# Prototyping an Iterative Combinatorial Exchange

Airline Industry Wye River Workshop, June 23, 2004

David C. Parkes
Harvard University



# Why Markets?

- Takeoff and landing slots are a constrained resource
- Markets can facilitiate the efficient (re)allocation of slots:
  - administrative processes cannot do this
  - rationing cannot do this
  - multilateral negotiation cannot do this
- Markets can expose the true value of a slot and change strategic investment decisions:
  - new airport capacity
  - new technology, ...

# Why an Exchange?

- · Keep incumbents whole:
  - allocate initial property rights
  - don't force anyone to sell
  - not a new "taxation"
- · Allow new-entrants to compete
- Extensible:
  - bring in additional resources (in-route capacity, gates, other airports, etc.)
  - bring in additional players (e.g. airports)

#### vs. one-sided markets & vouchers

- more expressive, simpler

#### Why a Combinatorial Exchange?

#### Slots are complements:

- {9.00am landing & 10.00am takeoff} vs. {9.00am landing and 9.15am takeoff}
- {9am takeoff@Logan, 10am landing@LGA} vs.
   {9am takeoff@Logan, 9.20am landing@LGA}

#### Slots are substitutes:

- {9am landing, 9.05am landing} vs {9am landing}

#### • Business constraints:

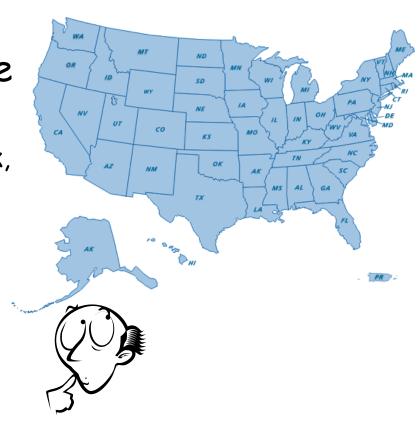
- "need at least 5 landing slots during peak M-F time"
- "need at least 2 landing slots between 10am and noon"

#### Alternative business plans:

- a) sell all slots, b) sell some slots, c) buy more slots

# Why an **Iterative** Combinatorial Exchange?

- Multiple rounds
  - allow participants to revise bids
- Important when good space is large and complex
  - 18 hrs, 4 blocks/hr, 10 slots/block,
     2 runways, M-F, Sat, Sun:
     approx 4320 items/airport
  - ten's of arlines, each with hundred's of flights a day
  - multiple airports



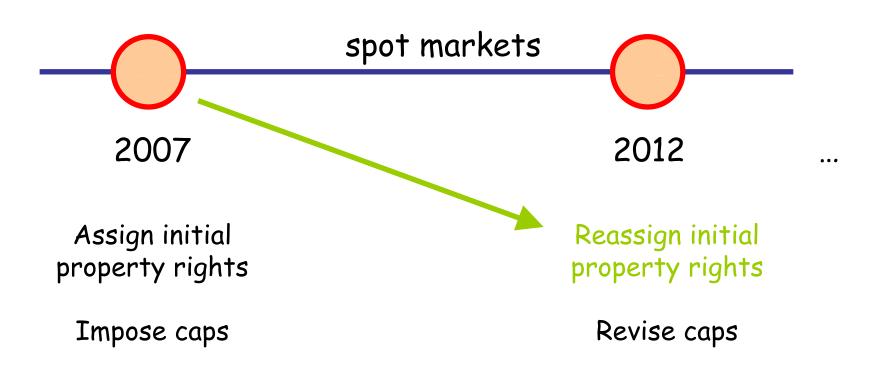
#### How might this work?

- Fix goals (safety, efficiency, regional access, ...)
- · Define goods, & assign initial property rights
- Host an exchange:
  - goods: landing (takeoff) slots for one plane
  - attributes: time of day, day of week, plane size, flexibility
- Can impose additional constraints:
  - maximal market share
  - minimal level of competition
  - minimal level of regional service



policy tools

#### Long-term vs. Spot markets



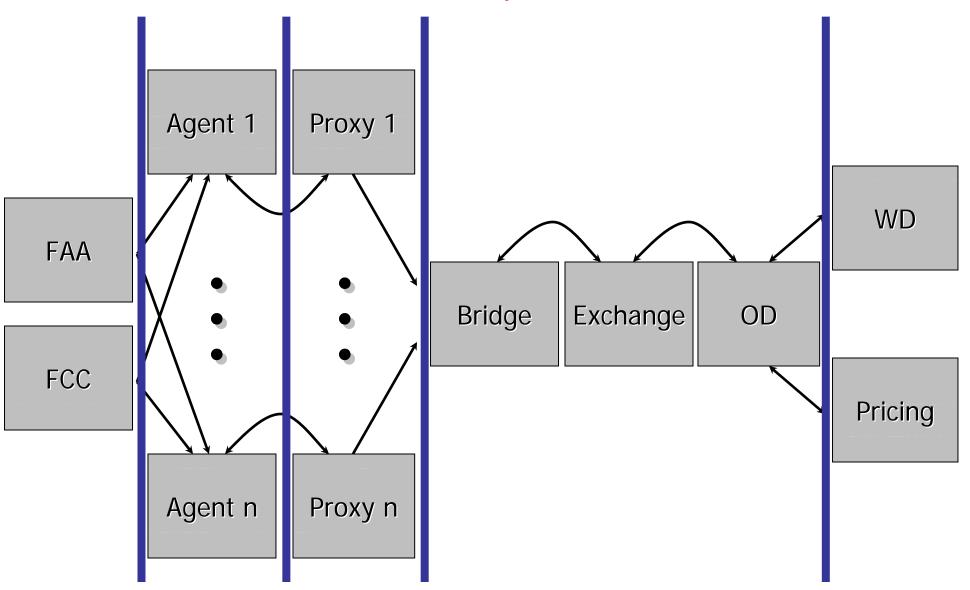
# Prototyping an Exchange

- Summer, 2001
  - proposed clearing rules for a one-shot combinatorial exchange (Parkes, Kalagnanam, Eso, IJCAI'01)
- October, 2001
  - presented exchange design to FCC-Wye river conference
- Spring, 2003
  - experiments on incentive properties of "Threshold" rule
- Summer, 2003
  - initial design for an iterative exchange
- November, 2003
  - presented iterative design to FCC-Wye river conference
- Spring, 2004
  - CS 286r: Project class focused on "Iterative Combinatorial Exchanges"
  - www.eecs.harvard.edu/~parkes/cs286r
  - study FCC and FAA domain problems

