18:50
VOR	80	FL	6/5/02 15:19	6/5/02 20:30
VOR	80	FL	7/9/02 14:55	7/9/02 16:44

Analysis Results - VOR

Runway Configuration	Weather Condition	Time Interval (local)	Estimated Affect of Throughput	t-value	P-value (Significance level 0.05)	Observations
28L, 28R I	VFR	14:00 pm-21:00 pm	0.7859	1.72	0.0855 (not significant)	2667
1L, 1R	VFR	14:00pm-21:00 pm	0.2202	0.37	0.712 (not significant)	2667
28L, 28R I	VFR	14:00 pm-21:00 pm	-1.161	-1.19	0.2356 (not significant)	1232
28L,28R	VFR	14:00 pm-21:00 pm	-0.4502	-0.6	0.5505 (not significant)	1232

Reconstruction - VOR



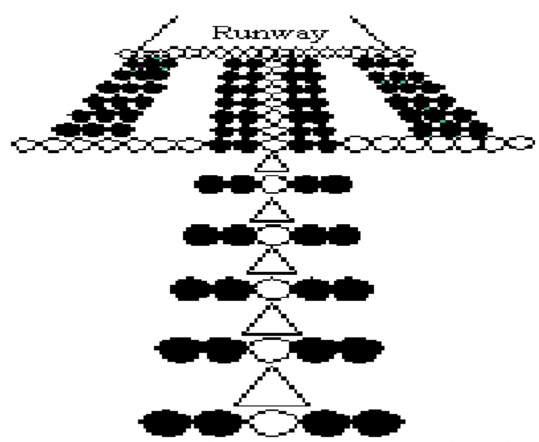
Reconstruction - VOR

Airport Adaptability:

ability to shift to different air traffic procedures or a set of equipment facilities in order to accommodate new circumstances related to equipment outages.

Analysis – ALSF-2

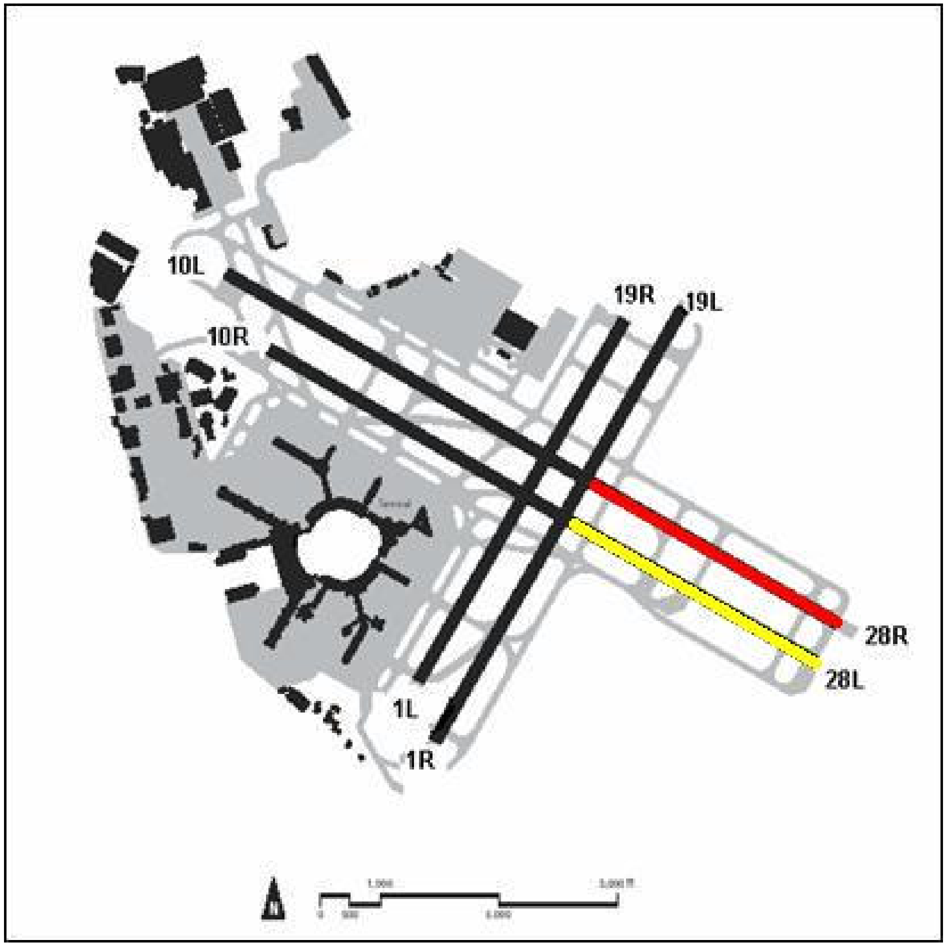
Approach Lighting System with Sequenced Flashing Lights: impact depends on the visibility, located on runway 28R at SFO



Analysis – ALSF-2

List of ALSF-2 Outages at SFO					
Facility Type	Runway	Code Category	Interrupt Condition	Outage Local Start Date and Time	Outage Local End Date and Time
ALSF-2	28R	80	RS	7/28/2000 18:00	7/28/2000 20:00
ALSF-2	28R	80	RS	8/9/2000 20:00	8/9/2000 22:00
ALSF-2	28R	80	FL	9/2/2000 19:30	9/2/2000 20:30
ALSF-2	28R	80	RS	11/15/2000 16:55	11/15/2000 17:30
ALSF-2	28R	80	FL	12/9/2000 1:30	12/9/2000 2:30
ALSF-2	28R	80	RS	3/19/2001 18:47	3/19/2001 20:20
ALSF-2	28R	80	FL	3/19/2001 18:47	3/19/2001 20:20

Reconstruction – ALSF-2



IFR: 2 arrival streams → 1 stream on 28R

ALSF-2 outage & IFR: single arrival stream on 28R → 28L

Analysis Results for ALSF-2s

Runway Configuration (arrivals | departures)	Weather Condition (IFR of VFR)	Time Interval (local)	Dummy Variable	Estimated Affect on Throughput **	t-value	Significance at 0.05 Level	Number of Observations ***
28L, 28R |	VFR	18:00 pm-22:00 pm	Outage* (occurred)	0.2904	0.30	0.7628 (not significant)	1684
1L, 1R	VFR	18:00 pm-22:00 pm	Outage	1.127	1.16	0.2452 (not significant)	1684
28L, 28R |	IFR	18:00 pm-22:00 pm	Outage	19.4989	0.00	0.9999 (not significant)	5759
1L, 1R	IFR	18:00 pm-22:00 pm	Outage	-3.2371	-0.92	0.3590 (not significant)	5759

* Outage = 1 if there was an ALSF-2 outage during the period j; otherwise Outage = 0.
** Estimated change in quarter-hour throughput.
*** Each observation is 1 quarter-hour period.

