

The Economic Impact and Value of Aviation Infrastructure

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Motivation

Continuing pressure to justify investments in R&D and public aviation capital

- Peripheral involvement in some of these episodes
- □ What do we really know?





Questions

What is the value of our aviation infrastructure?

- Do current studies correctly represent that value?
- What does the aviation infrastructure do that's worth doing?





Outline

 Economic Impact Studies
 Aviation Infrastructure and Economic Growth
 Economic Benefits of Aviation Infrastructure Investment





Economic Impact Studies

Recent examplesThought experimentsConclusions





Aviation's Economic Impact







Price-rise scenario: GDP

Aviation Contribution to GDP







Aviation Economic Impact (Wilbur Smith Version)

- Primary Direct Impacts: Activity of firms providing aviation services, such as airlines, FBO's, aircraft manufacturers, flight schools, ATC, etc.
- Primary Indirect Impacts: Activity of firms serving aviation visitors
- Secondary Impacts
 - Intermediate: Activity of suppliers to firms providing aviation services or serving aviation visitors
 - Activity generated by households who derive income from the primary and secondary impacts





- Spending (Economic Activity)
 Total expenditures by all economic units
 Same \$ counted multiple times: for example pax→airline→manufacturer
 Earnings
 - Personal income generated
 - □ Not subject to double counting
 - Comparable to GDP
- Jobs





Aviation Economic Impact (DRI-McGraw Hill Version)

- Direct goods and services provided by the civil aviation industry such as:
 - Scheduled and unscheduled commercial passenger and cargo operations, and general aviation (including business aviation and air taxi) operations
 - Manufacturing, servicing, and support including pilot and maintenance technician training
 - Services carried out at airports and flying fields, including maintenance and storage of aircraft, handling of air cargo and passengers, and air traffic control. Also included are all other government operations related to civil aviation.
- Indirect goods and services bought from the rest of the economy by the civil aviation industry
- Induced goods and services, which include:
 - Those purchased from the income provided to employees in the direct and indirect provision of goods and service
 - Direct (from the industry itself), indirect (from supporting industries) and induced impacts (from the spending of income generated) of industries for which air transportation provides an enabling function (e.g., travel and tourism and, broadly, economic development)





Economic Multipliers

The multiplier concept provides an excellent method of summarizing economic impacts. It is an effective way of relating one impact analysis to another.

The production multiplier is straightforward, and there is little controversy concerning its magnitude. For most industries, the indirect impact is generally about the same as the direct impact, giving a production multiplier—the ratio of the combined direct and indirect impacts to the direct impact alone [(Direct + Indirect)/Direct]—of about 2.0. However, both the aviation and tourism industries are characterized by high labor costs. As a result, the direct impact is higher than for most other industries, which reduces the production multiplier for civil aviation to 1.7 in this study.

The income multiplier is more controversial. It was once thought that the induced impact was about the same size, or even larger than, the combined direct and indirect impacts. The relation of these concepts is summarized in the income multiplier—the ratio of the combined direct, indirect, and induced impacts to the combined direct and indirect impacts [(Direct + Indirect + Induced)/(Direct + Indirect)]. Economists once thought that the income multiplier was limited only by savings and imports. This was one of the important results of Keynesian economics for public policy.

More recently, economists have recognized that supply-side limitations are more important. Thus, income multipliers, thought to be close to 2.0 or even higher in the past, have been found to be considerably lower. The DRI•WEFA U.S. macro model indicates an income multiplier of 1.5 for this industry.

As a result, the combined multipliers for this study give $1.5 \times 1.7 = 2.6$ rather than close to 4.0 found in other studies. Brief descriptions of the models and methodology are provided in Appendix A





- A family spends \$2500 on a trip to Disney world.
- □ That \$2500 includes
 - □\$1000 for the air fare
 - \$1500 for hotel, restaurants, rental car, park admission, etc.





How would this Impact on Impact?

	Spending	Earnings/Jobs
Primary Direct	Expenditures of airlines and other aviation firms resulting from \$1000 payment	Earnings/jobs of airline and aviation firm employees and owners resulting from \$1000 payment
Primary Indirect	Expenditures of hotels, restaurants, etc resulting from \$1500 payment	Earnings/jobs of hotel and restaurant employees and owners resulting from \$1500 payment
Secondary Intermediate	Expenditures of industries supporting airlines, hotels, etc resulting from primary expenditures	Earning/jobs of employees and owners of supporting industries resulting from primary expenditure
Secondary Induced	Increased household consumption of those gaining income from primary and secondary impacts	Personal earnings/jobs throughout economy resulting household consumption of those gaining income from primary and secondary impacts. 13





What is the Counterfactual?