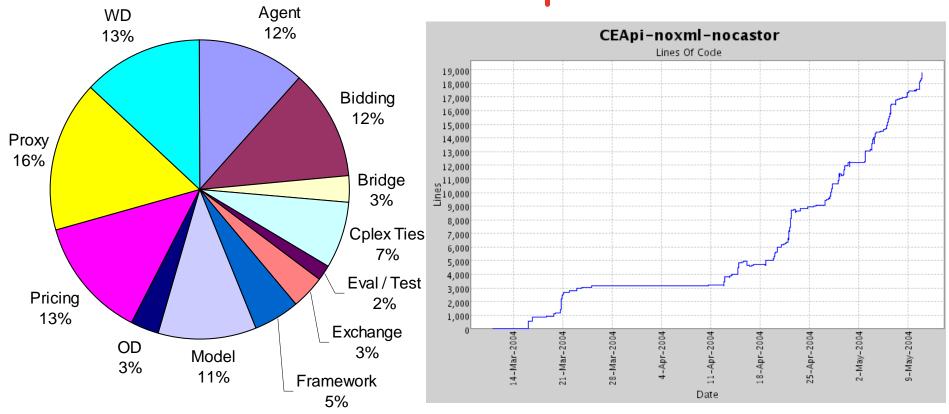
# Exchange Design

- Bidding Language
  - expressive, compact
- Winner determination
  - scalable
- Feedback
  - prices
- Activity rules, termination
  - drive progress
- Distribution
  - final payments

# Four Components



#### Code Development



Eclipse development environment, Java, CVS support Interfaces between components, Design for threading and distributed processing

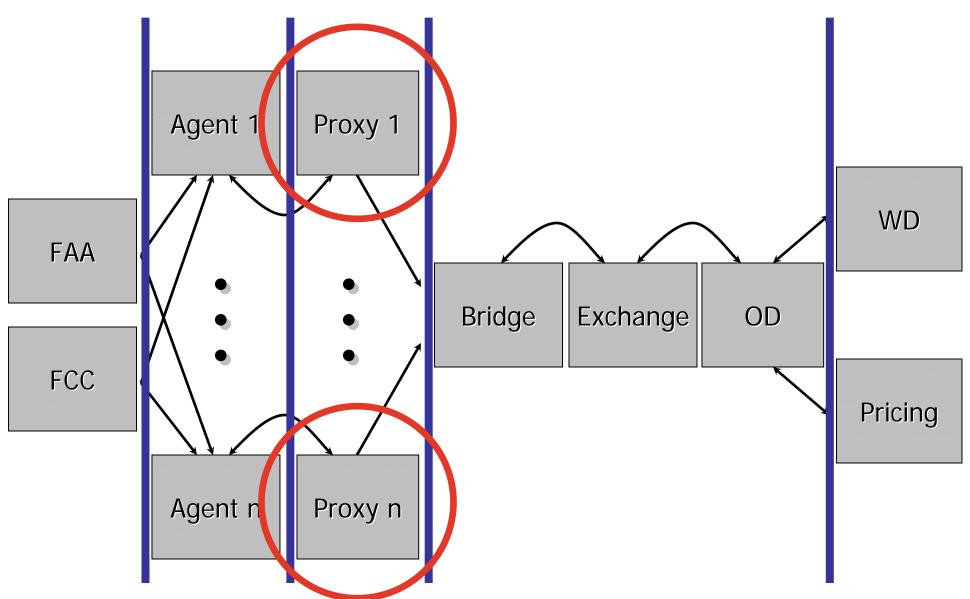
CPLEX RMI servers, sitting behind a load balancer

XML language for component specifications, and simulation infrastructure Run on two, four-processor, Blade machines

#### Class mantra

"No enumeration of goods..."

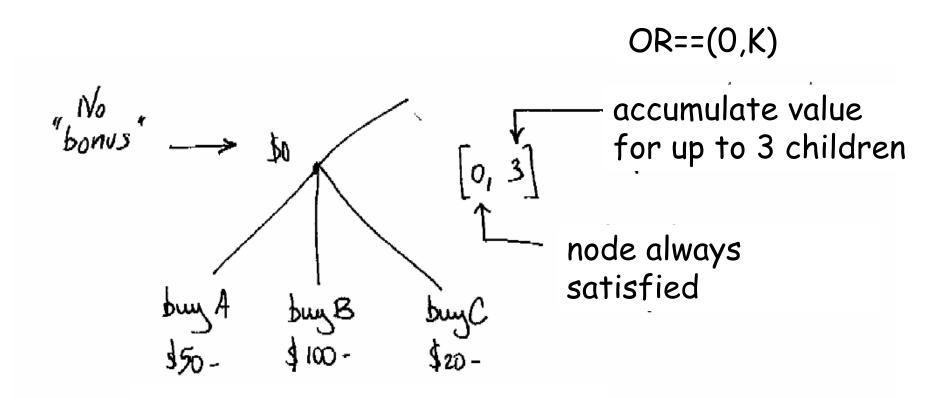
#### First Component: Bidding Language



# Bidding in the Exchange (related to Boutilier's LGB language)

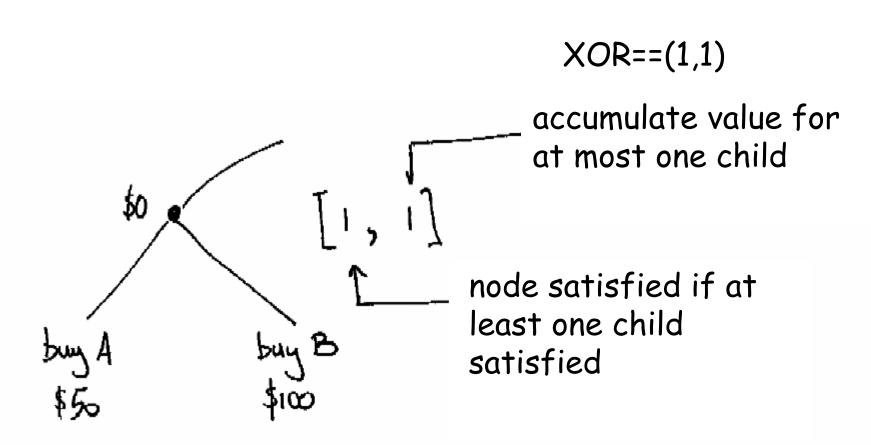
- Compact and Expressive bidding language
  - logical structure ("one of", "all of", "some of",...)
  - goods at leaves ("buy A", "sell B")
- Buyer:
  - define value for acquiring new slots
- Seller:
  - define value (negative) for no longer holding slots
- Mixed buyer/sellers
  - define value (+ve, -ve) for a "bundled" trade

# E.g. Buy any number of slots.



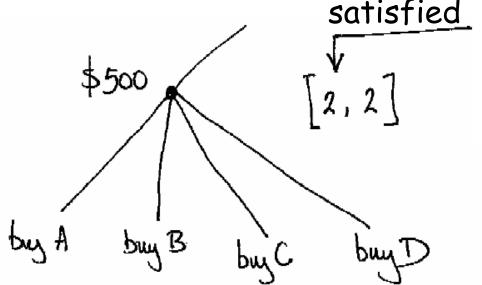
"buy A" leaf satisfied if item A allocated to that node.