Reconstruction – ALSF-2

Airport Re-configurability:

airport's ability to switch operations to a different runway in case of equipment outages, or utilize a set of equipment facilities with similar functions to maintain a desired level of performance.

Conclusions

VOR and ALSF-2 unscheduled outages do not have significant impact on arrival and departure throughputs at SFO

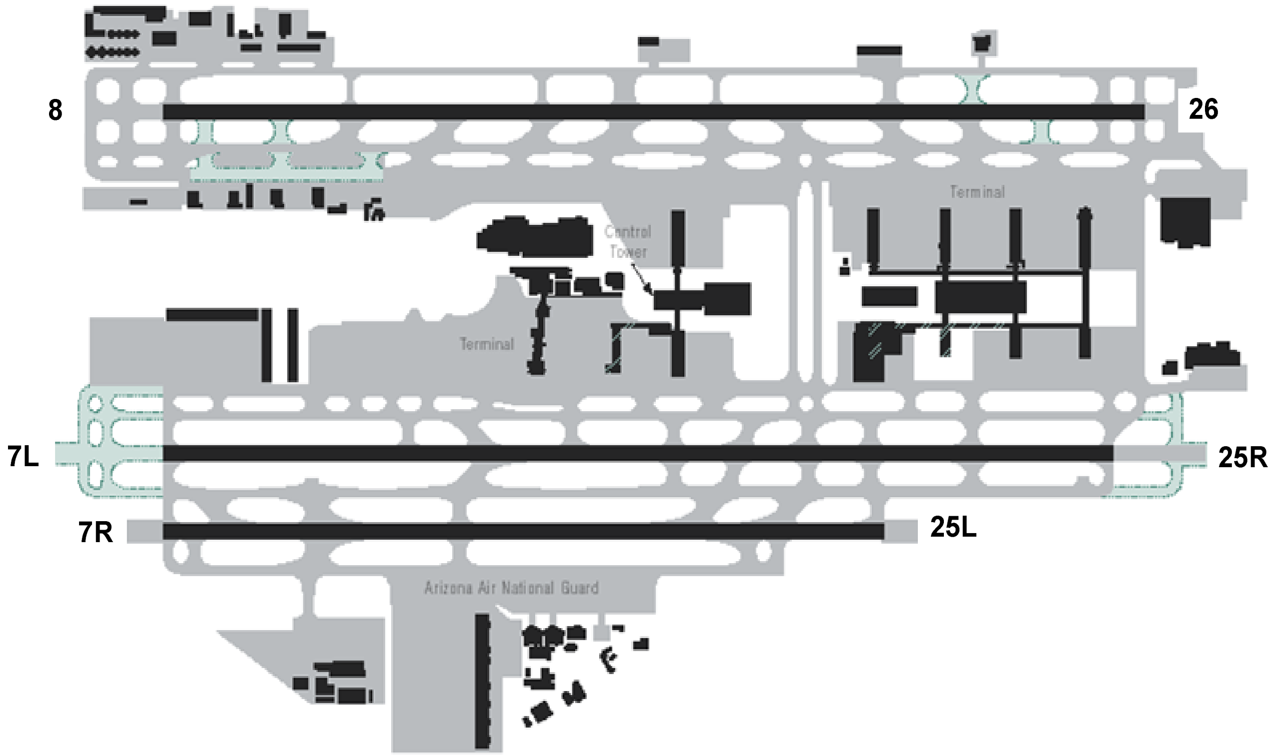
Airport is highly adaptable and re-configurable regarding VORs and runway lights.

Consequences of equipment outages are very much airport specific.

SFO is not sensitive to VOR unscheduled outages during IFR and VFR conditions.

ALSF-2 unscheduled outages during the IFR conditions do not cause capacity degradation.

Analysis of PHX Airport



Phoenix Sky Harbor International Airport

ATCRBS Results

Runway Configuration (arrivals l departures)	Weather Condition (IFR of VFR)	Dummy Variable	Estimated Affect on Throughput**	t-value	Significance at 0.05 Level	Number of Observations***
25L, 26 l	VFR	Outage* (occurred)	-0.94	-7.51	<.0001 (significant)	34486
25R	VFR	Outage	-0.53	-1.72	0.0856 (not significant)	34486
7R, 8 l	VFR	Outage	-1.14	-4.64	<.0001 (significant)	29404
7L	VFR	Outage	-1.13	-3.86	0.0001 (significant)	29404

* Outage = 1 if there was a ATCRBS outage during the period j; otherwise Outage = 0.
** Estimated change in quarter-hour throughput.
*** Each observation is 1 quarter-hour period.

Analysis Results for ATCRBS at PHX

ATCRBS Results

West flow (25L,26 | 25R):

• the arrival throughput decreased by 0.94 operation per quarter-hour,

East flow (7R, 8 | 7R):

• the arrival throughput decreased by 1.14 operations per quarter-hour

• departure throughput decreased 1.13 operations per quarter-hour

We found the quantitative evidence of the reduction in arrival and departure throughputs due to the outages of the main ATCRBS system.

Mode S Results

Runway Configuration (arrivals | departures)	Weather Condition (IFR of VFR)	Dummy Variable	Estimated Affect on Throughput**	t-value	Significance at 0.05 Level	Number of Observations***
25L, 26 |	VFR	Outage* (occurred)	-0.6	-4.88	<.0001 (significant)	34486
25R	VFR	Outage	-0.94	-5.08	<.0001 (significant)	34486
7R, 8 |	VFR	Outage	-0.81	-4.26	<.0001 (significant)	29404
7L	VFR	Outage	-0.96	-6.09	<.0001 (significant)	29404

* Outage = 1 if there was a Mode S outage during the period j; otherwise Outage = 0.
** Estimated change in quarter-hour throughput.
*** Each observation is 1 quarter-hour period.

Analysis Results for Mode S at PHX

Mode S Results

When PHX airport operated in the West flow (25L, 26 | 25R)
in the VFR conditions:

- the arrival throughput decreased by 0.6 operations per quarter-hour
- the departure throughput decreased by 0.94 operations per quarter-hour.

In the East flow, during the VFR conditions,
with aircraft arriving on runways 7R and 8:

- the throughput decreased by 0.81 operations per quarter-hour.

Under the same conditions, when aircraft departed from runway 7L:

- the throughput decreased 0.96 per quarter-hour.

The full outages of Mode S, due to the loss of the overlapping radar coverage, resulted in both arrival and departure throughput deteriorations.

√ **SERVICE AVAILABILITY**

√ **FAULT TREE ANALYSIS**

AIRPORT PERFORMANCE ASSESSMENTS:

√ **Censored Regression – Tobit Model**

Deterministic Queuing Model

Methodology

Deterministic modeling

(1)

A deterministic aircraft separation model is used to estimate airport/runway capacity. This method is useful for quick estimates of the number of aircraft operations per facility under some predefined conditions (i.e., mile-in-trail separation and aircraft mix). However, these methods do not provide delay estimates.

