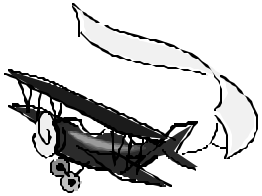

“An Airspace Planning and Collaborative Decision Making Model (APCDM) Under Safety, Workload, and Equity Considerations”

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NEXTOR Research Seminar - Falls Church, VA

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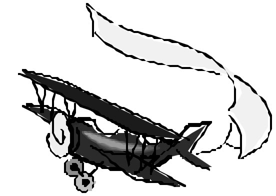
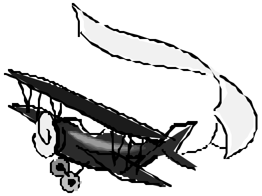
Overview

- Motivation & Background
- APCDM
  - Conflict Resolution
  - Sector Workloads
  - Collaborative Decision Making (CDM) Considerations
  - Overall Model Formulation
- Model Analyses
  - Probabilistic Aircraft Encounter Model (PAEM) Analysis
  - Parameterization & Sensitivity Analysis
  - Conflict Constraint Formulation Analysis
- Research Directions

Motivation & Background

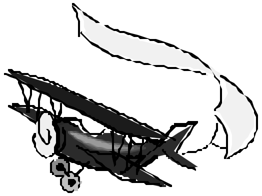


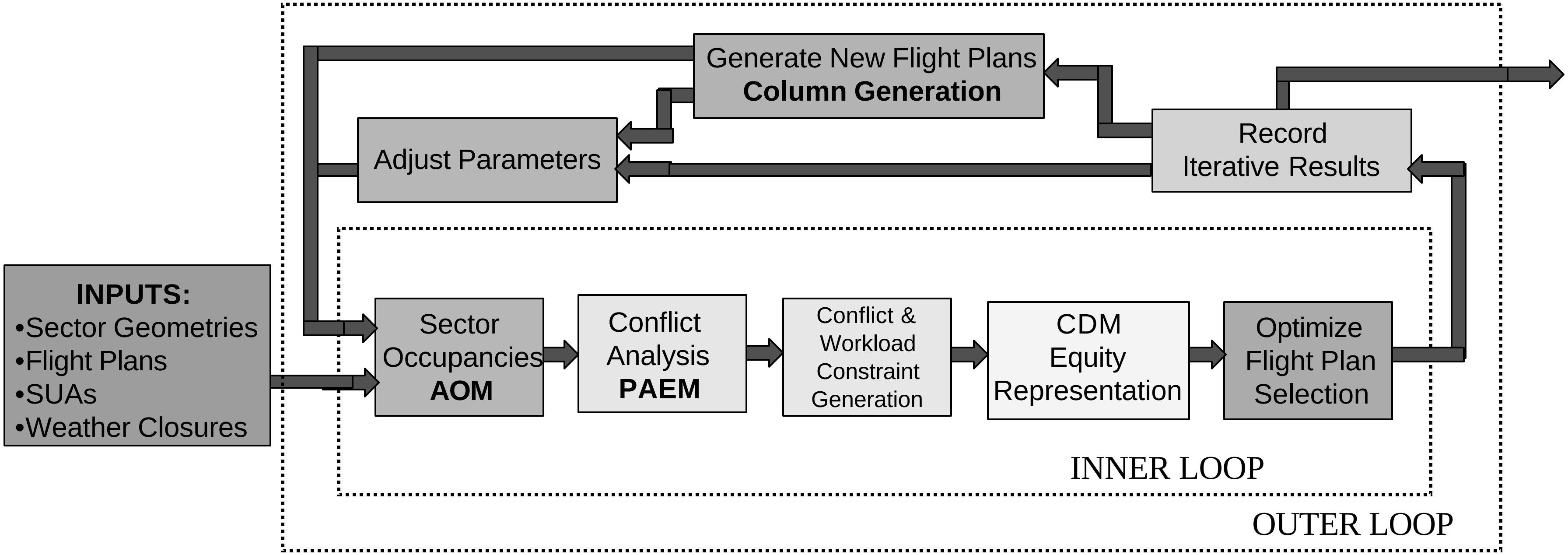
- Delays: Space Launch, Weather Systems
- Congested Airspace: Safety and ATC Workload
    - Distribute sector workloads
    - Minimize en-route aircraft conflicts
- Airline Competition
    - Fair allocation of constrained resources
    - New entrants and small/medium community service
    - Disparity in distribution of costs
    - Consumer expectations



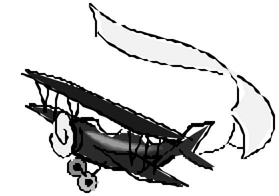
- Flight Plan Selection
  - For each flight, select one flight plan from among alternatives
  - Minimize Flight Costs (Objective Function)
  - Subject to Considerations (Penalty Terms in Objective Function):
    - Sector Workload
    - Safety (Conflict Resolution)
    - Decision Equity

APCDM




Generate New Flight Plans
Column Generation
Adjust Parameters
Record Iterative Results
INPUTS:
•Sector Geometries
•Flight Plans
•SUAs
•Weather Closures
Sector Occupancies AOM
Conflict Analysis PAEM
Conflict & Workload Constraint Generation
CDM Equity Representation
Optimize Flight Plan Selection
INNER LOOP
OUTER LOOP

APCDM



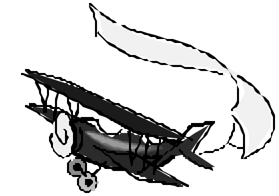
- Sector Occupancies

- Aircraft Conflict Analysis
  - Stochastic with respect to aircraft trajectory
  - Conflict risk thresholds

- Conflict & Workload Constraint Generation
  - Continuous time formulation
  - Two new classes of valid inequalities
  - Sector workloads--average and peak workloads

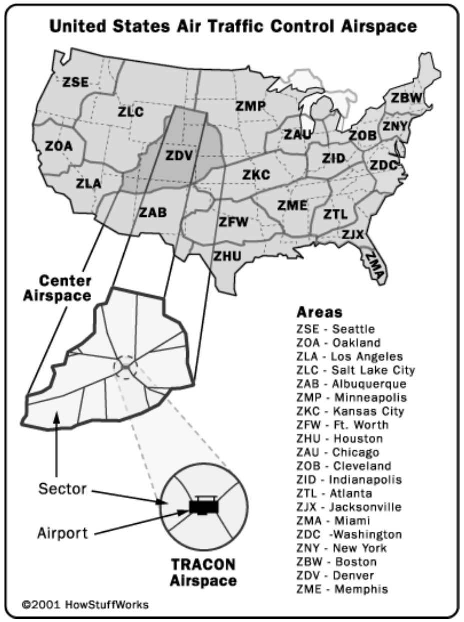
- CDM Equity Representation
  - Cost Model
  - Collaboration Efficiency & Equity