- To define impact we must compare two alternative scenarios
- What is the alternative scenario in the previous example?
 - The household does not make the trip
 - The money spent on the trip is hidden under the mattress





More Realistic Counterfactuals

- Some of the \$2500 is spent on other consumption (also generates spending, earnings, and jobs)
- Some of the \$2500 is invested (also generates spending, earnings, and jobs)
- Lacking the need for the \$2500, the household works less (thus generating less spending, earnings, and jobs)
- Some of the time spent for the trip is used to work (thus generating more spending, earnings, and jobs)





GDP Implications of Counterfactual Scenario

- GDP=Consumption+Investment+Gvt.Expenditures+ Exports-Imports
- Under unchanged earnings scenario
 - Consumption+Investment unchanged
 - □ Imports may increase or decrease
 - Induced consumption will increase or decrease
- Under changed earnings scenarios
 - Consumption+Investment may either increase or decrease
 - Imports may increase or decrease
 - Induced consumption will increase or decrease





Conclusion

The family trip to Disneyland has no clear implication for aggregate economic activity in terms of spending, earnings, jobs, or GDP.





Business Trips

- GDP includes sum of value added of production units in the economy
- □ If a \$2500 business trip occurs
 - Total direct and indirect value-added of firms providing travel and their suppliers will increase \$2500
 - Purchases of intermediate goods by traveler's firm will increase at least \$2500, reducing the value-added of the firm by \$2500
 - If trip is successful, \$2500 purchase will be more than counteracted by benefits (such as increased sales) resulting in net increase in value-added
 - But value-added of competing firms may decrease





Conclusion

The family trip to Disneyland has no clear implication for aggregate economic activity in terms of spending, earnings, jobs, or GDP.





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Growth Theory

- Why does the GDP grow?
- Classic formulation:
 - □ Actual GDP depends upon
 - □ Productive capacity (Potential GDP)
 - Demand
 - □ If demand < potential GDP
 - Recession
 - Labor and capital underutilized
 - Fiscal policies to encourage growth in demand
 - □ If demand > potential GDP
 - Demand temporarily satisfied by "overproduction"
 - Inflation
 - Fiscal policies focus on keeping demand close to potential GDP in short run
 - Productivity growth and increases in available inputs allow potential GDP to increase in long run





Aviation Economic Impact Studies Revisited

- Impact studies focus on the demand side of GDP
- If impacts were real, they have little policy significance

Impacts of policies would be long term

Demand-side issues are short term

The real question is: how do aviation infrastructure investments affect productive capacity of the economy?





Aviation and the Growth of Potential GDP: Two Perspectives

Aviation as an input to production

Aviation as a stimulus to innovation





Aviation as an Input to Production

- Aviation Infrastructure as social overhead (public) capital
- Studies examine relationship between GDP (output) and inputs including
 - Labor
 - Private capital
 - Public capital





Aviation Infrastructure as Production Input

"The ultimate aim as a means of communication must be to reduce not the costs of transport, but the cost of production." Jules Dupuit, "On the Measurement of Utility in Public Works," 1844



GDP Production Function

$$Y = A \cdot F(K_P, K_G, L) = A K_P^{\alpha} K_G^{\beta} L^{\gamma}$$

Where: *Y* is GDP K_P is private capital K_G is public capital *L* is labor





Aschauer Analysis

- Time series analysis of post-War US data
- Effect of public capital found to be very strong
- \$1 of public capital yields \$.60 of increased GDP
- Implied underinvestment in public infrastructure
- Spawned much controversy and subsequent analysis
- □ See FHWA web site for summary





Issues

- □ Are statistical results realistic?
- □ What is the direction of causality?
- Public investment as a stimulus for private investment.
- Heterogeneity of public capital
 - Different infrastructures
 - Good investments and bad investments
 - No studies specifically look at aviation infrastructure





- Used aviation activity variables (passengers and freight enplaned) in state-level production functions
- Found that, all else equal, states with more aviation activity have higher output
- Freight effect is stronger and more statistically significant than passenger effect