Methodology

(2)

A deterministic *queuing approach* is then used to estimate capacity and delays due to single outages for a hypothetical airport (i.e., to estimate the impact of outages on runway throughput) and terminal airspace area.

Deterministic queuing analysis is used for calculating aircraft *delays*, numbers of aircraft experiencing queuing, and queue duration. This method can handle traffic conditions where both the *arrival and service rates vary over time*.

Deterministic Aircraft Separation Model:
considers arrivals only, and assumes that the runway occupancy time is not the bottleneck in the system

T_i : time when lead aircraft *i* passes over runway threshold

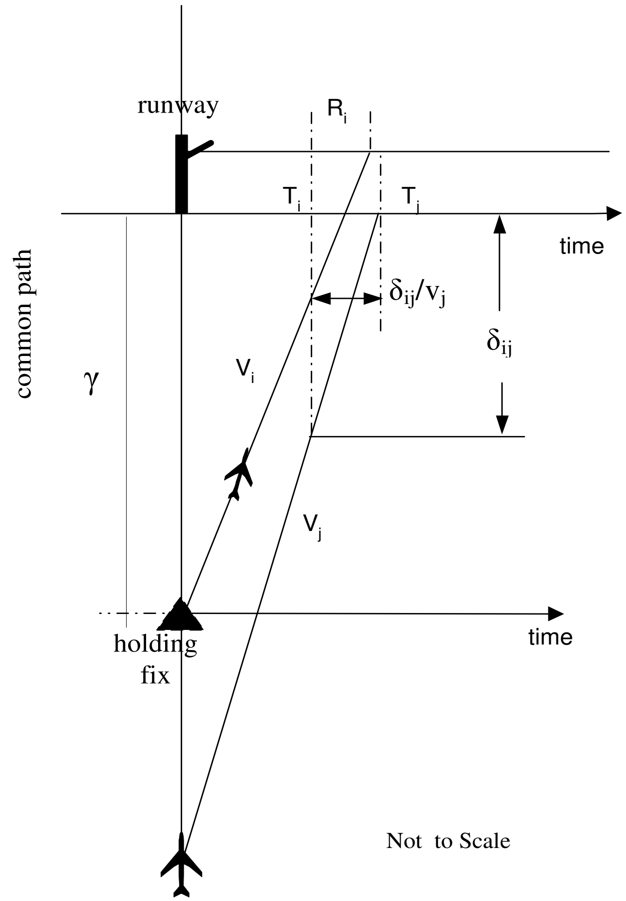
T_j : time when following aircraft *j* passes over runway threshold

$[T_{ij}] = T_j - T_i$: matrix of actual time separations at runway threshold for two successive arrivals, an aircraft of speed class *i* followed by an aircraft of speed class *j*

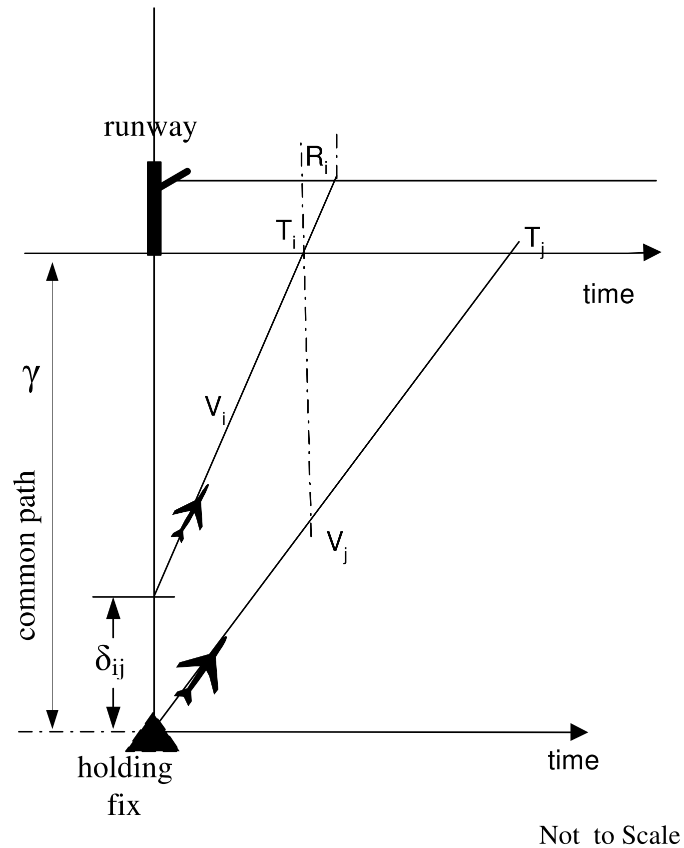
p_{ij} : probability that a lead aircraft of class *i* will be followed by a trail aircraft of class j

$E[T_{ij}] =$: expected value of T_{ij}, i.e., mean service time

Deterministic Aircraft Separation Method



Case: $V_i < V_j$



Case: $V_i > V_j$

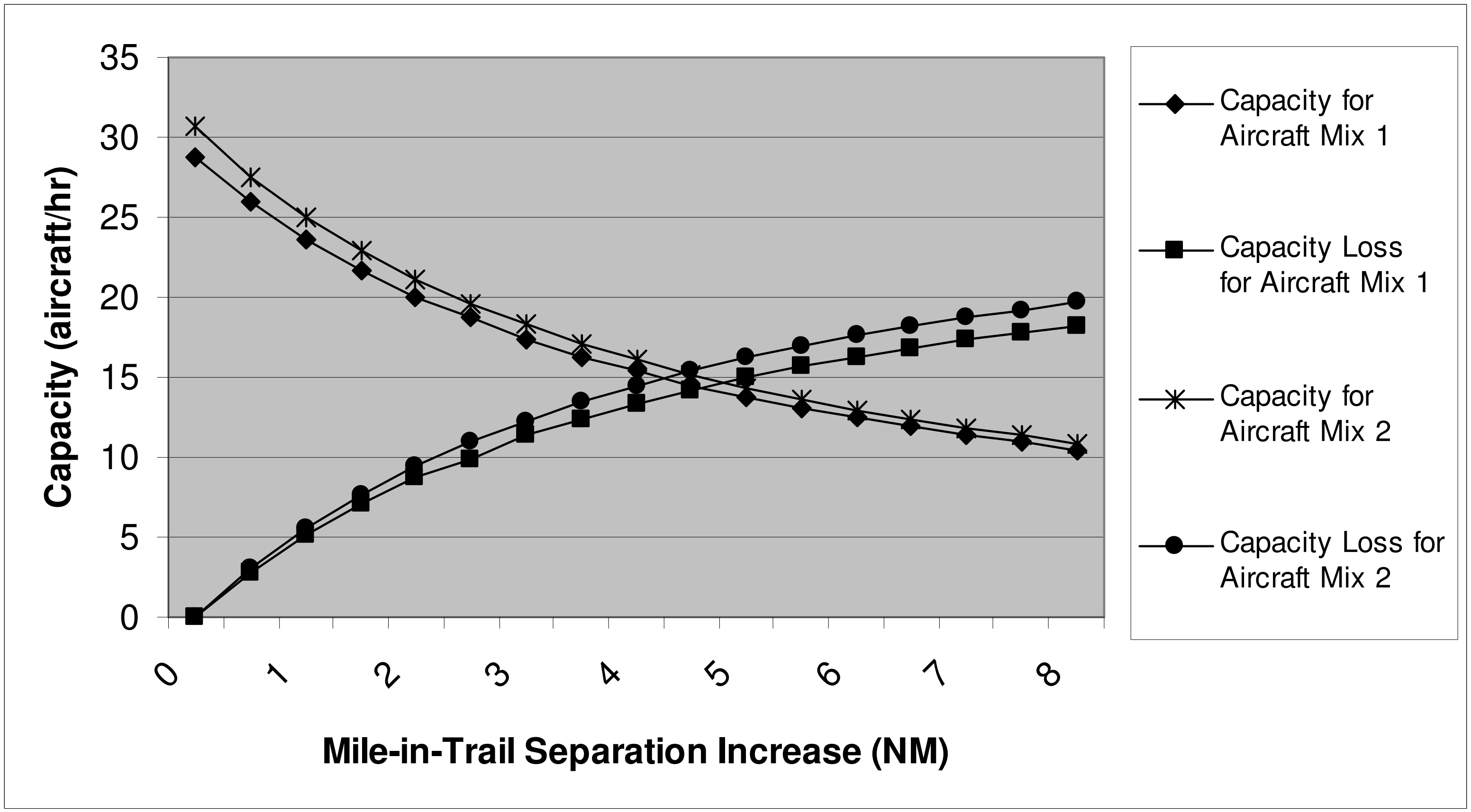
Source: Horonjeff (1994)