- Mixed-Integer Programming Model

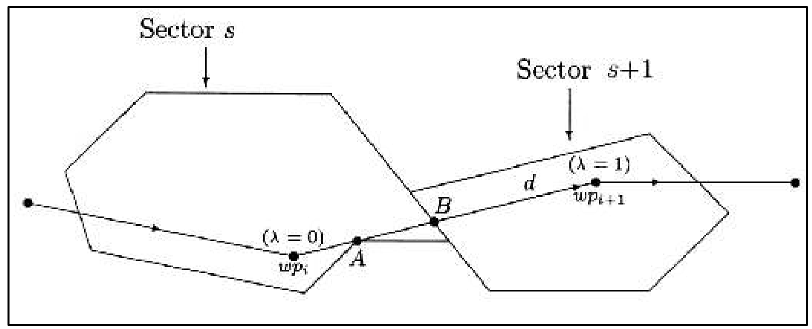


Sector Occupancy

AIRSPACE OCCUPANCY MODEL

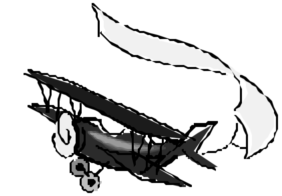


- Mathematical NAS representation
- 20 centers each divided into sectors



- Flight plans processed to determine sector occupancy time intervals
- Occupancy data used:
  - To characterize sector occupancy workloads
  - As pre-processing data for PAEM conflict analysis

Sector Workload



- Workload Characterization
  - Average Occupancy throughout some horizon (w_s)
  - Peak Occupancy (n_s)

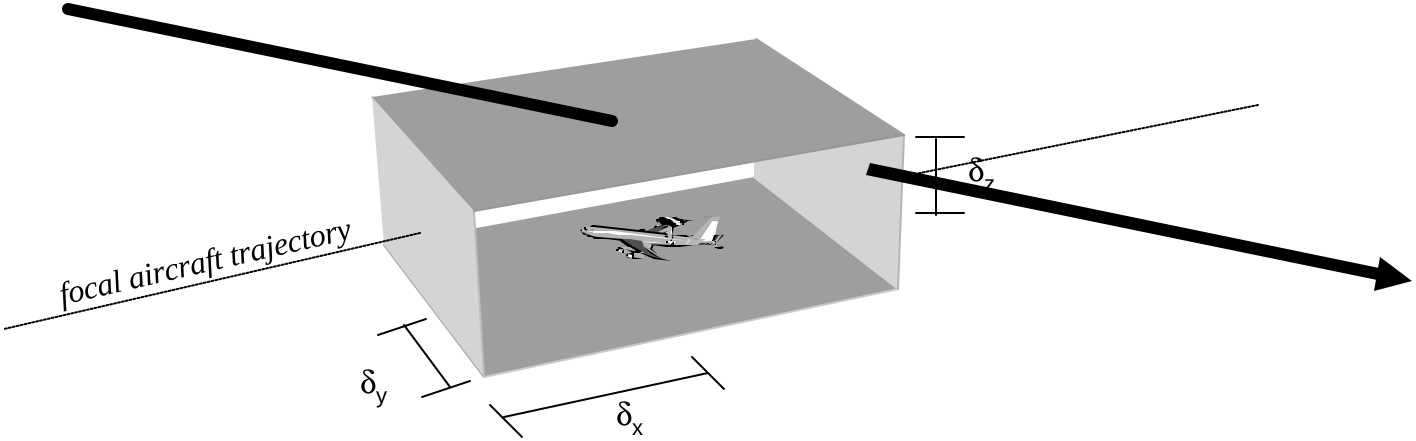
- Occupancy Constraints:
  - Prohibit selection of combinations of flight plans that cause sector capacity ($\bar{n}_s$) to be exceeded

- Penalty Functions:
  - Average Workload: $\sum_{s=1}^{S} g_s w_s$
    - Constant penalty
  - Peak/Average Differential: $\sum_{s=1}^{S} \sum_{n=0}^{\bar{n}_s} m_{sn} y_{sn}$
    - Piecewise linear representation of increasing quadratic function

Conflict Analysis

PROBABILISTIC AIRCRAFT ENCOUNTER MODEL

- Proximity Shell Around Each Focal Aircraft

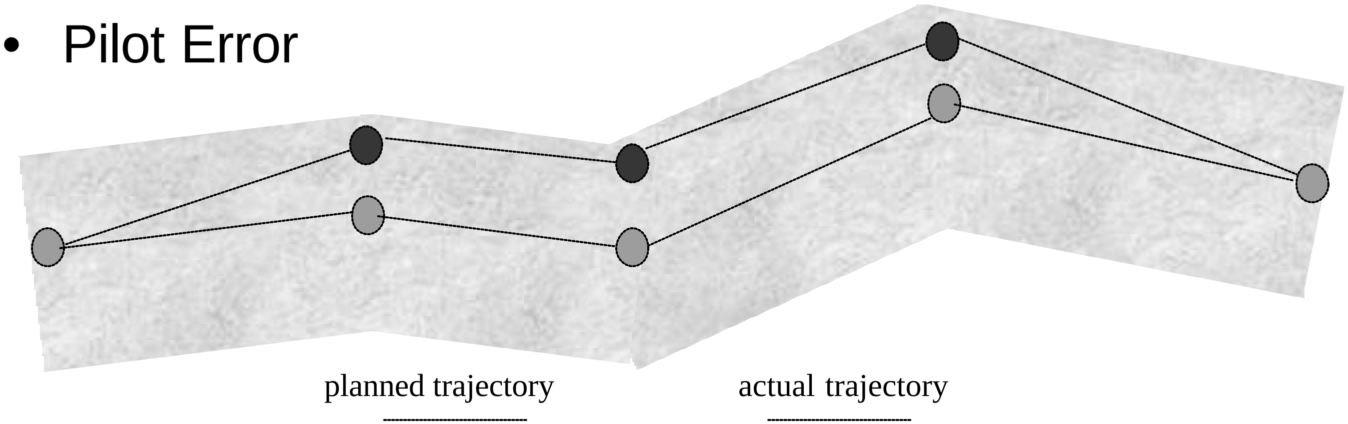


- Moves with aircraft as it traverses its flight trajectory
- Conflict occurs when another intruder aircraft pierces the focal aircraft's proximity shell