- Examined relationship between foreign direct investment in the United States and the initiation of international air service
- Found evidence that foreign direct investment increases after initiation of air service to the investor country



Aviation as a Stimulus to Innovation

- Initial impact of improvements is to do old things better
- Ultimate value rests on combining improved transport with other things
 - Do old things in new ways
 - Do new things
- These "companion innovations" by users of transportation systems drive growth and economic benefit





Examples

- Bi-coastal households and extended families
- Theme parks with nation/international market areas
- One-day meeting
- International corporations
- Organ donor networks





Technological Life Cycle

- System goes through processes of birth, growth, and maturity
- Predominant technology and initial uses of system established during birth phase
- Growth phase features rapid increases in traffic and scaling up of system, accompanied by continued discovery of new uses
- Maturity phase features slowing traffic growth
 - Uses fully explored and diffused throughout society (stable demand curve)
 - Scale and structure makes meaningful innovation and performance improvement difficult (stable supply curve)





Logistic Curve (S-Curve) (see Grubler, The Rise and Fall of Infrastructures)

- Relates life-cycle to long term evolution of traffic and other system status variables
- Growth in traffic proportional to product of existing traffic and potential additional traffic:

$$\frac{dX}{dt} = \frac{\alpha}{K} X(K - X)$$

Solution is Lotka equation:

$$X = \frac{\kappa}{1 + \exp(-\alpha(t - t_0))}$$

 \boldsymbol{V}

Interpretation

 \Box *K* is saturation traffic level

 \Box *t*⁰ is time when traffic reaches half of *K*





Applications to Air Transport







Outline

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 Economic Benefits of Aviation Infrastructure Investment




Willingness-to-Pay

 Fundamental concept in assessing benefits
Net benefit of an infrastructure investment is (arguably) positive if:

 $\sum WTP > 0$

everyone

- □ In this case can find way to distribute benefits so that everyone is better off
- Premise for benefit-cost analysis



Issues with CBA/WTP

- □ Some WTP's may be negative
- WTP not equal to what is paid
- Thus projects with net benefit can be costly or harmful to some
- Best viewed as a "constitutional principle" that everyone accepts knowing that, over many projects, they will come out ahead



WTP, Utility, and Demand

- Consumers and firms acquire goods and services, mostly through purchase
- Derive benefit, welfare, utility ... from these goods and services
- Have preferences among different "bundles" of goods and services





Trends in Personal Consumption

No. 1433. Personal Consumption Expenditures, by Type: 1929 to 1997

Year	Total ¹	Food and tobacco	Cloth ₂ ing ²	Per- sonal care	Hous- ing	House- hold opera- tion	Medical care	Per- sonal busi- ness	Trans- porta- tion	Recre- ation
1929 1933 1935 1940 1945 1945 1950 1955 1960 1955 1960 1970 1975 1980 1985 1990 1995 1997	77.5 45.9 55.9 119.9 192.7 259.1 332.2 444.3 648.1 1,029.1 1,760.4 2,704.8 3,839.3 4,953.9 5,493.7	21.2 12.8 17.6 22.0 43.5 58.1 73.6 89.2 108.8 154.6 238.2 376.2 497.3 672.5 780.4 832.3	11.2 5.4 7.0 8.9 19.6 23.7 28.4 32.7 41.4 57.6 85.6 132.3 188.3 262.7 321.8 353.3	1.1 0.7 0.8 1.0 2.0 2.4 3.7 5.6 8.1 11.8 16.7 26.6 39.1 57.3 71.8 79.4	11.7 8.1 7.9 9.7 12.8 21.7 34.4 48.2 65.4 94.0 147.0 255.2 407.1 586.3 750.4 829.8	10.7 6.4 7.7 10.4 15.5 29.1 37.3 46.7 62.1 84.8 135.4 232.6 342.0 436.2 559.4 620.7	3.1 2.4 3.2 9.4 14.2 22.1 34.1 60.0 107.9 206.4 366.7 615.6 875.0 957.3	3.9 2.5 2.8 3.2 6.6 10.1 14.6 20.9 32.0 53.0 101.2 182.6 290.1 388.8 459.1	7.7 4.0 5.4 7.2 6.8 25.4 34.9 42.9 59.1 81.1 130.2 238.4 372.8 463.3 574.1 636.4	4.4 2.2 2.6 3.8 6.2 11.2 14.6 18.5 26.8 43.1 70.5 116.3 185.9 281.6 404.2 462.9
1929 1950 1970 1990 1997	100.0 100.0 100.0 100.0 100.0	27.4 30.2 23.9 17.5 15.2	14.5 12.3 6.9 6.8 6.4	1.4 1.2 1.8 1.5 1.4	15.1 11.3 14.5 15.3 15.1	13.8 15.1 13.1 11.4 11.3	4.0 4.9 9.3 16.0 17.4	5.0 3.4 4.9 7.6 8.4	9.9 13.2 12.5 12.1 11.6	5.7 5.8 6.7 7.3 8.4