Capacity is $C_r = \dfrac{1}{E[T_{ij}]}$

Degraded Capacity

$$c_r^d = \begin{cases} 0 & \text{, if ILS fails} \\ \dfrac{1}{E[\textit{affected matrix}\ T_{ij}]} & \text{, if other equipment fails} \end{cases}$$

Capacity Loss $c_r^l = \begin{cases} all\ (C_r) & \text{, if ILS fails} \\ C_r - \dfrac{1}{E[\textit{affected matrix}\ T_{ij}]} & \text{, if other equipment fails} \end{cases}$



Airport Capacity and Capacity Loss for Different Mile-in-Trail Separations and Aircraft Mixes

Varying Service Rate Case

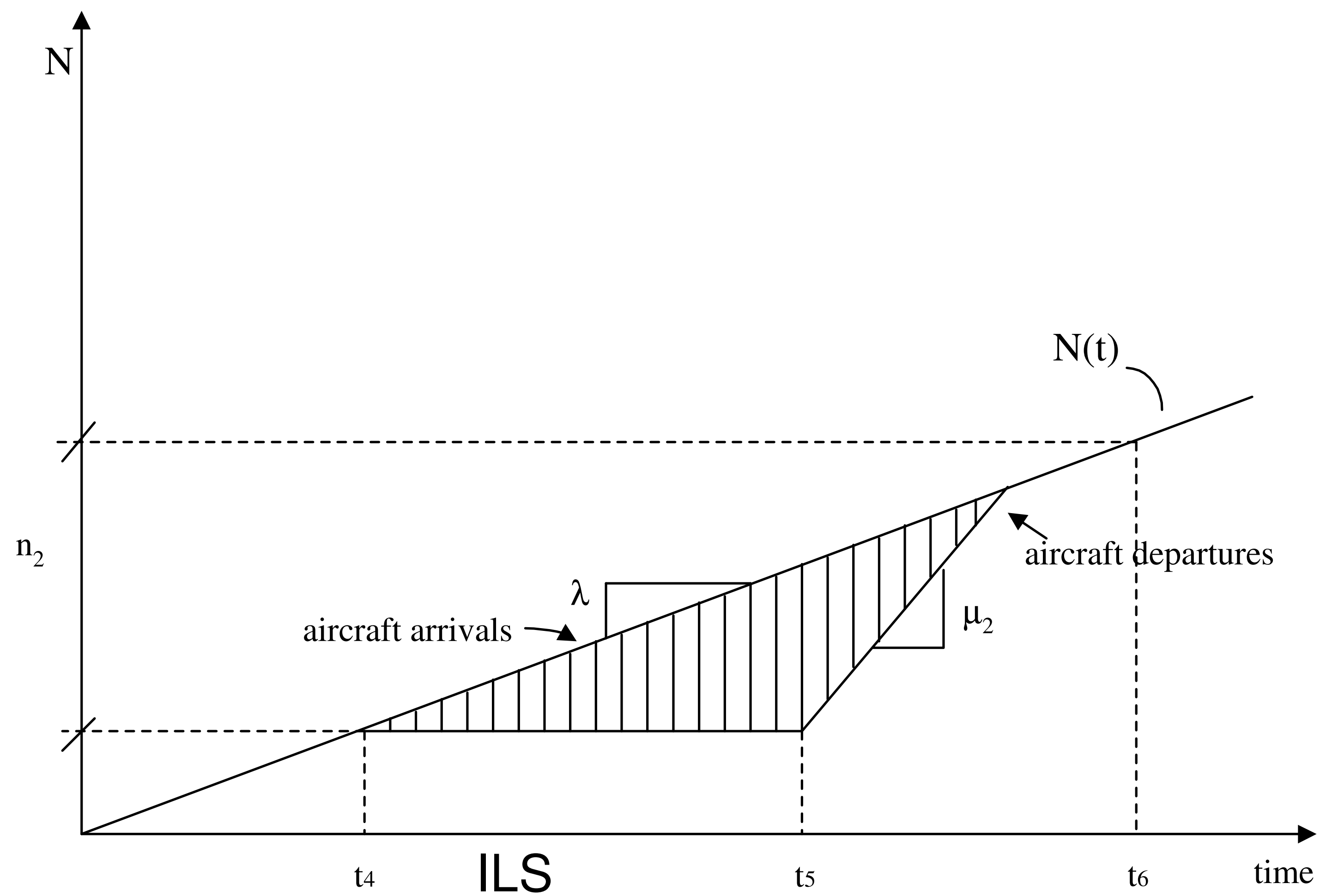
Deterministic queuing analysis is applied at a macroscopic level, i.e. by modeling continuous aircraft flows rather than individual aircraft.