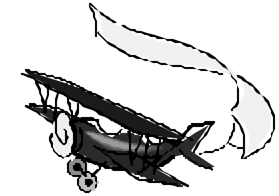
Conflict Analysis

PROBABILISTIC AIRCRAFT ENCOUNTER MODEL

- Aircraft Position & Trajectory Not Known With Certainty
  - Weather Effects
  - Navigation System Inaccuracy
  - Pilot Error



- Bounded Error Regions → Probabilistic Trajectory Corridor
  - Randomized Displacement Errors
  - Wind-induced Displacement Errors

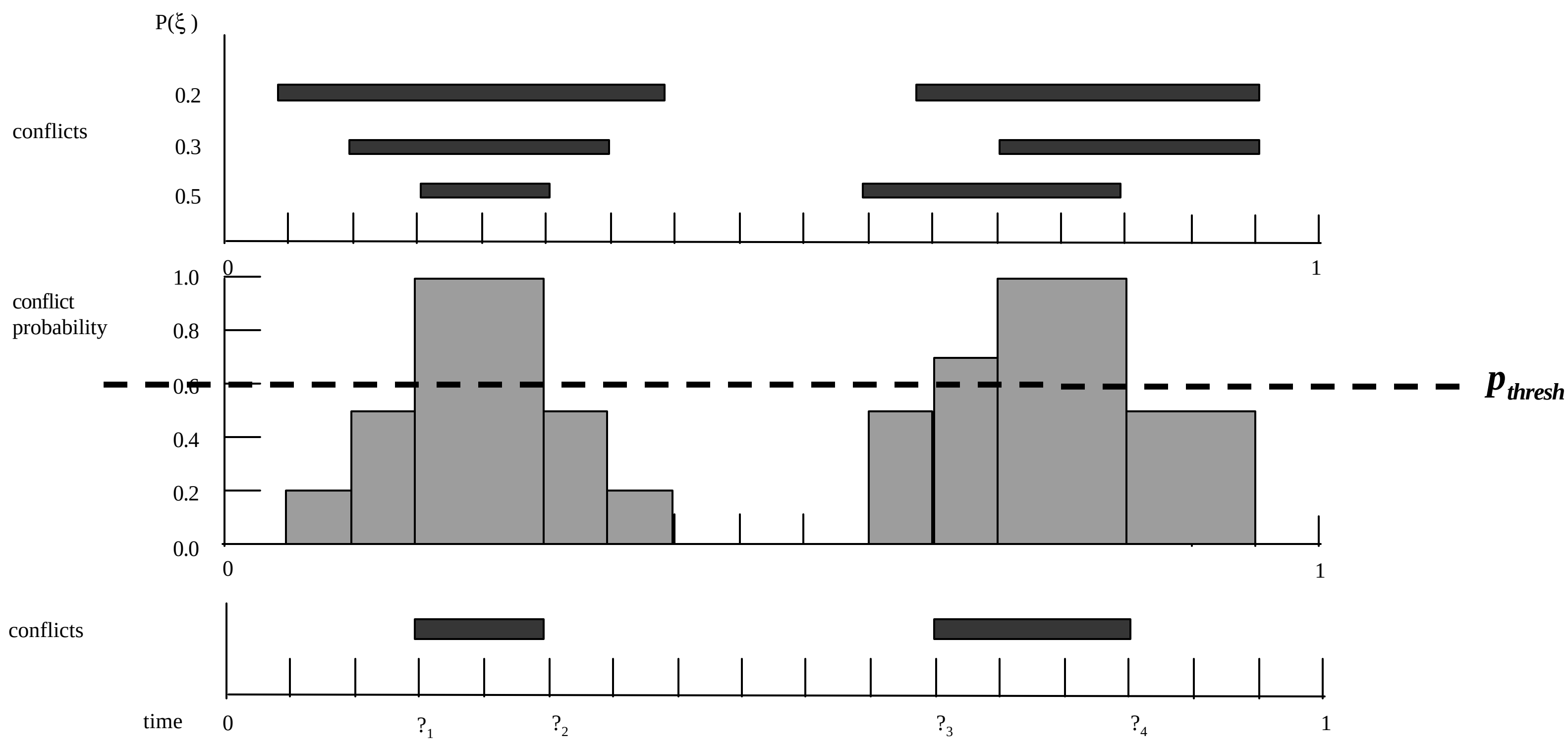


Conflict Analysis

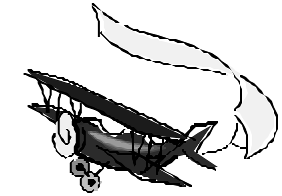
PROBABILISTIC AIRCRAFT ENCOUNTER MODEL

- For each pair of discretized error trajectory realizations

(for focal and intruder aircraft) we can compute the conflict risk:



Conflict Resolution Constraints

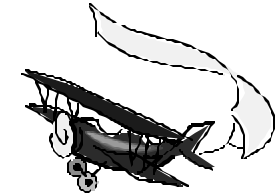


- Probabilistic conflicts generated by PAEM are fit into the constraint structure of APCDM

- Constraints prohibit the selection of particular combinations of flight plans
  - Flight pairs that have a “fatal” conflict
  - Flight combinations that exceed sector ATC conflict resolution capability during any specified time interval

- Penalty function: $\sum_{(P,Q)\in A} j_{PQ} z_{PQ}$
  - Constant j_{PQ} is determined by conflict geometry

- Polyhedral analysis of conflict constraint structure
  - Derived classes of valid inequalities to tighten representation

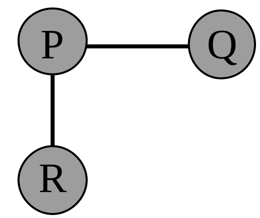


Conflict Resolution Constraints

C_3 REPRESENTATION

- Define T_{NC} as:

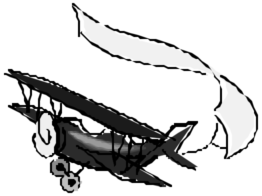
$$\left\{\begin{array}{l}(P,Q,R),\ P<Q<R: \quad \text{for some } (s,k)\text{, a subgraph of } G_{sk} \text{ that is} \\ \text{induced by the nodes } P, Q\text{, and } R \text{ contains precisely two edges,} \\ \text{but no such subgraph for any } (s,k) \text{ contains three edges}\end{array}\right\}$$



- $$C_3 = \left\{\begin{array}{lll}(x,z): & \displaystyle\sum_{(P,Q)\in M_{sk}} z_{PQ} \le 1, & \forall (s,k) \\ & x_P + x_Q + x_R \le 2, & \forall (P,Q,R) \in T_{NC} \\ & z_{PQ} \ge x_P + x_Q - 1, & \forall (P,Q) \in A \\ & z \ge 0, \quad x \text{ binary} & \end{array}\right\}.$$

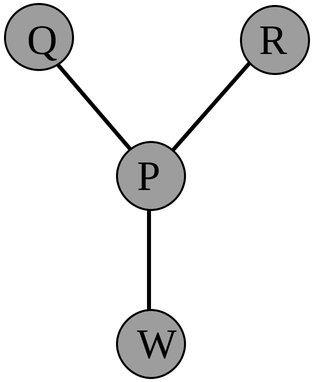
- C_3 tightens representation

Conflict Resolution Constraints



C_4 REPRESENTATION

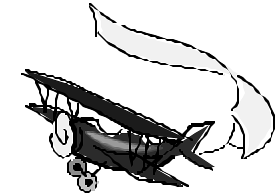
- Focus is on underlying star-graph convex hull constraints



- $$C_4 = \left\{ \begin{array}{ll} (x,z): & \sum_{(P,Q)\in M_{sk}} z_{PQ} \le 1, \quad \forall (s,k) \\ & \sum_{Q\in J_r(P)} z_{(PQ)} \le x_P, \quad \forall r = 1,\ldots,r_P, \forall P \in I^* \\ & z_{PQ} \ge x_P + x_Q - 1, \quad \forall (P,Q) \in A \\ & z \ge 0, \ x \text{ binary} \end{array} \right\}.$$

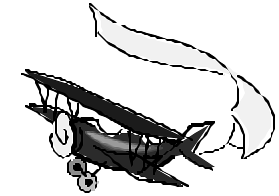
- C_4 is a provably tighter representation than C_3

CDM Considerations



- Optimal Individual Decisions vs. Optimal Group Decision
  - Each participating airline’s decisions represent conflicting objectives
  - Possibly no feasible satisfying solution for these conflicting objectives
  - Inefficient overall use of the NAS
- Collaboration Efficiency
  - Ratio of costs incurred by an airline due to resolution between the group’s conflicting objectives to costs obtainable using the airline’s individually optimized strategy
- Collaboration Equity
  - Aggregate measure of disparity of costs incurred via group decision

CDM Considerations



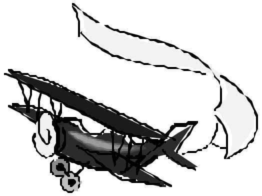
FLIGHT PLAN COST MODEL

- Fuel Cost: (F_{fp})
  - Base of Aircraft Data (BADA) Operations Performance Model
  - Fuel cost as a function of:
    - Aircraft type
    - Flight envelope
    - Engine thrust
    - Fuel consumption
    - Mass
    - Aerodynamics
    - Reduced power flight
    - Ground movement

- Delay Cost: $D_{fp} = \left(t_{fp}^{d}\right)\left(d_{f}^{c}\right)\left(l_{f}\right)\left(d\right)$
  - Length of delay $\left(t_{fp}^{d}\right)$
  - Connection delay cost factor $\left(d_{f}^{c}\right)$
  - Passenger load estimate $\left(l_{f}\right)$
  - Delay cost factor per passenger-minute $\left(d\right)$

CDM Considerations

FLIGHT PLAN COST MODEL

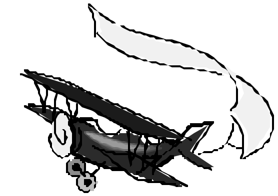


- Total Flight Plan Cost: $c_{fp} = F_{fp} + D_{fp}$

- Flight Cancellations
  - Each flight has a “cancellation” surrogate flight plan
  - Cancellation cost is greater than highest cost surrogate

$$c_{f0} = \max_{p \in P_f} \left\{ F_{fp} \right\} + \left(t_{f0}^{d} \right) \left(d_f^{c} \right) \left(l_f \right) \left(d \right)$$

CDM Considerations



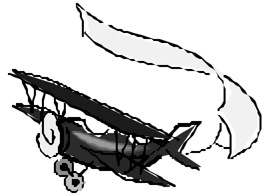
COLLABORATION EFFICIENCY

- Airline Collaboration Cost:
  - Ratio of total airline cost after resolving conflicting objectives between all airlines to airline’s individually optimized flight costs.

$$d_a(x) = \frac{\sum_{f \in A_a} \sum_{p \in P_{f0}} c_{fp} x_{fp}}{\sum_{f \in A_a} c_f^*}$$

  - We impose $d_a(x) \le D_{\max}$, a maximum CDM-based cost ratio, for all airlines $a = 1, \ldots, \bar{a}$

CDM Considerations



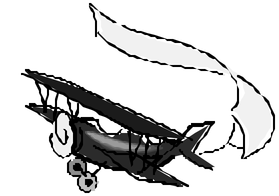
COLLABORATION EFFICIENCY

- Airline Collaboration Efficiency:
  - Function constructed such that $E_a(x) = \begin{cases} 1 & \text{if} \quad d_a(x) = 1 \\ 0 & \text{if} \quad d_a(x) = D_{\max} \end{cases}$
  - This yields: $$E_a(x) = \frac{D_{\max} \sum_{f \in A_a} c_f^* - \sum_{f \in A_a} \sum_{p \in P_{f0}} c_{fp} x_{fp}}{\left(D_{\max} - 1\right) \sum_{f \in A_a} c_f^*}, \ \forall a = 1, \ldots, \bar{a}$$
- ω-Mean Collaboration Efficiency: $\sum_a w_a E_a(x)$,

where $w_a = \frac{|A_a|}{F}, \forall a = 1, \ldots, \bar{a}, \quad \sum_{a=1}^{\bar{a}} w_a = 1, w_a \geq 0, \forall a.$