¹ Includes other categories, not shown separately. ² Includes accessories, and jewelny.

Source: U.S. Bureau of Economic Analysis, National Income and Product Accounts of the United States, 1929-94: Vol. 1; and Survey of Current Business, August 1998.





The 2-Good Case

Assume 2 goods

- One specific good that is of interest (air transport)
- One composite good that stands for all others

□Utility function becomes

$$U = U(X_1, X_2)$$









Non-Satiation: More is Preferred to Less

 $C, D, E, F, G \succ B$ $C, D, E, F, G \succ A$

 X_1





Indifference Curve Map







Perfect Substitutes and Complements











 X_1

 X_1^*



Utility Maximization

 X_2

- Maximize utility subject to a budget constraint
- Interior solution is point of tangency between budget line and indifference curve
- Corner solution if there is no such point for X₁,X₂>0
- Solution is unique if indifference curves are convex

-P2/P1







Normal and Inferior Goods

- Normal Good--As income (budget) increases, utility maximizing amount increases
- Inferior Good--As income (budget) increases, utility maximizing amount decreases
- Luxury Good—Consumes larger share of budget as income increases











Demand Curve







Compensated Demand Curve







Compensated Demand Curve











Welfare Measures--Compensating Variation





Consumer Surplus







Consumer Surplus







Change in Consumer Surplus from a Price Change $CS(P_2) = \int_{0}^{X_2^*} P(x) dx - P_2 X_2^*$ P_2 $\Delta CS(P_2 \rightarrow P_2) =$ $\left(\int_{a}^{X_{2}^{*'}} P(x)dx - P_{2}X_{2}^{*'}\right) - \left(\int_{a}^{X_{2}^{*}} P(x)dx - P_{2}X_{2}^{*}\right)$ P₂' X_2^* X_{2}^{*}





Rule of 1/2

- If price changes are moderate, then demand curve can be approximated as straight line between old price and new price.
- $\square \text{ Then } \Delta CS(P \rightarrow P') = (P P')(X + X')/2$







CS,EV, and CV

- Equivalent variation is CS using compensated demand curve at higher utility level
- Compensating variation is CS using compensated demand curve at lower utility level
- CS based on uncompensated demand curve is between EV and CV





Implicit Price Changes

- Change in service level can shift demand curve up or down
- Estimate price change that would produce the same shift
- Estimate benefits from change in service level as equivalent to from this price change





Implicit Price Change

Shift in demand from D to D' as a result of service improvement has same benefit as reduction in price from P to P' on original demand curve.







- Sensitivity of demand curve to price
- Dimensionless and thus insensitive to units in which price and demand are measured
- Assume "all else equal" including incomes, service quality, and other prices
- Two types Arc Elasticities $\eta_{arc} = \frac{\Delta Q}{\Delta P} \cdot \frac{\overline{p}}{\overline{q}}$ Point Elasticities $\eta_{point} = \frac{\partial Q}{\partial P} \cdot \frac{p}{\overline{q}}$





Summary of Elasticity Estimates

Category	Number	Median	First Quart.	Third Quart.
All	274	-1.15	-1.52	-0.68
Long-haul	105	-0.95	-1.43	-0.50
Short/Med. Haul	124	-1.15	-1.54	-0.73
Long-haul Inter.	69	-0.79	-1.40	-0.35
Long-haul Dom.	41	-1.34	-1.55	-0.85
Long-haul Inter. Bus.	16	-0.26	-0.48	-0.20
Long-haul Inter. Leis.	55	-0.99	-1.65	-0.54
Long-haul Dom. Bus.	26	-1.15	-1.43	-0.84
Long-haul Dom. Leis.	9	-1.26	-2.03	-1.09
Short-haul Bus.	18	-0.73	-0.80	-0.61
Short-haul Leis.	19	-1.52	-1.74	-0.88
Cross-section	85	-1.33	-1.52	-0.81
Time Series	156	-1.02	-1.46	-0.50
Income	132	1.39	0.84	2.17 ₅ .