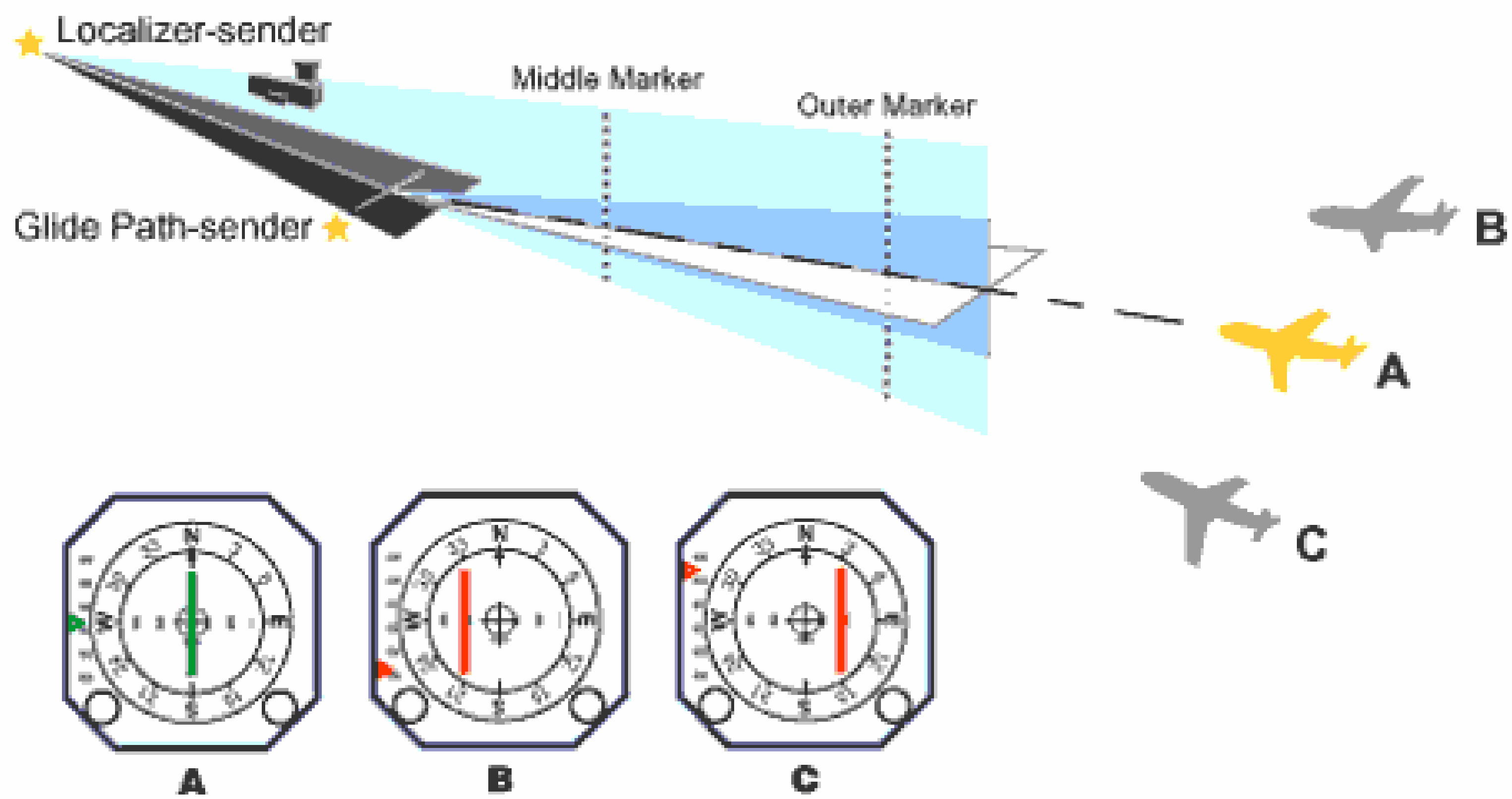
Pulsed Service Problem

The arrivals to the terminal area or an airport (i.e., runway) have a constant arrival rate (λ = aircraft/hour) but the service rate (μ = aircraft per hour) is "pulsed" (time-dependent) and may be defined as follows:

$$\mu = \begin{cases} 0 & \text{, if the ILS or any equipment that closes the server is out} \\ \mu_2 & \text{, if the equipment functions} \end{cases}$$

Deterministic Queuing Diagram for ILS Outages





Instrument Landing System (ILS)

On-board ILS Gauge from a Boeing 747-400 Aircraft

The following measures can be calculated for given

e: time equipment is functioning

r: outage time and

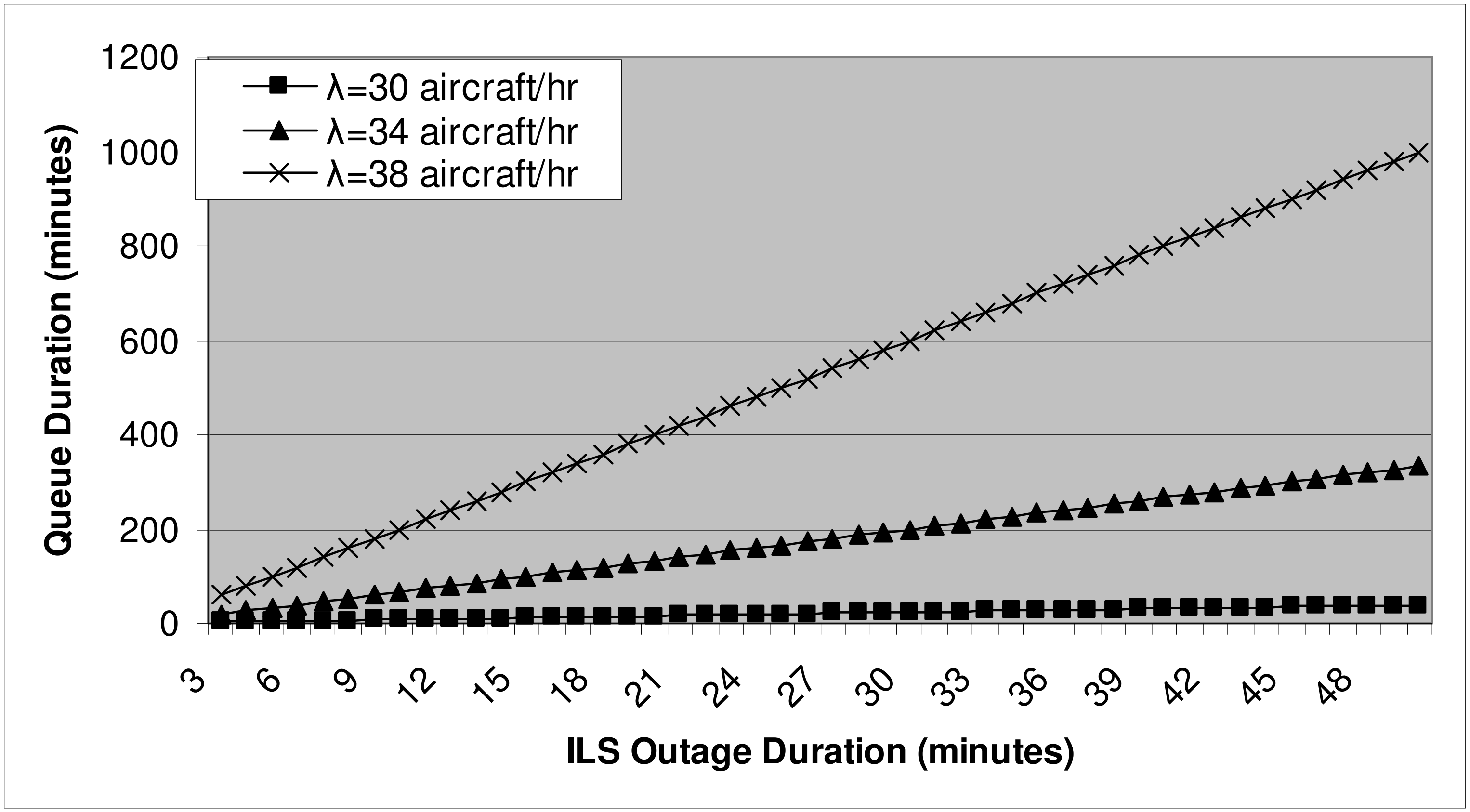
L: time length (L=e+r):