- ω-Mean Collaboration Inefficiency ($1 - \sum_a w_a E_a(x)$) is penalized in the objective function

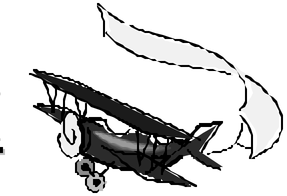
CDM Considerations



COLLABORATION EQUITY

- Airline Collaboration Equity: $E_a^e(x) = E_a(x) - \left(\sum_{a=1}^{\bar{a}} w_a E_a(x) \right)$
- ω-Mean Collaboration Inequity: $x^e \equiv \sum_{a=1}^{\bar{a}} w_a \left| E_a^e(x) \right|$
  - Formulation linearizes the absolute value terms
- Penalty function minimizes disparities in efficiencies (i.e. seeks a more equitable solution)
- $m^e = m^D = m_0 \sum_{f=1}^{F} c_f^* = (0.1) \sum_{f=1}^{F} c_f^*$

Model APCDM



$$\min \; \sum_{f=1}^{F}\sum_{p\in P_{f0}} c_{fp}x_{fp} + \sum_{s=1}^{S}\sum_{n=0}^{\bar{n}_s} m_{sn}y_{sn} + m^{D}\sum_{a=1}^{\bar{a}} w_a\left[1-E_a(x)\right] + m^{e}x^{e} + \sum_{s=1}^{S} g_s w_s + \sum_{(P,Q)\in A} j_{PQ}z_{PQ}$$

subj to: $$\sum_{p\in P_{f0}} x_{fp} = 1 \quad \forall f = 1,\dots,F$$

Workload Constraints

$$n_s \le \bar{n}_s, \quad \forall s = 1,\dots,S$$

$$\sum_{n=0}^{\bar{n}_s} y_{sn} = 1, \quad \forall s = 1,\dots,S$$

$$n_s - w_s = \sum_{n=0}^{\bar{n}_s} n y_{sn}, \quad \forall s = 1,\dots,S$$

$$w_s = \frac{1}{H}\sum_{(f,p)\in\Omega_s} t^{s}_{fp}x_{fp}, \quad \forall s = 1,\dots,S$$

$$\sum_{(f,p)\in C_{si}} x_{fp} \le n_s, \quad \forall i = 1,\dots,I_s,\; s = 1,\dots,S$$

Conflict Resolution Constraints (C_4)

$$\sum_{Q\in J_{sk}(P)} z_{(PQ)} \le r_s x_P, \quad \forall P\in N_{sk}:$$

$$\left|J_{sk}(P)\right| \ge r_s + 1, \quad \forall (s,k)$$

$$x_P + x_Q \le 1, \quad \forall (P,Q)\in FC$$

$$x_P + x_Q - z_{PQ} \le 1, \quad \forall (P,Q)\in A$$

$$\sum_{(P,Q)\in M_{sk}} z_{PQ} \le r_s, \quad \forall (s,k)$$

CDM Constraints

$$E_a(x) = \frac{D_{\max}\sum_{f\in A_a} c^{*}_{f} - \sum_{f\in A_a}\sum_{p\in P_{f0}} c_{fp}x_{fp}}{\left(D_{\max}-1\right)\sum_{f\in A_a} c^{*}_{f}}, \quad \forall a = 1,\dots,\bar{a}$$

$$E_a(x) \ge 0, \quad \forall a = 1,\dots,\bar{a}$$

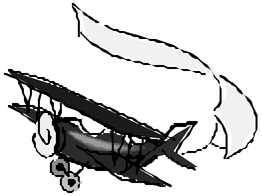
$$E^{e}_a(x) = E_a(x) - \left(\sum_{a=1}^{\bar{a}} w_a E_a(x)\right), \quad \forall a = 1,\dots,\bar{a}$$

$$-E^{e}_{\max} \le E^{e}_a(x) \le E^{e}_{\max}, \quad \forall a = 1,\dots,\bar{a}$$

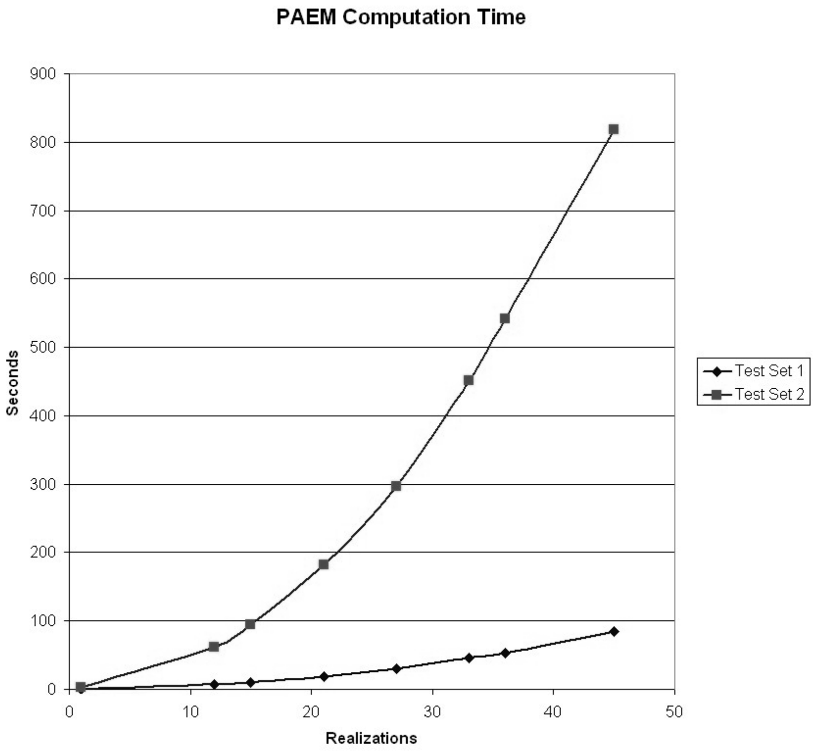
$$\sum_{a=1}^{\bar{a}} w_a n_a = x^{e} \le v^{e}$$

$$n_a \ge -E^{e}_a(x) \text{ and } n_a \ge E^{e}_a(x), \quad \forall a = 1,\dots,\bar{a}$$