Application: Benefits of Hubbing to Hub Regions

- Hansen (1998) estimates that local traffic has as an elasticity of 0.3 with respect to the hub traffic multiplier (total traffic/local traffic)
- Suppose hub region has originating traffic of 4 million and total traffic of 10 million (multiplier is 2.5)
- Assuming constant elasticity, this means that without hubbing, local traffic would be:

$$Q_{nohub} = 4 \cdot (1/2.5)^{0.3} = 3$$





Application (cont.)

- Suppose average fare per origination is \$200
- Using fare elasticity of -1, the fare would have to increase to \$267 cause traffic to go from 4 million to 3 million
- \Box By rule of $\frac{1}{2}$, benefit from hubbing is:
- \$67x(4 million+3 million)/2=\$234 million





Why Do Airlines Hub?

Logistics Perspective
Link Economies of Scale
Economies of Stage Length
Economies of Integration
Economics Perspective
Competitive Strategy
Structure-Conduct-Performance Paradigm





Link Economies of Scale

Elements of total logistics cost (TLC) for airline service Aircraft operation Passenger travel time □ Schedule delay Stochastic delay Accommodating increased flow on a link □ Increase load factor □ Increase frequency □Increase aircraft size





Link Economies of Scale

- Increase load factor
 - Unit operation cost decreases
 - Stochastic delay increases after a certain point
- □ Increase frequency
 - □ Schedule delay decreases
 - □ Stochastic delay decreases
- Increase aircraft size
 - □ Unit operation cost may increase or decrease
 - Stochastic delay decreases (for given load factor)
- It is generally possible to accommodate increased flow in a manner that decreases unit TLC







Implications of ESL



Is more efficient than







Economies of Integration

 One-airline itineraries better than twoairline itineraries
Transaction costs
Connection costs
Consumer confidence




Disaggregate Choice Models

- Model choices between discrete alternatives at individual level
- Assume choice behavior is utility maximizing
- Early applications in transportation, but now used (and abused) widely





Utility-Based Approach

- Assumes that individuals make rational choices
- Basis for choice is maximization of utility--level of satisfaction the traveler attains
- Utility is function of attributes of alternative, characteristics of choice maker/choice context





Decision Tree







Aviation Choice Alternatives

Routes
Airline+Route
Airline
Airport
Airport+Airline
etc





Characteristics and Attributes

- Traveler
 Characteristics
 Income
 Trip purpose
 Travel party size
 Frequent Flier Affiliation
- Alternative
 - Attributes
 - Fare
 - □# of stops
 - Circuity
 - Frequency
 - Aircraft Size





Logit Model

□ Utility=Deterministic Utility+Stochastic Utility $U_{im} = V_{im} + \mathcal{E}_{im}$ $= V_m(Z_i, S_{im}) + \mathcal{E}_{im}$

Where \$\mathcal{E}_{im}\$'S\$
 are independently, identically distributed
 have a Gumbel distribution:

$$P(\mathcal{E}_{im} < w) = \exp(-e^{-w})$$



With these Assumptions:

$$P(U_{im} = \max(U_{i1}...U_{in})|V_{i1}...V_{in}) = \frac{\exp(V_{im})}{\sum_{j} \exp(V_{ij})}$$

$$P(i's choice = m | V_{i1}...V_{in}) = \frac{\exp(V_{im})}{\sum_{j} \exp(V_{ij})}$$

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Route Choice Model

Table 1. Route choice model regression results			
Independent Variable (Parameter)	Estimated Value	Standard Error	
In(direct frequency) (ϕ_0) In(maximum hub frequency) (ϕ_1) In(minimum hub frequency) (ϕ_2) direct service utility (V_{det}) fare in S (α) circuity in miles (σ)	1.29 0.33 0.78 2.72 -0.0045 -0.0029	0.17 0.14 0.10 0.81 0.0010 0.00026	

 $R^2 = 0.74$ Number of Observations = 271





NAS Equilibrium Flow Model

- Given the OD traffic predict equilibrium
 - Segment and airport pax flows
 - Airport delays
- Assess how equilibrium affected by increase in ORD capacity