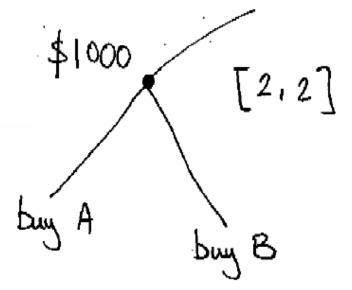
# E.g. Buy at most one slot



# Buy any two slots, Buy all slots...

satisfied if any two children are satisfied

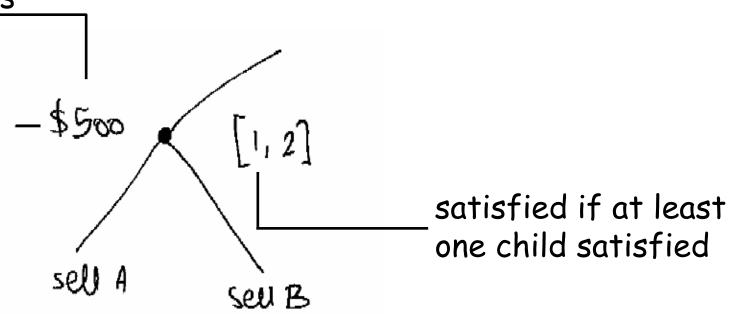




$$AND==(K,K)$$

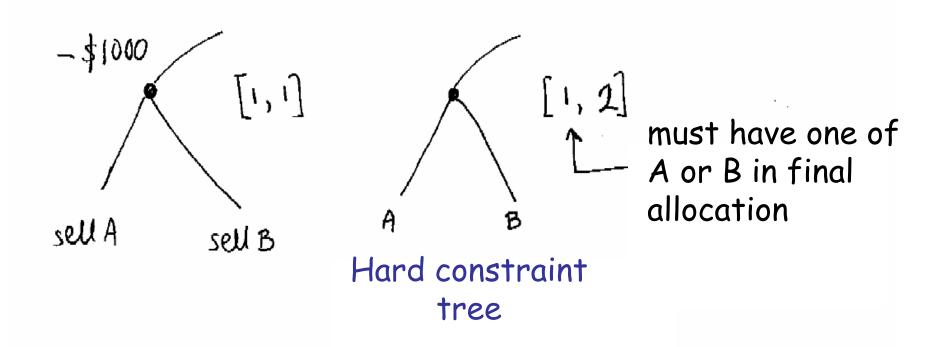
# E.g. Sell all slots

loss in value for selling one or both of these slots



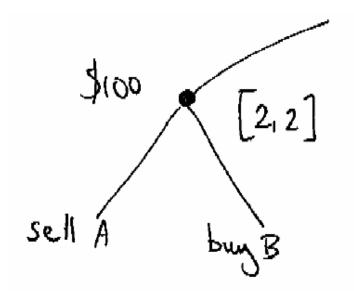
"sell A" leaf satisfied if item A not allocated to agent.

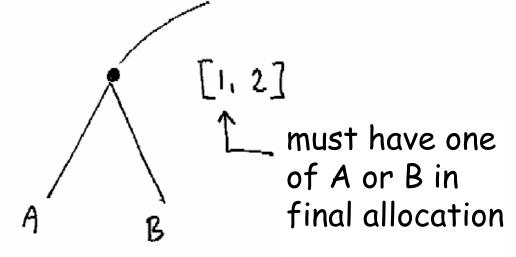
#### E.g. Sell at most one slot...



A seller can supplement valuation tree with hard constraints (satisfied by intial allocation)

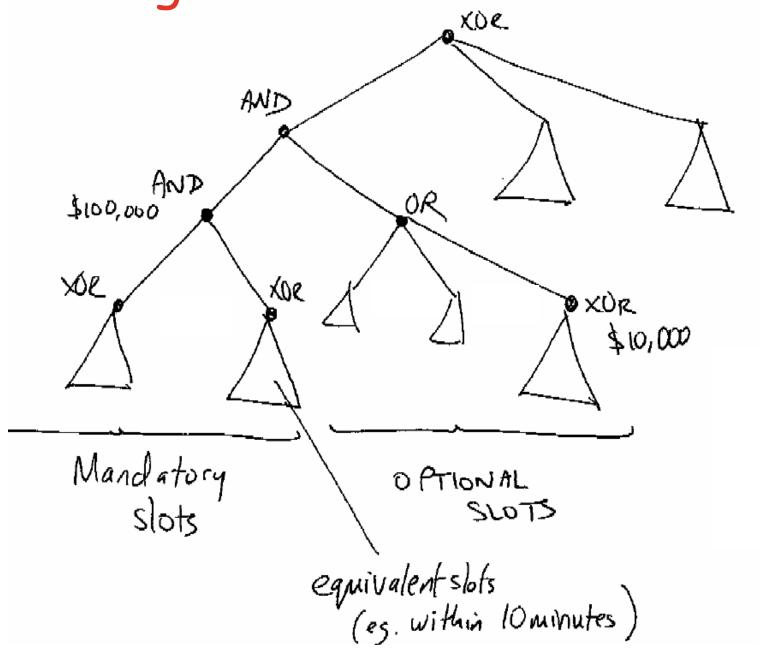
# E.g. Swap A for B



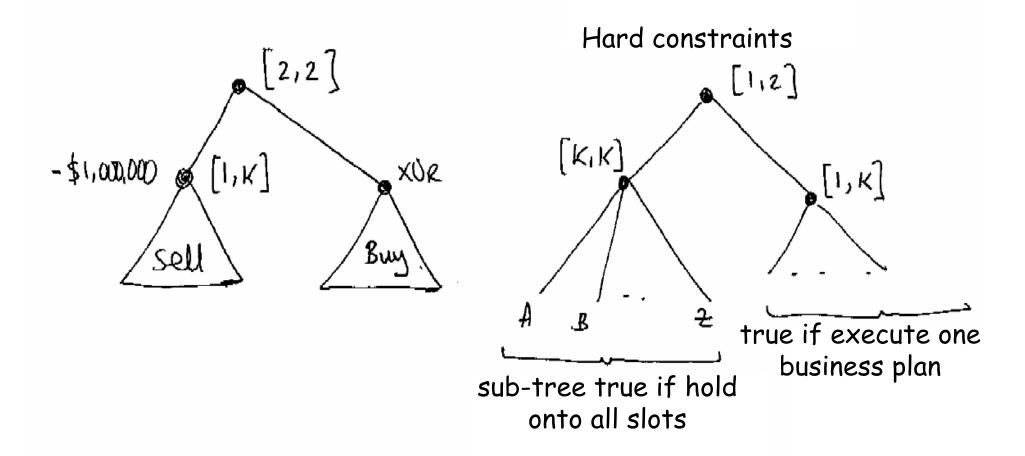


Hard constraint tree

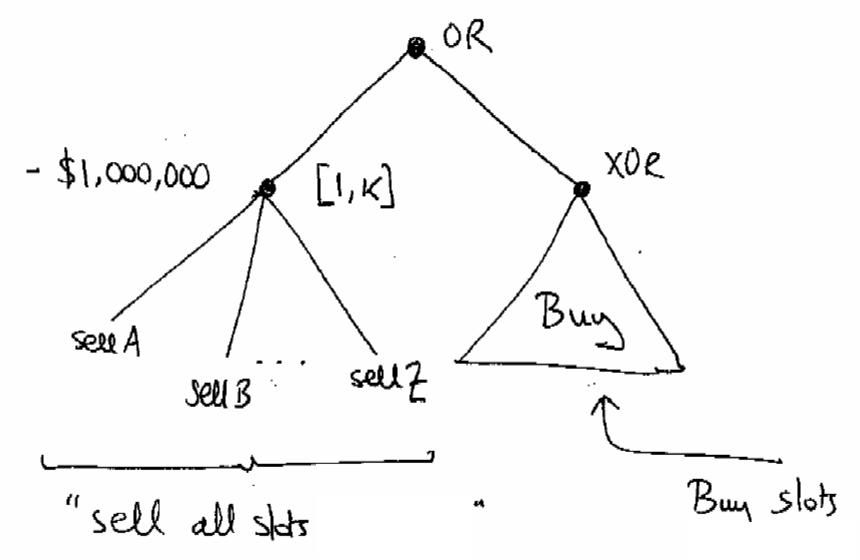
# E.g. More Elaborate Plans...