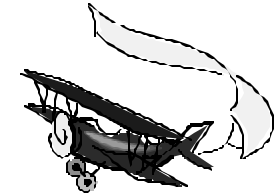
PAEM Analysis



- Eight Probabilistic Trajectory Displacement Sets
  - 5 Randomized
  - 3 Wind-induced
- 15 to 45 Realizations
- Nonlinear increasing relationship



PAEM Analysis

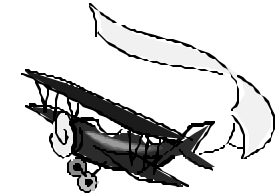


- Two-Dimensional Displacement Regions
  - Minimal conflicts generated with vertical displacements
  - FAA-imposed separation much greater than maximum vertical deviations (±400 ft)
  - Reduces number of realizations and computational effort

- Identified Intervals Versus Threshold Probability

	69 deterministic conflicts		559 deterministic conflicts	
	Test Set 1		Test Set 2	
p_1	n_A = 15	n_A = 21	n_A = 15	n_A = 21
0.50	20	20	178	157
0.45	27	22	264	230
0.40	45	44	373	369
0.35	59	60	495	527
1/3	68	64	583	600
0.30	78	80	728	726
0.25	103	115	991	974

PAEM Analysis

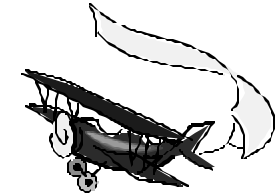


- Baseline Threshold Probabilities
  - Conservative
  - Identify a reasonable number of probabilistic conflicts
  - Comparable to probabilities observed for conflicts identified in previous deterministic analyses

$$p_{\text{thresh}} = \{p_1, p_2, p_{\text{fatal}}\} = \left\{ {}^{1}\!/_{3}, {}^{1}\!/_{6}, {}^{1}\!/_{18} \right\}$$

- Sensitivity Analysis: Vary p_1, p_2, and p_{fatal} proportionally
  - Perturbations to the structures of induced conflict subgraphs
  - Results demonstrate model insensitivity to moderate changes in threshold probabilities