1) Queue duration: $$t_Q = \frac{\mu_2 \times r}{\mu_2 - \lambda}$$

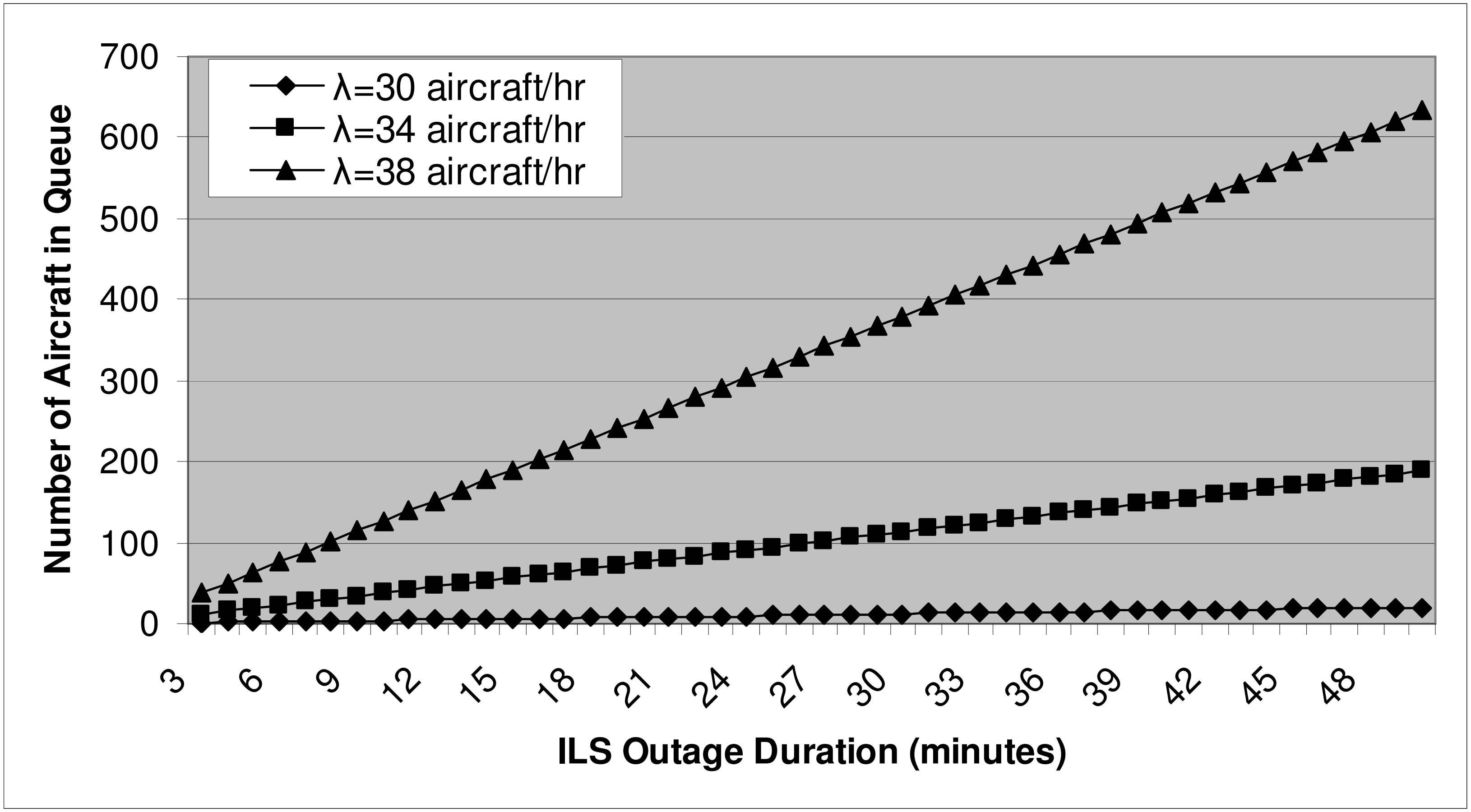
2) Number of aircraft experiencing queue: $$N_Q = (\lambda \times t_Q)/3600$$