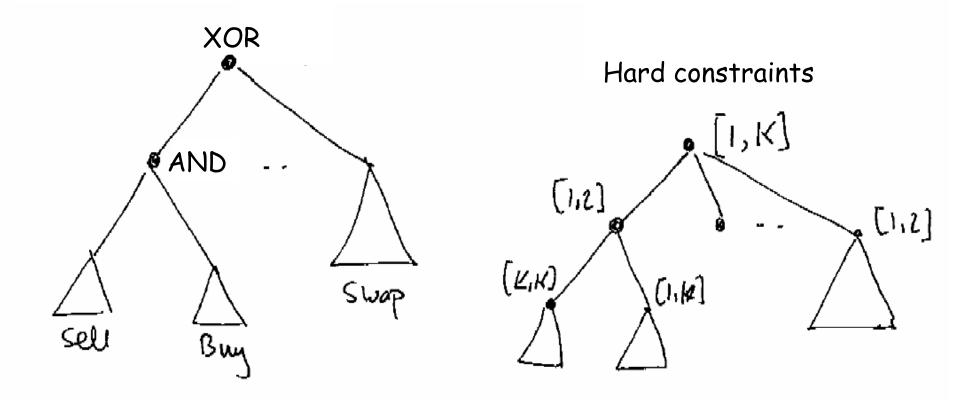
#### E.g. "Swap Peak Slots for Off-peak"



# E.g. Sell for sure, Try to Buy back



#### E.g. Multiple Business Plans...

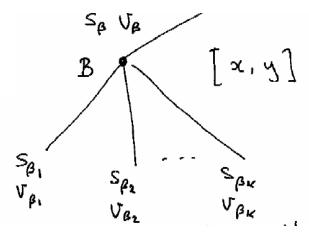


Second Component: Winner Determination Proxy 1 Agent 1 WD FAA Exchange OD Bridge FCC Pricing Agent n Proxy n

#### Winner-Determination

Formulate as a MIP. Number of variables XOR  $V_1, S_1$ scales as size of tree. V<sub>2</sub>, S<sub>2</sub> **AND AND V**<sub>3</sub>, **S**<sub>3</sub> 20 XOR 30 XOR OR V<sub>4</sub>, S<sub>4</sub> Z 50 C D D

#### Internal Node Constraints



if at least x children satisfied, then

$$S_p = 1$$

$$V_p = B + \max_{K \leq Sat_p} \sum_{J \in K} V_{p_J} + P_{en_p}$$

$$|K| \leq y$$

else,  $s_{\beta}=0$ ,  $v_{\beta}=Pen_{\beta}$ 

Pen<sub>B</sub> is total value across -ve valued children

 $Sat_{\beta}$  is set of satisfied children

$$\mathbf{x} \mathbf{c} \mathbf{s}_{\beta} \cdot \sum_{\mathbf{i}} \mathbf{s}_{\beta \mathbf{i}}$$

$$v_{\beta i}$$
 M  $t_{\beta i}$ , 8 i  $\sum_i t_{\beta i}$  y

$$\sum_{i} t_{\beta i} y$$

#### General MIP Formulation

$$\text{max } \textstyle \sum_{\text{i2 N}} \! \textbf{v}_{\beta}$$

s.t. internal node constraints

# hard constraints $\sum_{\beta 2 \text{Sell}(A)} s_{\beta} \cdot 1 - x_{i}(A)$ $\sum_{\beta 2 \text{Buy}(B)} s_{\beta} \cdot x_{i}(B)$ $\sum_{i 2 \text{N}} x_{i}(A) \cdot \text{supply}(A)$ ... $\sum_{\beta 2 \text{Buy}(B)} s_{\beta} \cdot x_{i}(B)$ $\sum_{i 2 \text{N}} x_{i}(A) \cdot \text{supply}(A)$ $\sum_{i 2 \text{N}} x_{i}(A) \cdot \text{supply}(A)$ $\sum_{\beta 2 \text{Buy}(B)} s_{\beta} \cdot x_{i}(B)$ $\sum_{\beta 2 \text{Buy}($

buy(A): have good, one true. don't have good, all false sell(A): have good, all false. don't have good, one true.

 $x(\phi)$  variables define allocation.

#### Leaves

- satisfied if "A-token" assigned to this leaf
  - satisifed if "not A-token" sell Aassigned to this leaf

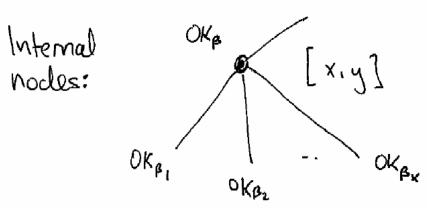
#### every node:

$$\mathbf{v}_{\beta}$$
 -  $\mathbf{B}_{\beta} \mathbf{\mathcal{C}} \ \mathbf{s}_{\beta}$  +  $\sum_{i} \mathbf{v}_{beta \ i}$ 

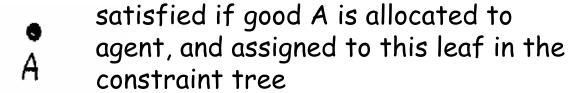
 $v_{\text{beta i}}$  · M¢  $s_{\beta i}$ , 8 i

#### Hard Constraints Tree

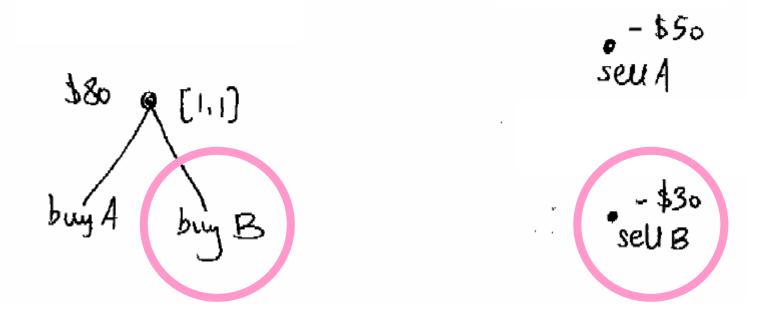
Feasible, Root node satisfied



 $OK_{\beta}$  = True, only when between x and y children are True.