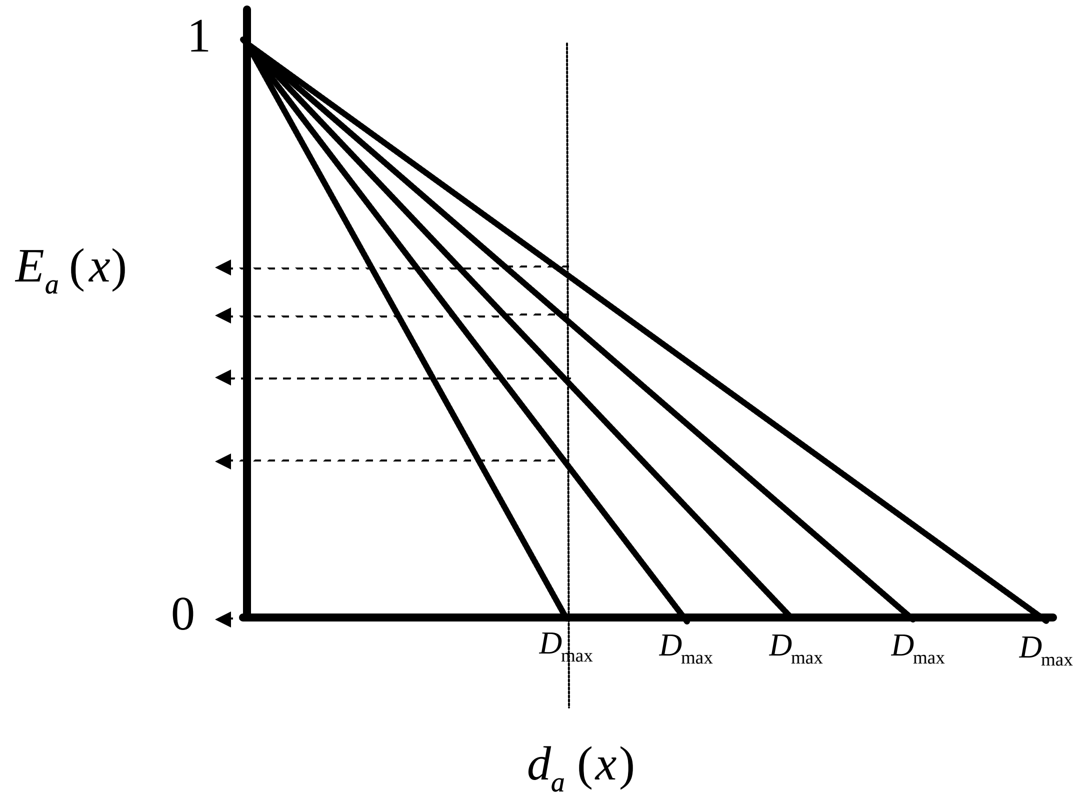
APCDM Parameterization



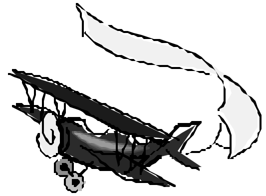
$D_{\max}$

- Recall: $d_a(x) \le D_{\max}$

- Collaboration Efficiency Curve



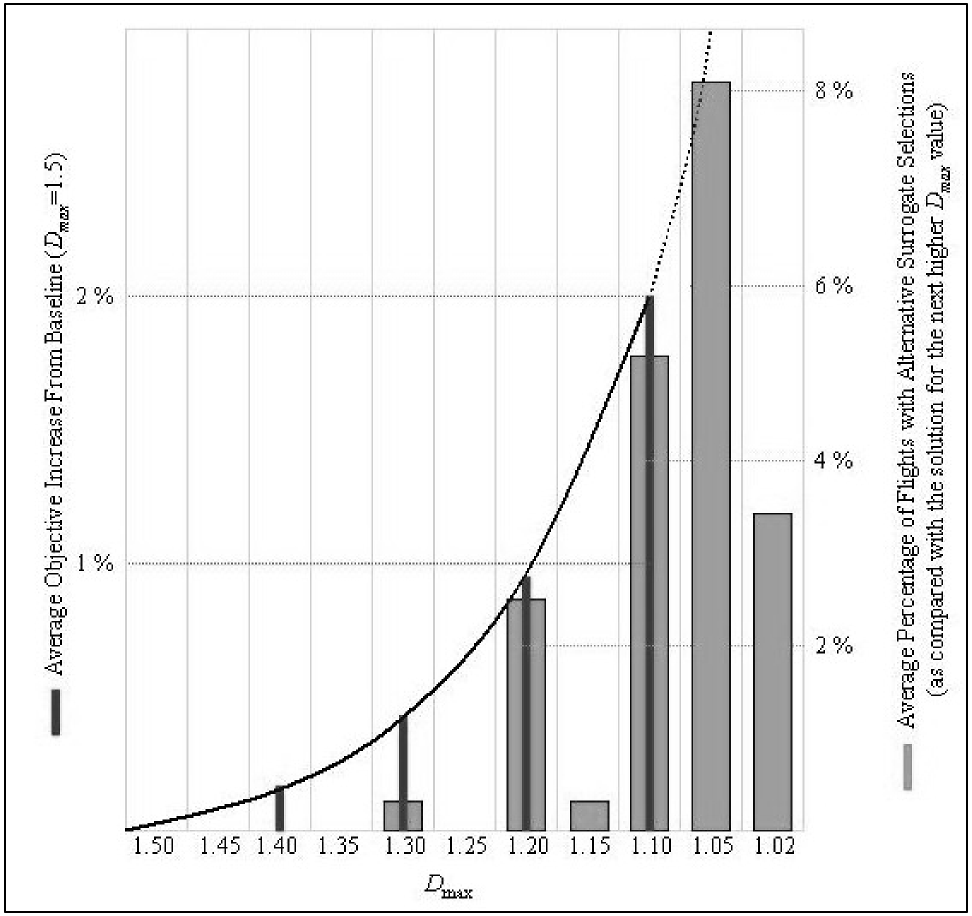
APCDM Parameterization



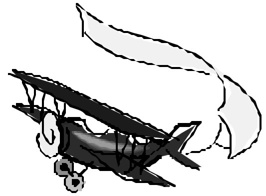
D_{max}

- Increasing Objective Value: Two Factors
  - Slope of Efficiency Curve
  - Surrogate Selections

- Parameter Influences Decision when $D_{max} \leq 1.20$



APCDM Parameterization

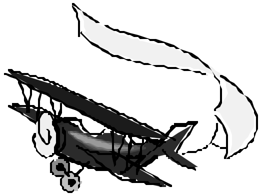


$E^e_{\max}$

- Four instances run with unconstrained airline collaboration equities

APCDM Instance	$\sum_{\alpha} \omega_{\alpha} E_{\alpha}(x)$	$\max_{a} \left\| E^e_a(x) \right\|$	x^e
CDM-1	0.9608	0.0187	0.0070
CDM-2	0.9474	0.0321	0.0087
CDM-3	0.5014	0.1867	0.0612
CDM-4	0.9928	0.0437	0.0024

APCDM Parameterization



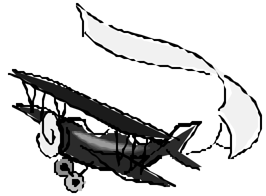
E^{e}_{max}

- CDM-3 Analysis

E^{e}_{max}	Unconstrained	0.15	0.10	0.08	0.06	0.04	0.02
$\sum_{\alpha} \omega_{\alpha} E_{\alpha}(x)$	0.5013	0.4989	0.4953	0.4955	0.4822	0.4818	0.4760
x^{e}	0.0612	0.0574	0.0523	0.0442	0.0337	0.0217	0.0042
% Objective Increase with Respect to Objective with Unconstrained E^{e}_{max}	0%	0.04%	0.09%	0.18%	0.32%	0.35%	0.41%

- More stringent equity requirements induce reduced collaboration efficiencies

APCDM Parameterization



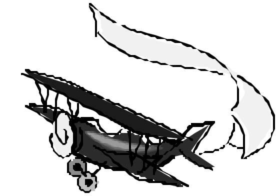
m_0

- Function of the CDM penalty terms in objective
  - Mathematical incentive for maximizing collaboration efficiencies and decision equity (i.e., mimimize “spread”)
  - Should not dominate solution

- Definition: “CDM Improvement”

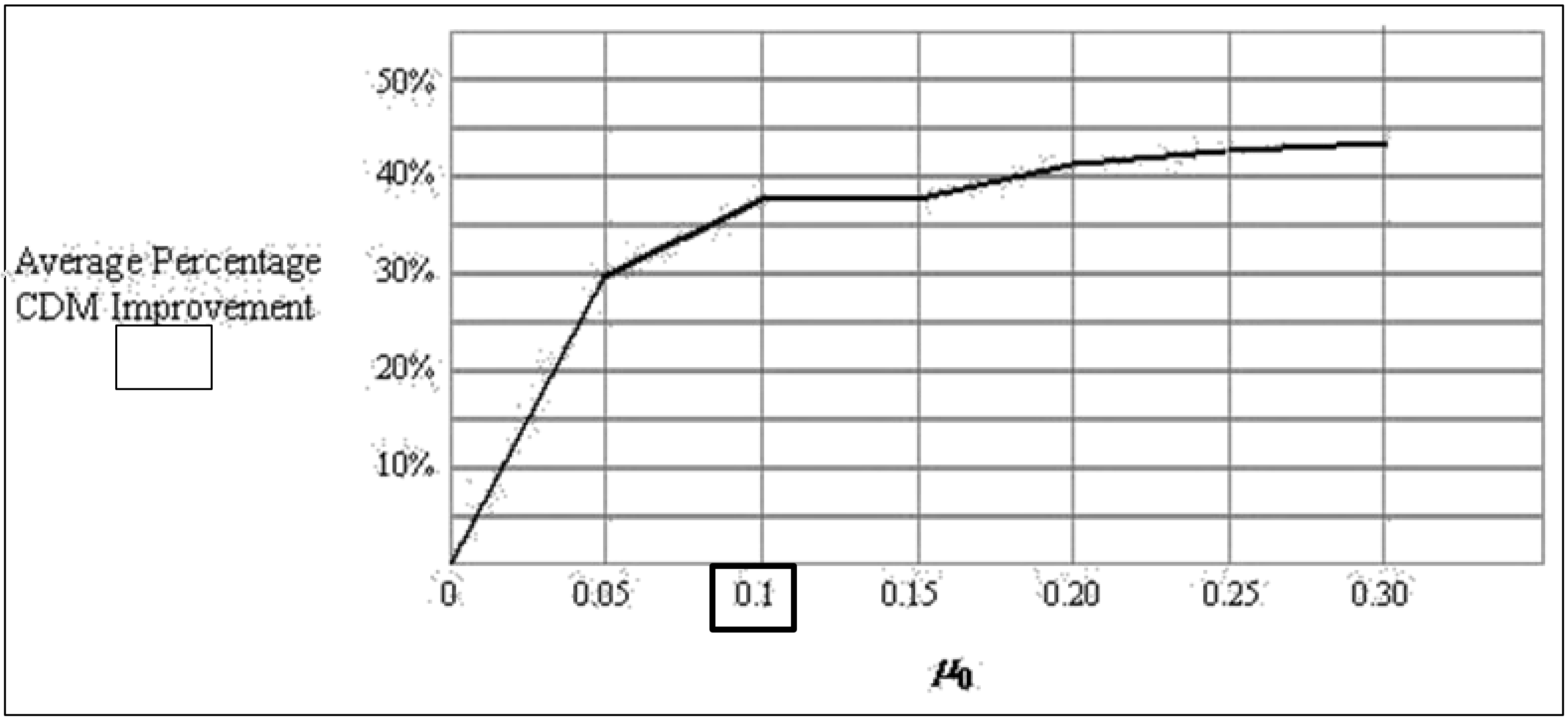
$$0.5\left[\left\{\sum_{a=1}^{\bar{a}} w_a E_a(x)\right\}_{m_0=\bar{m}_0} - \left\{\sum_{a=1}^{\bar{a}} w_a E_a(x)\right\}_{m_0=0}\right] + 0.5\left[\left\{x^e\right\}_{m_0=0} - \left\{x^e\right\}_{m_0=\bar{m}_0}\right]$$