Equilibrium Flow Model







Hub Choice Model

- Allocates OD Traffic to Segment Traffic— Route (hub) Choice
- Nested Logit Model
 - Direct or one-stop connecting
 - Conditioned on connecting, choose the connecting airport(hub)
- Specification

$$V_{direct} = c_0 + b_{01} dist D_{od} + b_{02} \ln(paxD_{od}) + b_{03} HHI_{od}$$

 $V_{od,i} = b_1 distC_{o-i-d} + b_2 \ln(\max pax_{oi/di}) + b_3 \ln(\min pax_{oi/di}) + b_4 Delay_i$





Model Estimation

Associated Factor	Estimate	Standard Error	
	Parameter	(*10 ⁻⁵)	P-value
Dist. of Connect	-2.931	9.159	[.000]
ln(Max Pax of Connect)	0.278	2.267	[.000]
ln(Min Pax of Connect)	0.821	2.250	[.000]
Delay of Connect	-0.006	0.057	[.000]
β , 1/(inclusive value)	1.121	2.658	[.000]
Constant of Direct	4.624	30.707	[.000]
Dist. of Direct	-3.160	8.497	[.000]
ln(Seg. Pax of Direct)	1.033	1.326	[.000]
HHI of Direct	-0.435	4.522	[.000]

 $\hat{\rho}^2 = 0.5559$

N=39,298,503 (100,951 routes)





Policy Experiment— ORD Delay Improvement

Delay:

$$\ln(Delay_{it}) = \alpha_0 + \sum_{i=1}^{30} \alpha_i * C_i + \beta_1 * \ln(Pax_{it}) + \varepsilon_{it}$$

□ Airport fixed delay effect improved:

$$\alpha_{ORD} = 1.8846 \longrightarrow$$

 $\alpha_{ORD}' = \alpha_{ATL} = 1.4923$











Policy Experiment— New Equilibrium Delays



 ORD delay: reduce 12.0 (Flt/1000 Flt), about 27%

Delays of other hubs also reduce



Depiction with Supply and Demand Curves



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Benefit from Improvement



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Benefit Assumed without Demand Response





Ρ



Losses from Capacity Constraint: Five Easy Pieces

S

Q



Congestion costs to existing users.

Potential users priced off system due to congestion.

Additional losses to existing users from failure to realize economies of scale.

Additional losses to users priced off as a result of congestion due to failure to realize economies of scale.

Potential users priced off system due to failure to realize economies of scale.





Optimal Pricing and Investment

Given

- □Inverse Demand Function—P(Q)
- □User Cost Function—U(Q,K)
- □Supplier Cost Function—S(Q,K)
- Find
 - Optimal Q and K
 - Optimal user charge





Objective Function

Total user's willingness to pay

$$\int_{0}^{Q} P(q) dq$$

- Total User Cost:
- □ Total Supplier Cost:

 $-Q \cdot U(Q, K)$ $-Q \cdot S(Q, K)$



First Order Conditions $Z(Q,K) = \int_{0}^{Q} P(q)dq - Q \cdot U(Q,K) - Q \cdot S(Q,K)$ $\frac{\partial Z}{\partial Q} = P(Q) - Q \cdot \frac{\partial U}{\partial Q} - U(Q,K) - Q \cdot \frac{\partial S}{\partial Q} - S(Q,K) = 0$ $\frac{\partial Z}{\partial K} = -Q \cdot (\frac{\partial U}{\partial K} + \frac{\partial S}{\partial K}) = 0$

- The user with the least willingness to pay should be willing to pay the cost his use will impose on other users the supplier, as well as on himself.
- This implies a charge of: $Q \cdot \frac{\partial U}{\partial Q} + Q \cdot \frac{\partial S}{\partial Q} + S(Q, K)$
- The savings in user cost from the marginal investment should just offset the increase in supplier cost.





Special Case

$$U(Q, K) = U_0 (1 + (\frac{Q}{K})^2)$$

$$S(Q, K) = a + b \frac{K}{Q}$$

$$P(Q) - U_0 (1 + (\frac{Q}{K})^2) - \frac{2Q^2 U_0}{K} - a = 0 \Rightarrow \text{Charge} = \frac{2Q^2 U_0}{K} - a$$

$$\frac{\partial Z}{\partial K} = (\frac{-2U_0 Q^3}{K^3} + b) = 0 \Rightarrow K = \left(\frac{2U_0}{b}\right)^{1/3} Q$$