# Simple Example

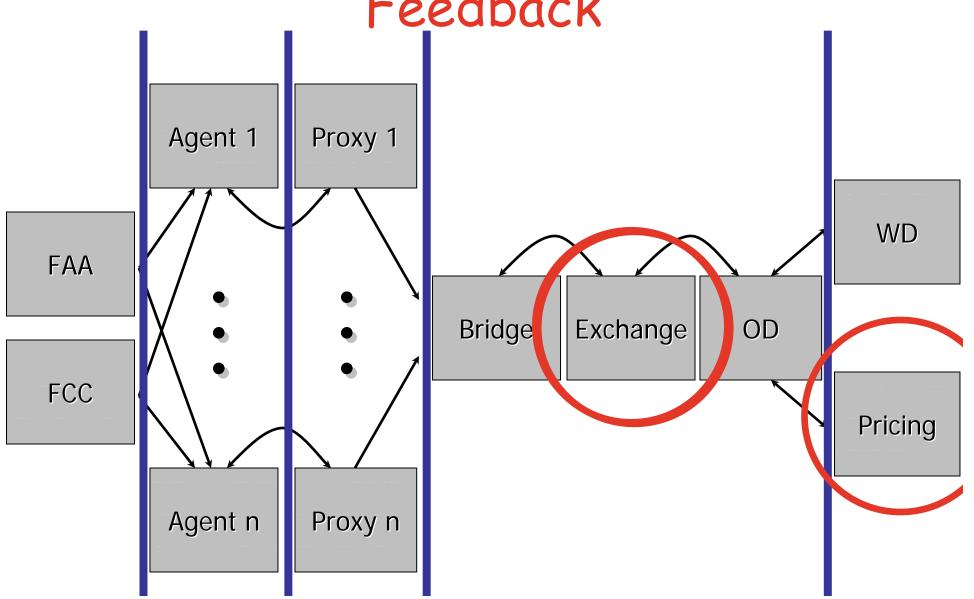


Final allocation: Agent 3 sells B to agent 1.

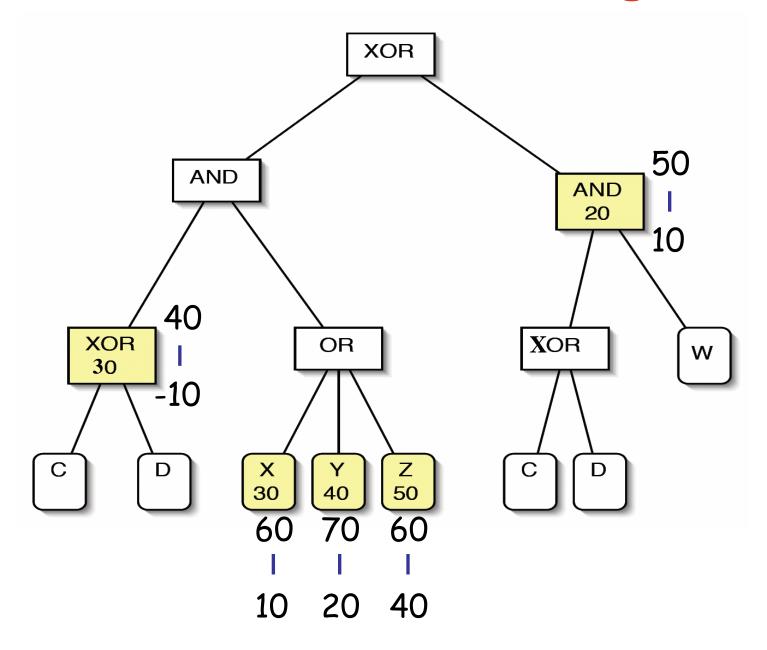
Surplus: \$50

"Threshold" payments: Agent 1 pays \$40 to agent 3.