APCDM Parameterization

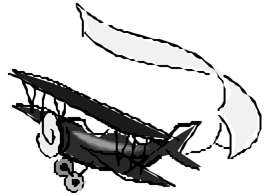


m_0

- Four APCDM instances tested using seven m_0 values



Conflict Constraint Formulation Analysis

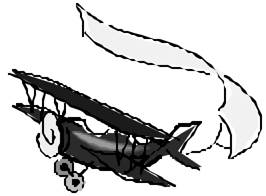


- Compactness of Representation: C_3 versus C_4

Label	Test Set *	Edges in A	Maximum Number of Overlapping Sets of Conflicts in any Sector	Maximum Number of Conflicts in any Overlapping Set	Number of C_3 Constraints Generated (Beyond C_2)	Number of C_4 Constraints Generated (Beyond C_2)
CFT-1,2	1	44	3	9	59	25
CFT-3	1	283	18	20	453	207
CFT-4,5	2	477	23	17	725	262
CFT-6	2	573	24	18	933	314
CFT-7	2	1130	57	45	2621	667
CFT-8	2	1407	68	56	3601	830
CFT-9	2	1448	70	42	3537	895
CFT-10	3	1458	37	140	6351	653
CFT-11	4	1215	74	70	3675	711
CFT-12	4	1436	72	65	4230	870

* All instances use randomized trajectory displacements, except CFT-7, which uses cylindrical SSW wind-induced displacements, and CFT-8 and CFT-12, which use N wind-induced displacements.

Conflict Constraint Formulation Analysis

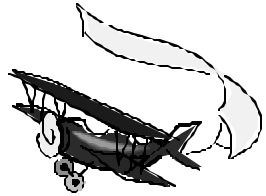


- CPLEX default cut generation disabled
- Analysis
- Recommend C_4 Formulation
- Specialized C_4 cuts superior to CPLEX general-purpose cuts w.r.t. APCDM

Label (1)	$\overline{n}_s$	C_2 Solution Time (seconds)	C_3 Solution Time (seconds)	C_4 Solution Time (seconds)
CFT-1	15	0.781	0.821	0.771
CFT-2 (4)	15	2.473	1.652	1.822
CFT-3	15	1.041	1.321	1.191
CFT-4	15	9.303	9.623	7.661
CFT-5 (4)	15	9.754	21.690	20.088
CFT-6	15	11.276	9.834	9.053
CFT-7	15	28.711	44.423	30.253
CFT-8	15	45.835	83.516	61.338
CFT-9	15	26.978	55.147	29.752
CFT-10 (4)	15	65.844	(2)	73.666
CFT-11 (4)	15	314.952	(2)	274.504
CFT-12 (4)	15	357.574	(2)	295.240
CFT-1a	3	0.570	0.540	0.600
CFT-2a (4)	3	8.422	9.613	9.105
CFT-3a	3	0.831	0.931	0.971
CFT-4a	3	53.286	51.774	66.856
CFT-5a (4)	3	65.774	74.457	69.420
CFT-6a	3	59.475	71.793	60.206
CFT-7a	3	71.012	83.570	53.056
CFT-8a	3	188.581	294.418	160.661
CFT-9a	3	195.571	402.200	230.690
CFT-10a (4)	7	723.670	(2)	772.506
CFT-11a (3)(4)	9	632.138	(2)	276.407
CFT-12a (3)(4)	9	301.493	(2)	207.888

(1) "a" label indicates an identical data set with sector capacity constrained as shown in the next column
(2) Resulting constraint matrix size exceeded the computer's memory capacity
(3) Solution obtained using an LP/IP gap tolerance of 5%
(4) The instances were examined using final prescribed parameter values

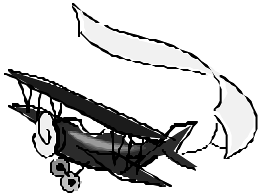
Research Contributions



The APCDM with the following characteristics:

- Probabilistic Conflict Analysis
  - Two alternative representations for trajectory errors
- Continuous time formulation for conflict risk intervals
- Two new classes of valid inequalities
- Flight plan cost model
- CDM Representation
  - Examines distribution of costs as well as maximum spread of costs
- Practical Applications

Research Directions



- Alternative Utility Theory based equity considerations
- Flight plan generation
- Dynamic Airspace Issues
  - Weather Systems
  - Space Launch SUAs
  - Dynamic Resectorization
- Strategic and tactical scenario tests

Q U E S T I O N S ?

Early experiments in transportation