3) average aircraft delay: $$d = \frac{r \times t_Q}{2L}$$

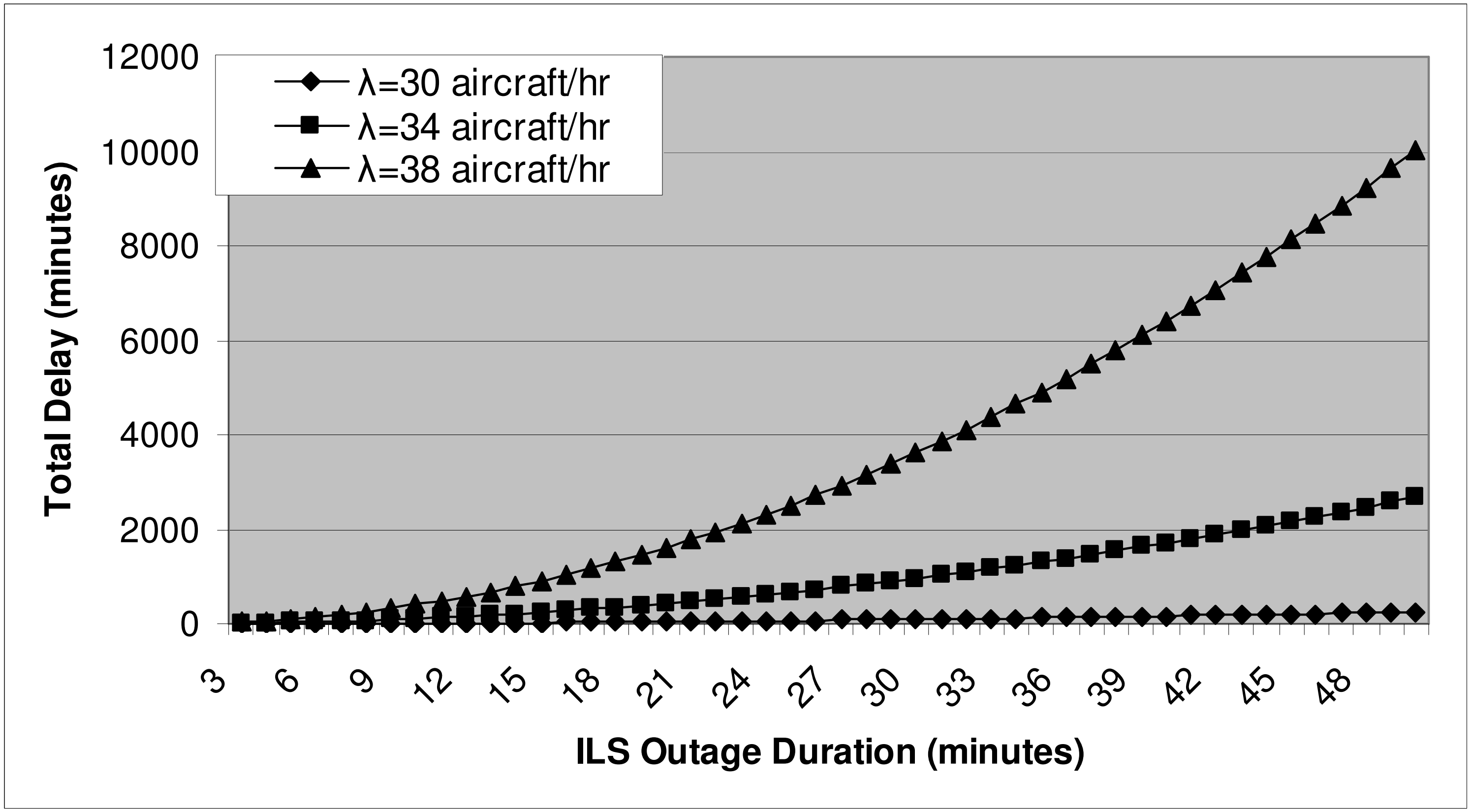
4) total delay: $$T_d = \frac{r \times t_Q \times \lambda}{2}$$



Aircraft Time in Queue for Various ILS Outage Durations



Number of Aircraft in Queue for Various ILS Outage Durations



Total Delay for Various ILS Outage Durations

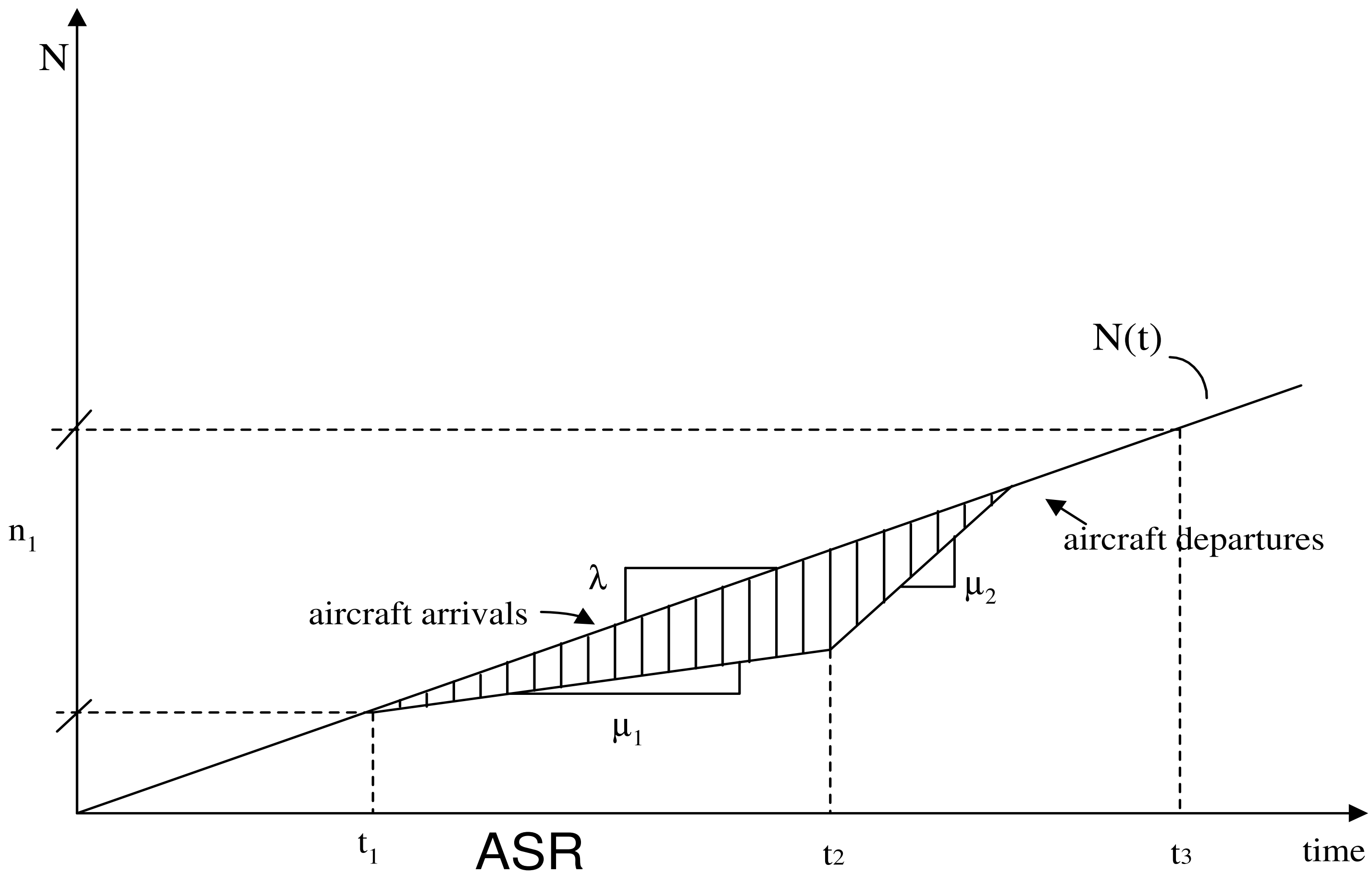
This model is applicable to the precision approaches for CAT I, II and III.