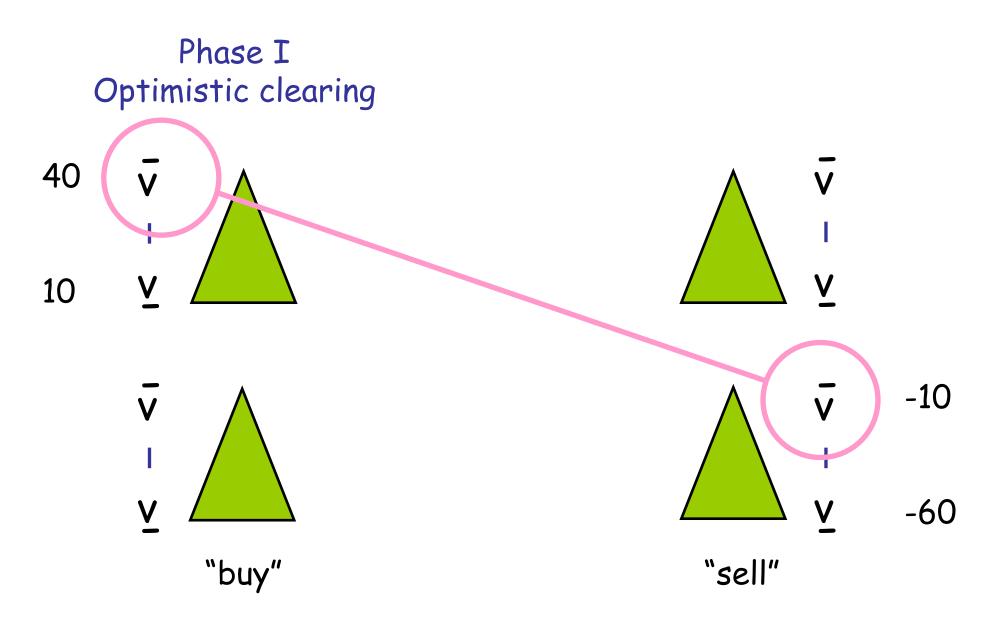
Third Component: Feedback



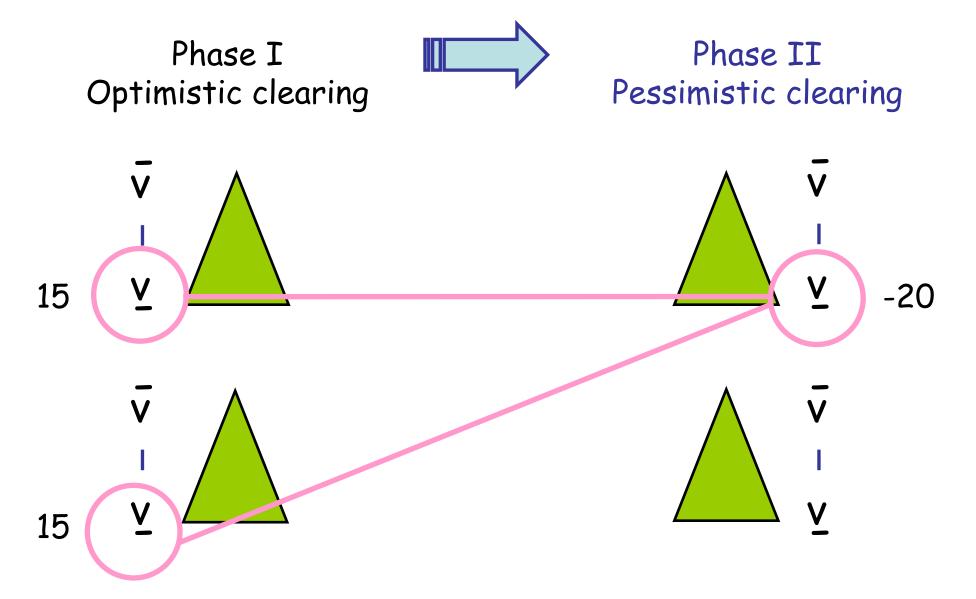
# Incremental Bidding



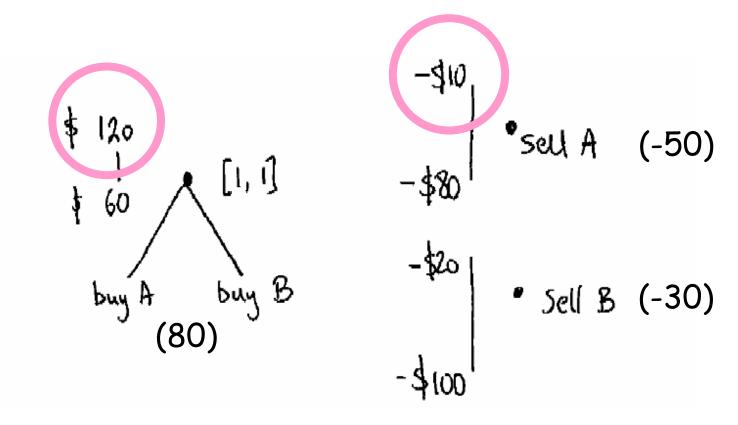
#### Exchange Phases



# Exchange Phases

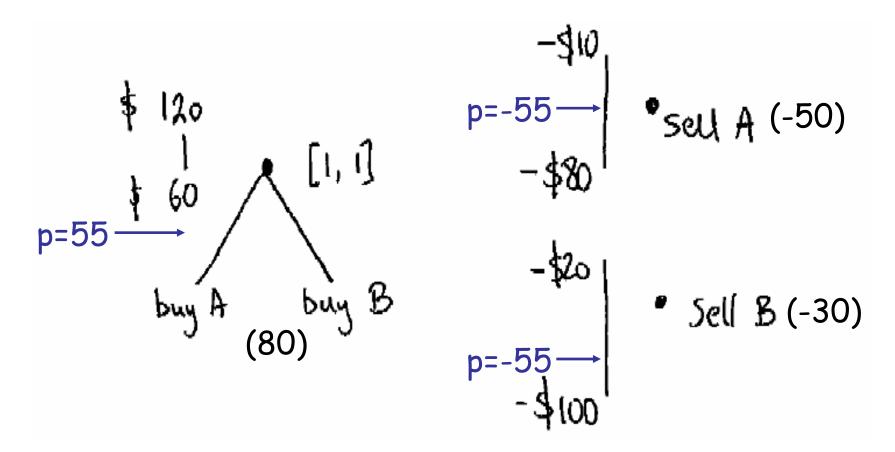


# Example: Round 1



10-  $p_A$ - 120,  $p_A$ -  $p_B$  (winner optimistic values)  $p_B$ - 100 (loser pessimistic value) )  $p_A$ = $p_B$ =55

### Bid Refinement: Round 1

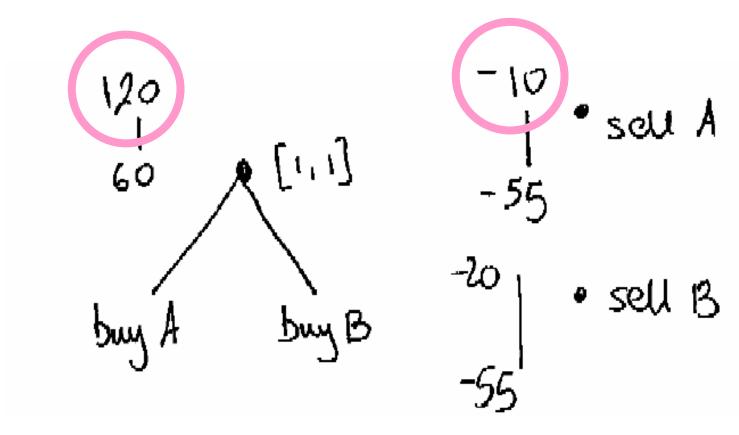


#### Activity rules:

Winners: ask winners to refine u.b.'s to meet price

Losers: ask losers to refine l.b.'s to meet price

### Round 2

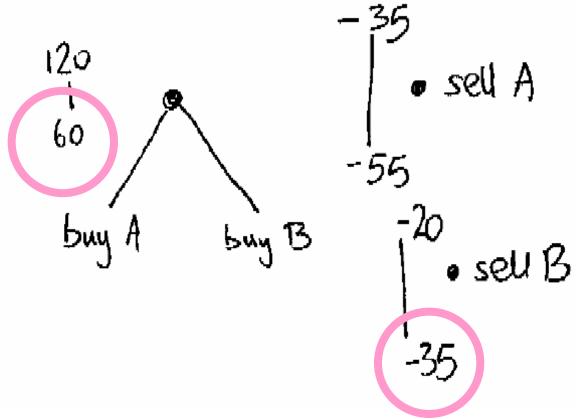


10.  $p_A$ . 120,  $p_A$ .  $p_B$  (optimistic values)

p<sub>B</sub>· 55 (pessimistic value)

$$p_{A}=p_{B}=35$$

### Round 3

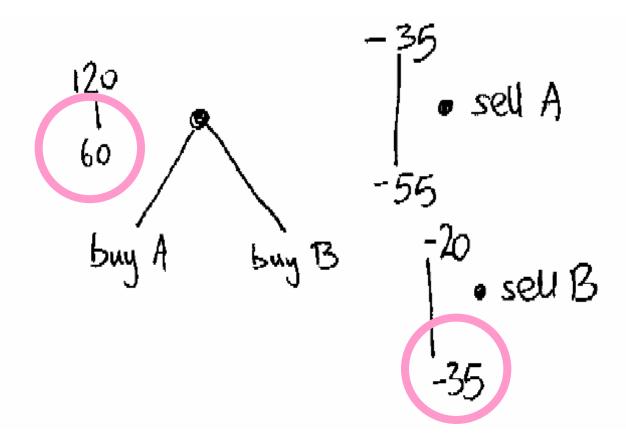


35.  $p_B$ . 60,  $p_B$ .  $p_A$  (pessimistic values)

 $p_A$  35 (optimistic value)

) 
$$p_A = p_B = 35$$

### Last & Final Round



Final allocation: Agent 3 sells B to agent 1.

"Threshold" payments: Agent 1 pays \$40 to agent 3.

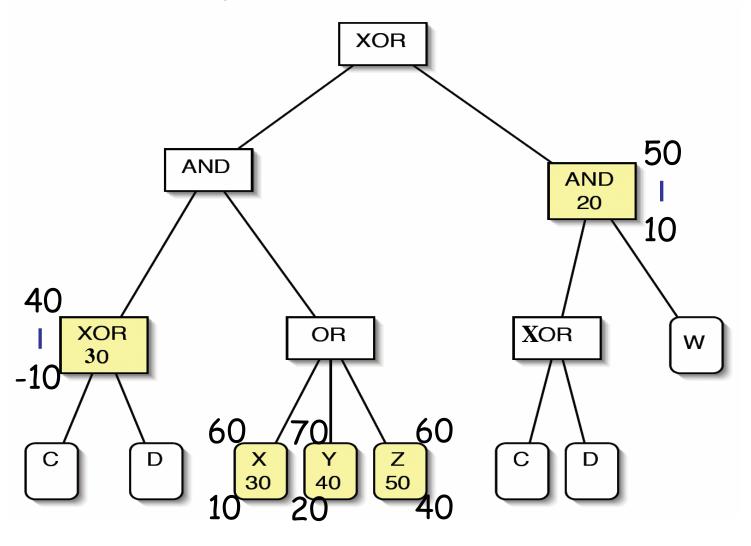
# Prices and Activity Ryles

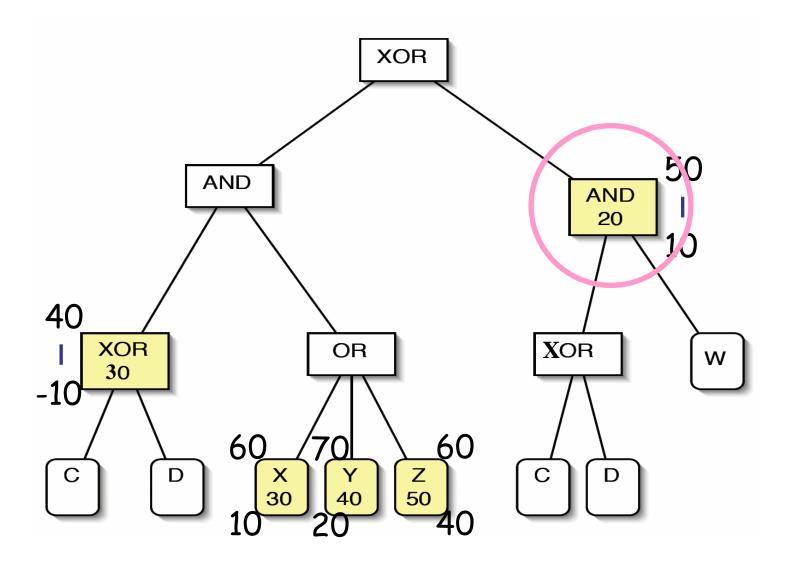
- Phase 1: optimistic outcome
  - drive price feedback in early rounds
  - use winner u.b's and loser l.b's to set prices
- Phase 2: pessimistic outcome
  - drive price feedback in later rounds
  - use winner l.b's and loser u.b's to set prices

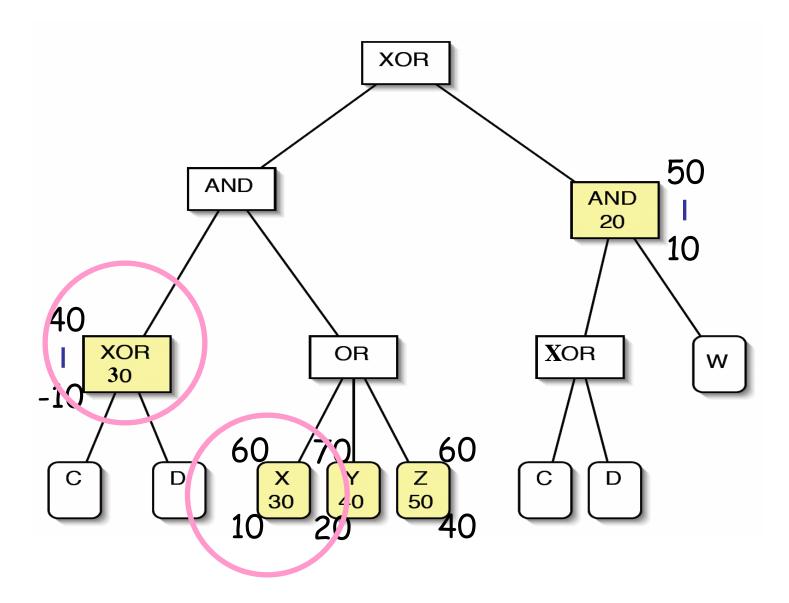
#### Activity rules:

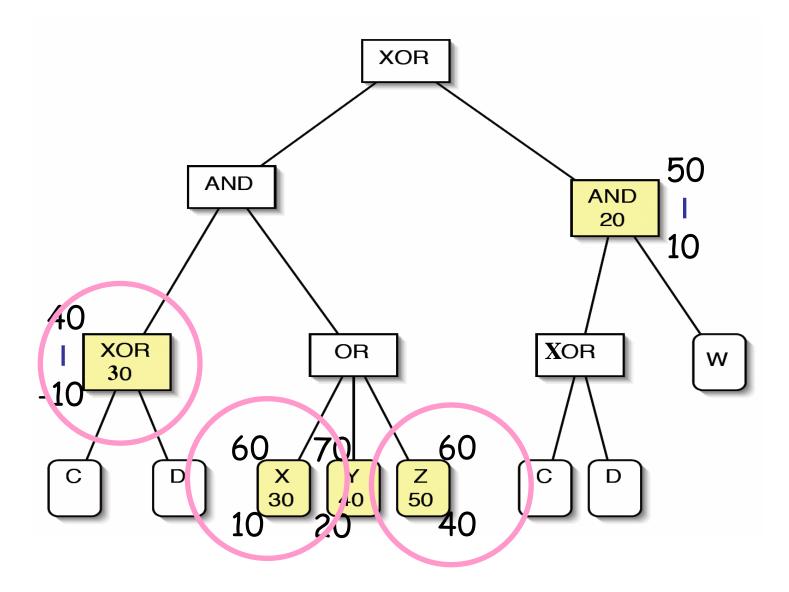
- winners must lower u.b's to meet activity
- losers must increase l.b's to meet activity

# Activity Rules work on Tree

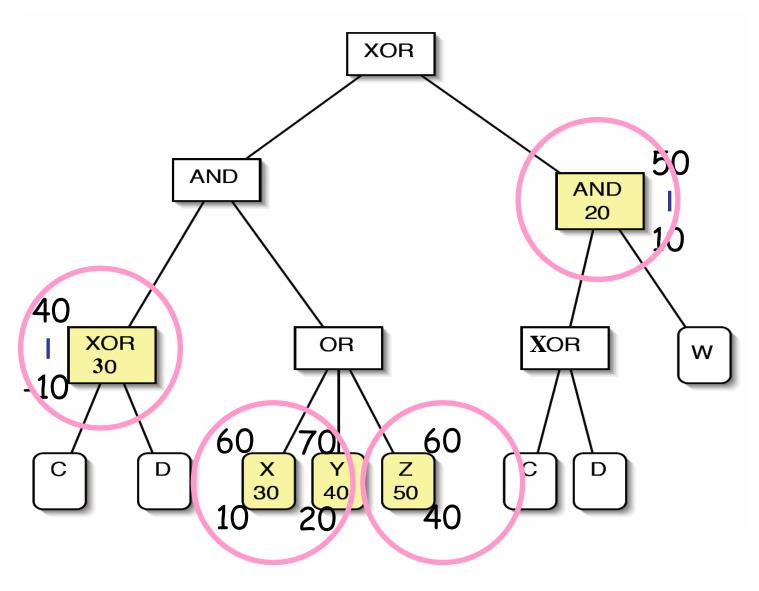








### Default Action



### Approximate Linear Prices

• Given WD outcome  $\lambda^*$ , compute prices to solve:

```
min \delta + \Delta(p,p^{t-1})
s.t. v_i(\lambda^*_i)-p\phi \lambda^*_i, v_i(\lambda')-p\phi \lambda', 8 \lambda' 2 M, 8 i
where \lambda_{ij}=1 if i buys j, =-1 if i sells j
```

•  $\Delta(p,p^{\dagger-1})$  is a price smoothing term (Hoffman et al.)