Varying Service Rate

- the arrival rate is constant

- the service rate is varied (i.e., degraded) due to the equipment failures but the server (i.e., runway) is not completely closed

Deterministic Queuing Diagram for ASR Outages



The service rate is defined as:

$$\mu = \begin{cases} \mu_1 & \text{, if ASR or any other equipment that degrades server fails} \\ \mu_2 & \text{, if equipment functions properly} \end{cases}$$

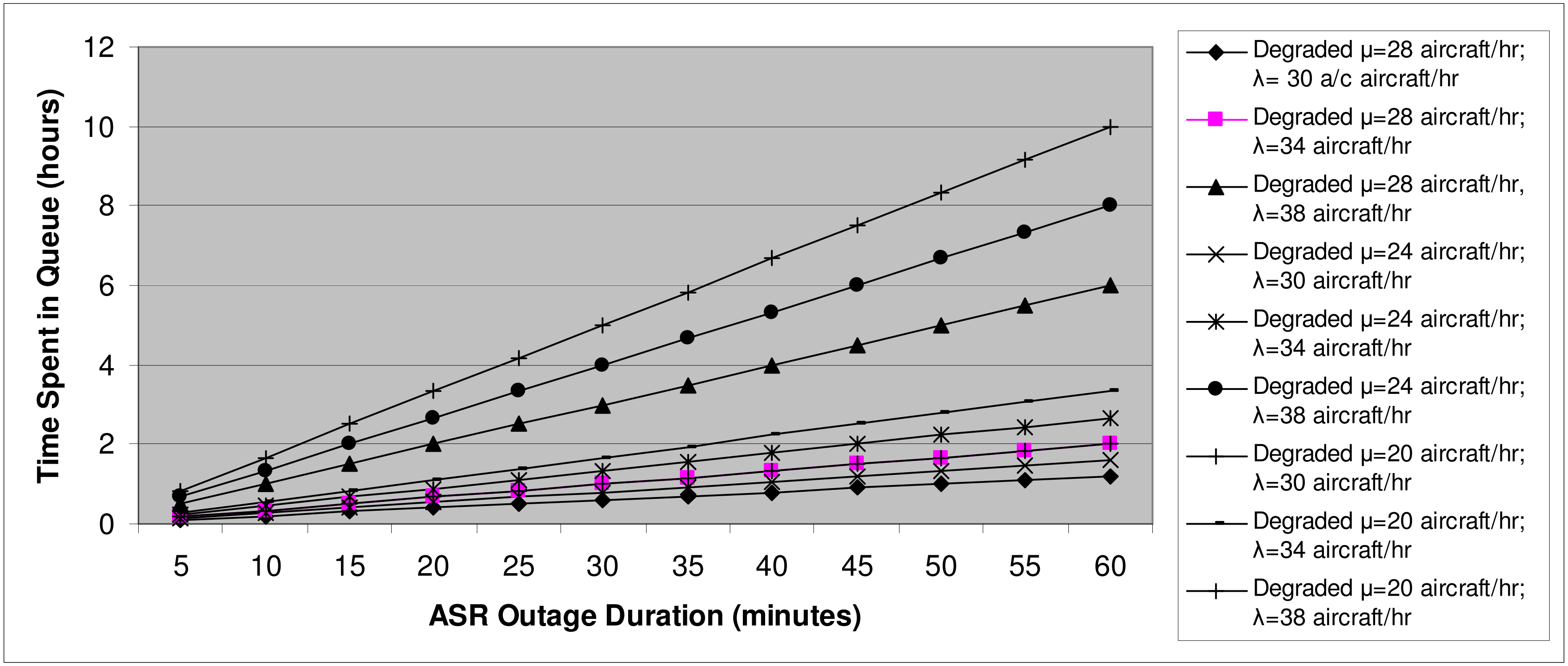
The same measures could be calculated:

1) Queue duration

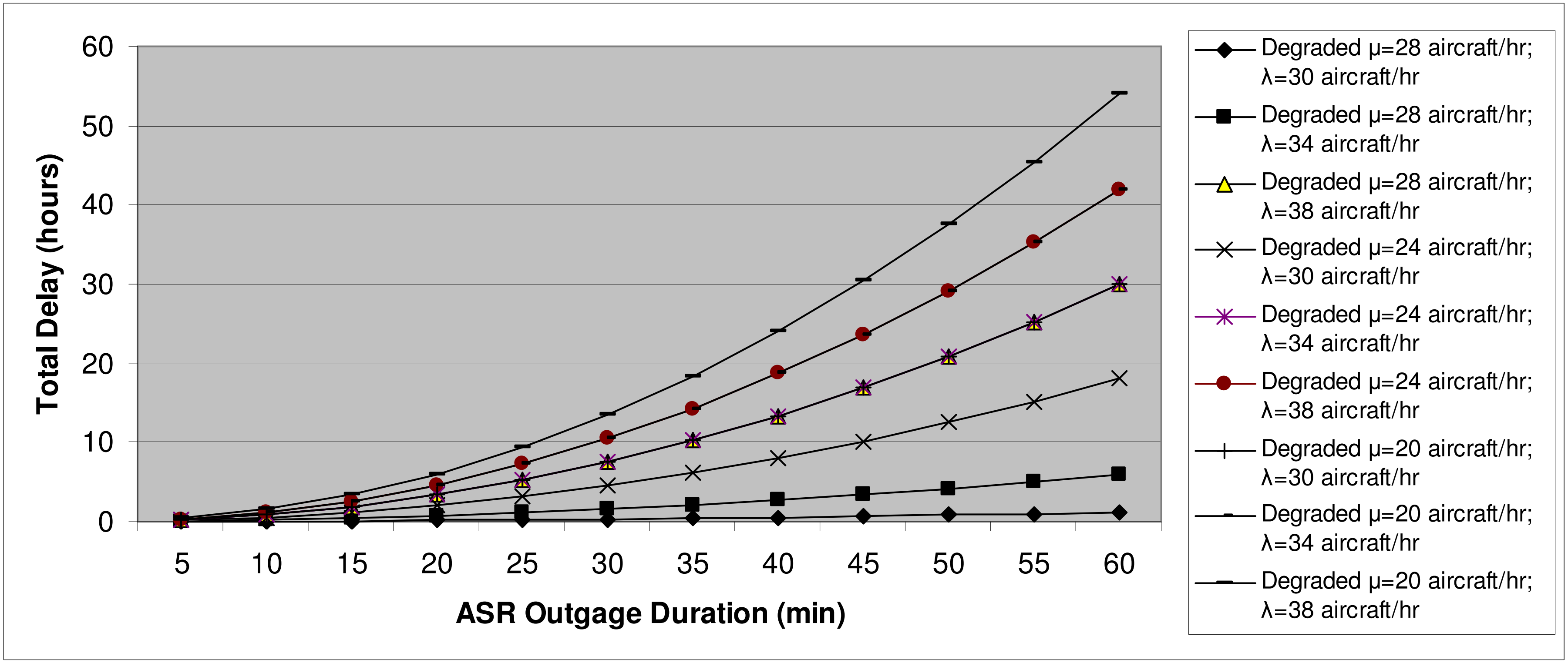
2) Number of aircraft experiencing queue

3) Average aircraft delay

4) Total delay



Aircraft Time Spent in Queue for Various ASR Outage Durations and Degraded Service Rates and Arrival Rates



Total Aircraft Delay for Various ASR Outage Durations, Degraded Service Rates and Arrival Rates