Solve with *column generation* to avoid enumeration.

### Linear prices: Column Generation

Consider restricted master problem:

- · Get a feasible, but perhaps suboptimal solution
- Solve restricted problem:

max 
$$\left[v_{\beta i} - p \phi \lambda_i\right] - \left[v_{i}^{\alpha} - p \phi \lambda_i^{*} + \delta\right]$$
 (RP)  
s.t. (constraints describing i's valuation tree)

• If (RP > 0) then add new bundle to (MP), and resolve.

# Dynamic Feedback to Participants

- Current provisional allocation and payments
- Prices to guide bid refinement
- · Can also provide "smart quoting"
  - how should I improve my bid to be a winner?
- Participants don't see:
  - other bids
  - allocation of other participants

**-** ...

### Final Outcome

- Move to last and final round
- Give participants last chance to refine valuations
- · Clear exchange to maximize reported value
- Allocate surplus
  - no-one pays more than bid,
  - no-one receives less than ask
  - distribute surplus to mitigate bargaining and improve efficiency

### Summary: Key Features

- Compact and expressive bidding language
- Staged proxy design w/ linear price feedback between stages
  - prices to guide value refinement
  - activity rules to drive progress
- Final "proxy round"
  - expressive bids
  - final clearing, final payments

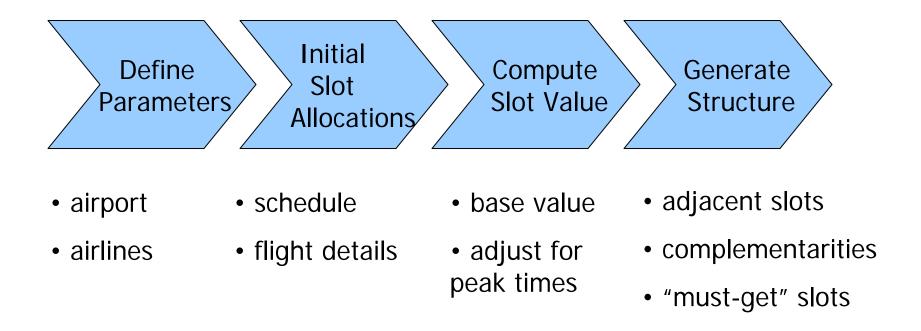
# Simulation and Testing

- Model FAA problem domain
  - problem generator
- Simulate bidding strategies
  - truthful & straightforward
  - ...
- Test Exchange
  - economic and computational properties

# FAA: Domain Modeling

	LDC2004 "Donohue"	LPS2000 "CATS"	CS286r
Value of slots	<ul> <li>Proportional to size of aircraft</li> </ul>	• Random utility level	<ul> <li>aircraft size</li> <li>miles flown</li> <li>unit cost, revenue</li> <li>airline type</li> <li>peak/non-peak</li> </ul>
Deviation from optimal slot	<ul><li>Current schedule preferred</li><li>same value</li></ul>	<ul><li>Current schedule preferred</li><li>value scaled</li></ul>	<ul><li>Current schedule preferred</li><li>value scaled</li></ul>
Expressiveness	OR	Atomic bids	XOR-(AND,OR)-XOR
Source of schedule data	Real data (from ATL)	Randomly generated	Randomly generated (can mimic real schedule)

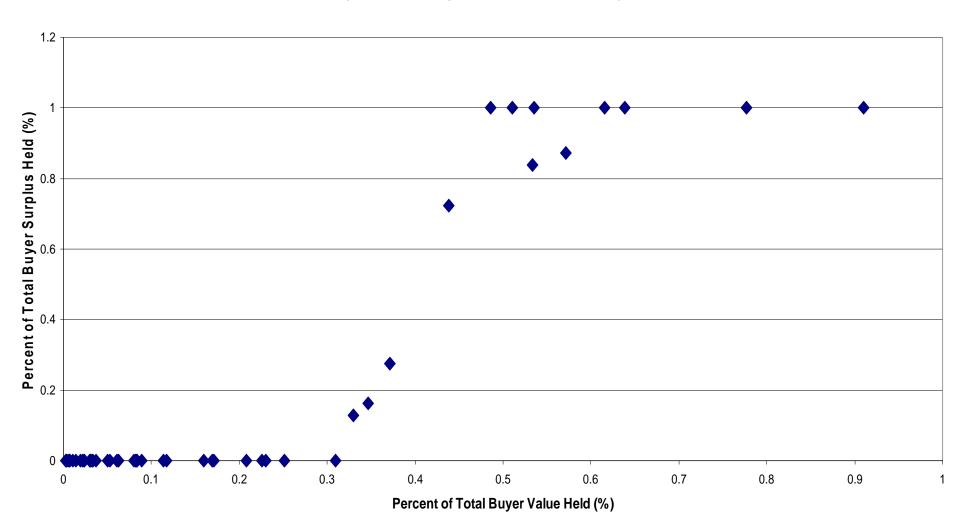
### Stages in Domain Modeling



Results in a complete specification of desired slots and valuation for each airline at an airport

### Econ Analysis

Surplus versus Value for Buyers in One-Shot, Truthful WD-only Mechanism (over 10 Runs parameterized to 6,1,1,1)



#### Continued Work

- Experiments, to study:
  - speed of convergence
  - informativeness of linear prices
  - scalability
  - opportunities for strategic behavior
  - economic impact of exchange
  - policy tools (e.g. assignment of incumbent rights)
- Appeal for help:
  - guide this process!
  - policy goals for design
  - models of participants

### Summary

- Combinatorial market technology is real
  - used every day for complex procurement problems
- · Expressive languages simplify:
  - allow participants to "say what want"
- Proxied & iterative exchange:
  - expressive bidding language, constraints for sellers
  - linear prices to guide bidding
  - bidding through refinement of value
  - final sealed-bid round

# Acknowledgements

- · Students in CS 286r, especially
  - S.Lahaie, R.Cavallo, N.Elprin, A.Juda, A.Kirsch, A.Kulesza, B.Lubin, L.Michael, E.Ou, J-F.Raymond, J.Shneidman, B.Szekely, H.Sultan, A.Sumiyama, K.Venkatram.

### Threshold Rule

